Compton effect

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Contents

I. History of the quantization of matter

5th century B.C

17th century

Democritus

Pierre Gassendi

Robert Hooke

Isaac Newton

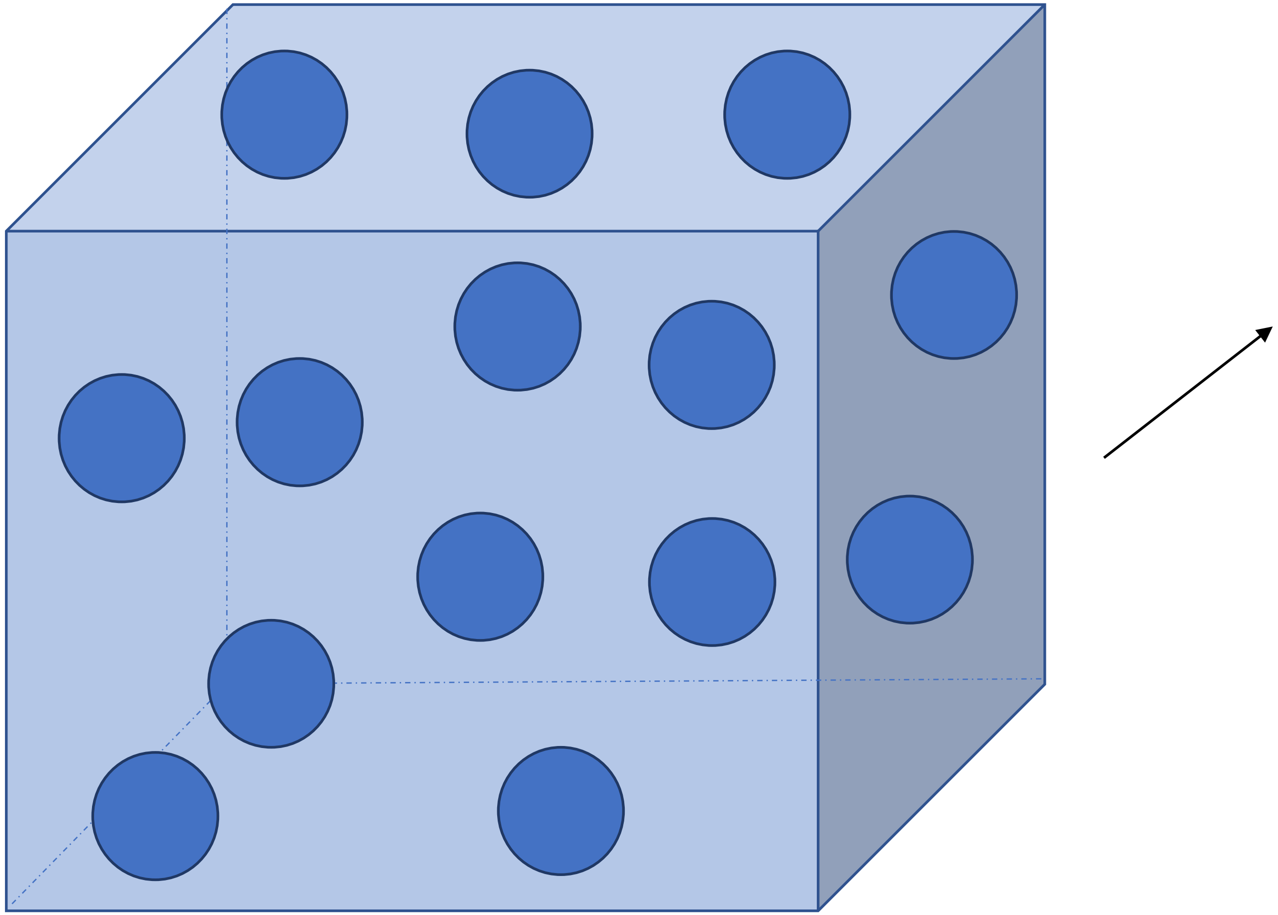
18th century

John Dalton

Amedeo Avogadro

Antoine Lavoisier

Dalton’s atomic model



All mater consists of
Tiny indivisible particles

II- The discovery of the electron

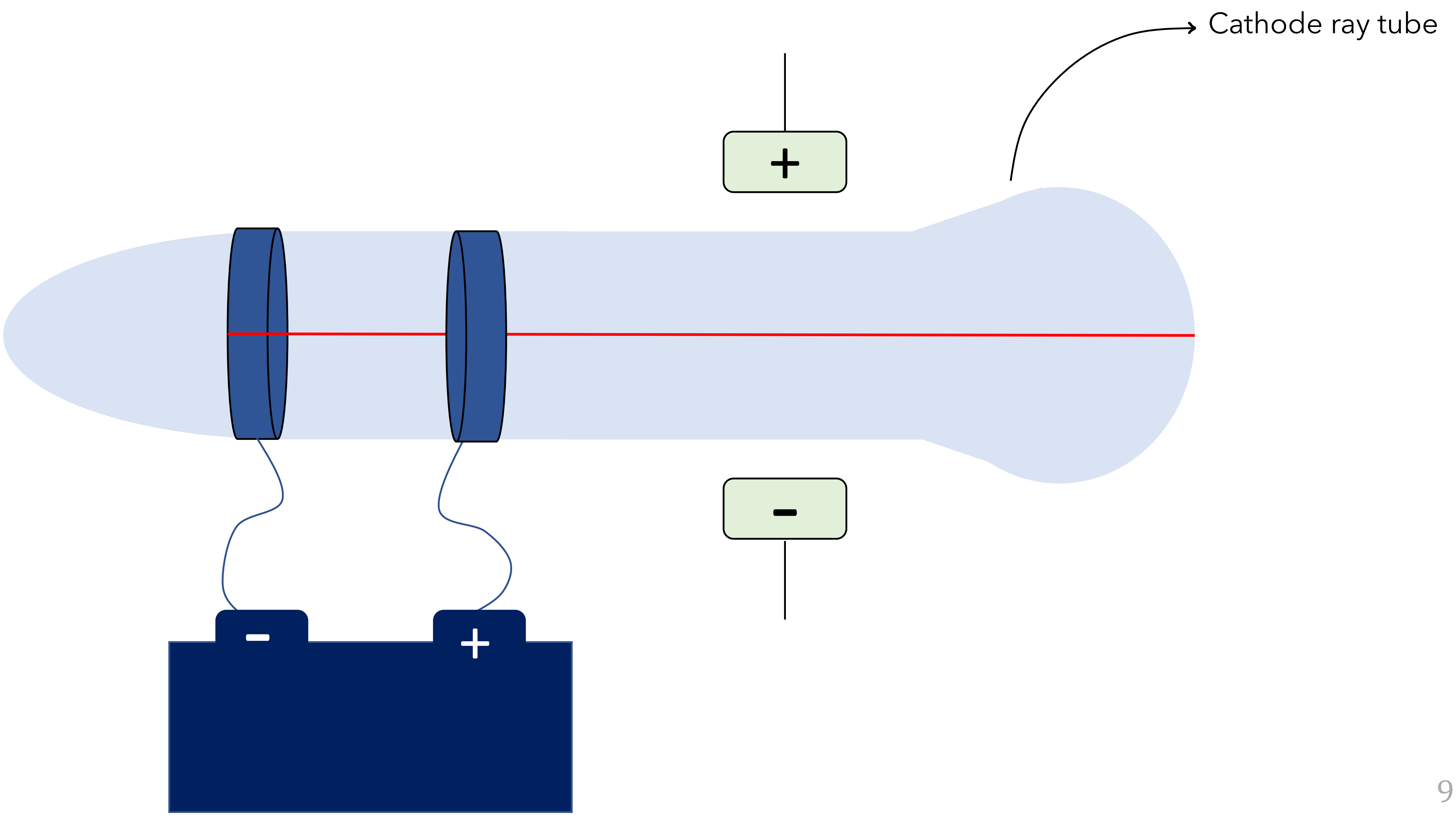
Joseph John Thomson's experiment

The cathode ray tube experiment:

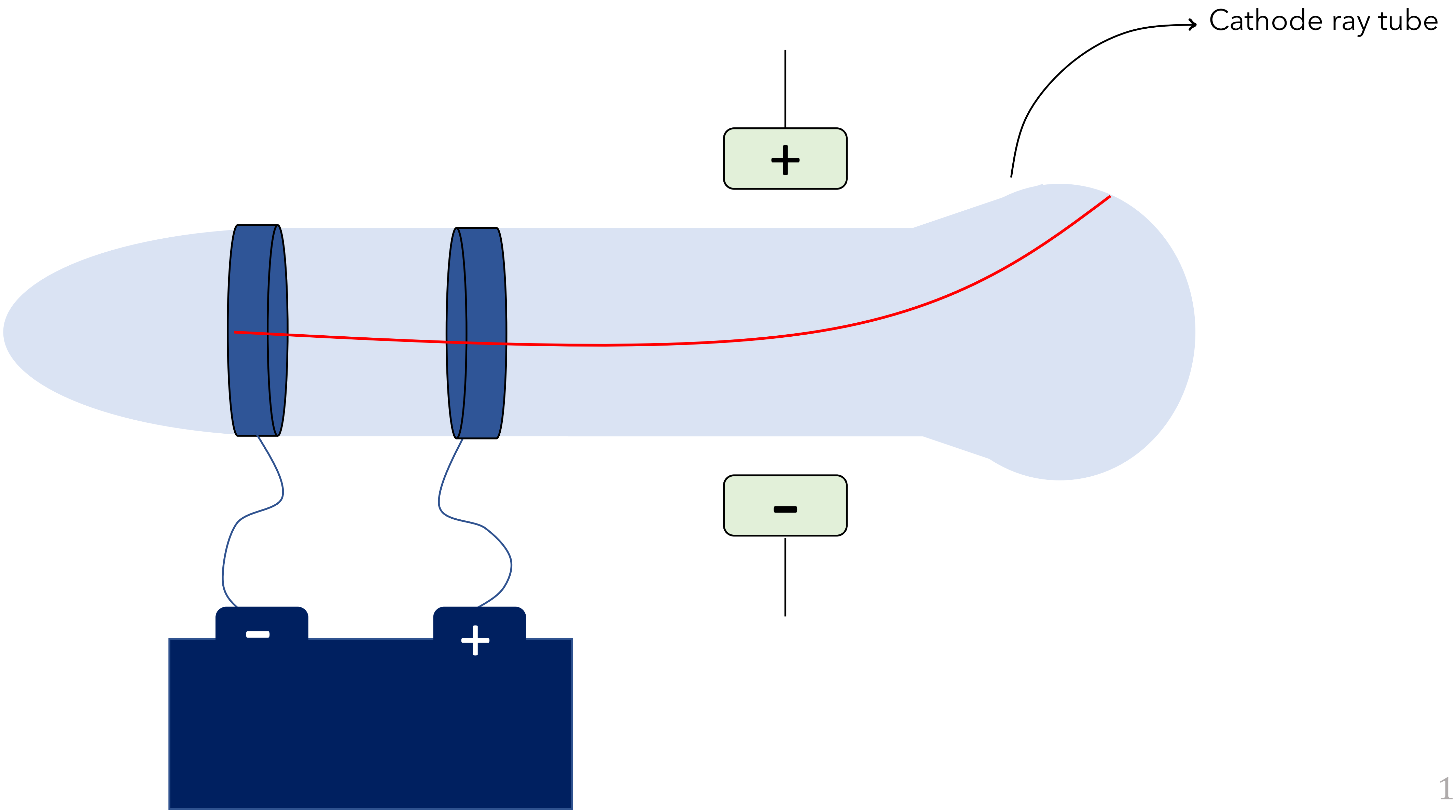
Discovery of the electron

Joseph John Thomson
1897

Experimental set-up of Thomson's experiment



Experimental set-up of Thomson's experiment



- Cathode rays must be made up of substance that is negatively charged
- Particles that make up cathode rays are 1000 times smaller than a Hydrogen atom
- All different metals give off cathode ray

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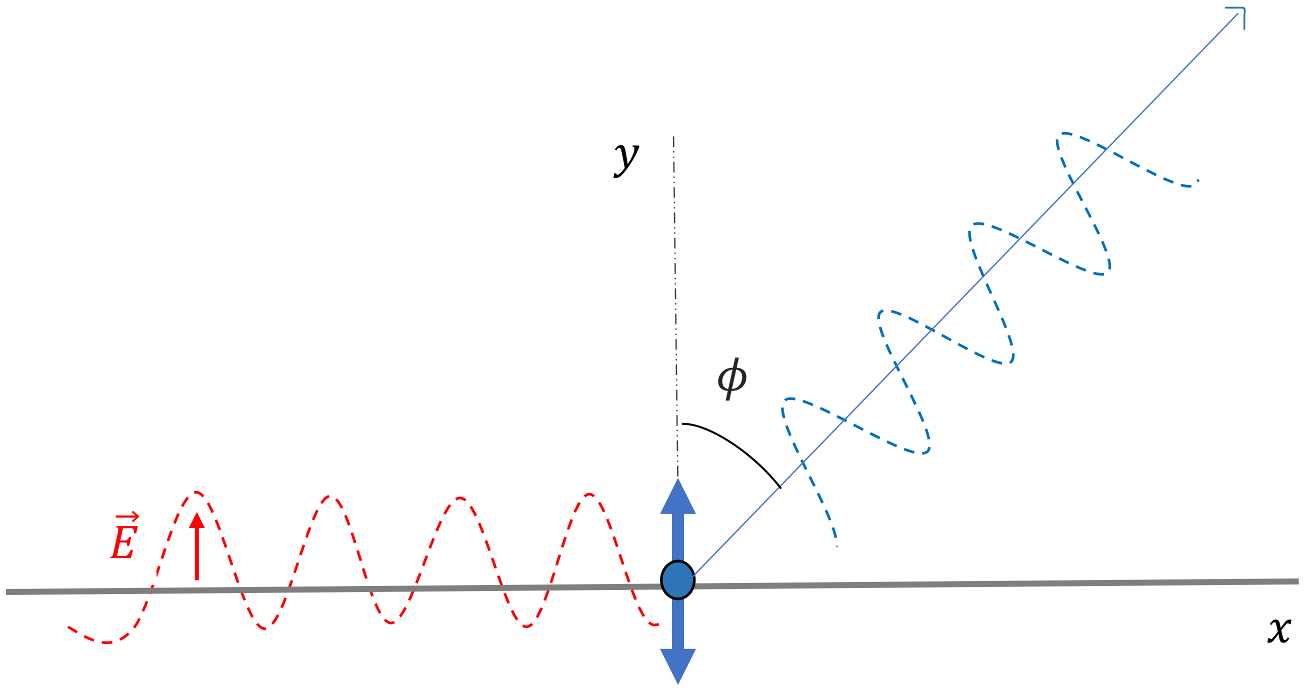
Atoms have tiny, negatively charged particles inside them:

Electrons

III- Scattering of electromagnetic radiation by electrons

Thomson's scattering **1904**

Thomson scattering is the elastic scattering of low energy electromagnetic radiation by a free charged particle



$$\vec{E}(\mathrm{r,t}) = \mathrm{e}\left(\frac{\vec{n}-\vec{\beta}}{\gamma^2\left(1-\vec{\beta}.\vec{n}\right)^3 r^2}\right) + \frac{\mathrm{e}}{\mathrm{c}}\left(\frac{\vec{n}\wedge\left[\left(\vec{n}-\vec{\beta}\right)\wedge\dot{\vec{\beta}}\right]}{\gamma^2\left(1-\vec{\beta}.\vec{n}\right)^3 r}\right)$$

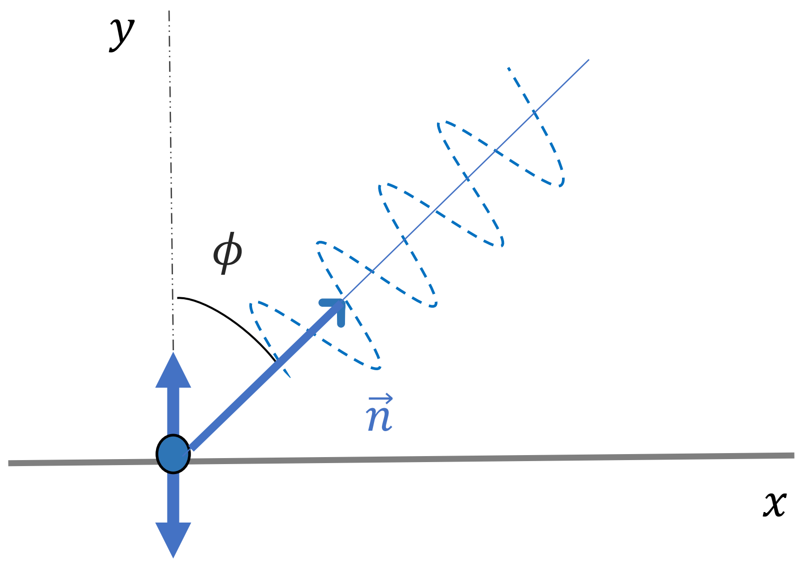
$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad \dot{\vec{\beta}} = \frac{\vec{a}}{c}$$

$$\sim \frac{1}{r^2} \ll 1 \qquad \vec{\beta} = \frac{\vec{v}}{c} \ll 1$$

$$\vec{B}(\mathrm{r,t}) = \frac{\vec{n}}{c}\wedge\vec{E}(r,t)$$

$$\vec{E}(\mathrm{r,t}) = \frac{e}{c}\frac{\vec{n}\wedge(\vec{\mathrm{n}}\wedge\dot{\vec{\beta}})}{r}$$

$$\vec{B}(\mathrm{r,t}) = \vec{n}\wedge\vec{E} = \frac{e}{c^2}\frac{(\vec{\mathrm{n}}\wedge\dot{\vec{\beta}})}{r}$$



- Equation of motion

$$m_e \ \vec{a} = e\,\vec{E} + \underbrace{e\,\vec{v} \wedge \vec{B}}_{\ll 1}$$

$$\Rightarrow \quad \vec{a} = \frac{e}{m_e}\vec{E}$$

$$\vec{E} = E_0 \sin(\omega_0 t)\,\vec{y}$$

$$\Rightarrow \quad a_y = \frac{e}{m_e} E_0 \sin(\omega_0 t)$$

Acceleration

$$\Rightarrow \quad y = \iint \frac{eE_0}{m_e}\sin(\omega_0 t) = \frac{eE_0}{\omega_0^2 m_e}\sin(\omega_0 t)$$

$$\Rightarrow \quad \vec{d} = e\,\vec{y} = \frac{e^2 E_0}{\omega_0^2 m_e}\sin(\omega_0 t)$$

Dipole moment

- Cross section

$$\sigma = \frac{R}{I}$$

σ = Cross section

R = Number of reactions per unit time per nucleus

I = Number of incident particles per unit time per unit area

$$\sigma = \frac{P}{< S >}$$

P = Power radiate by the charged particle

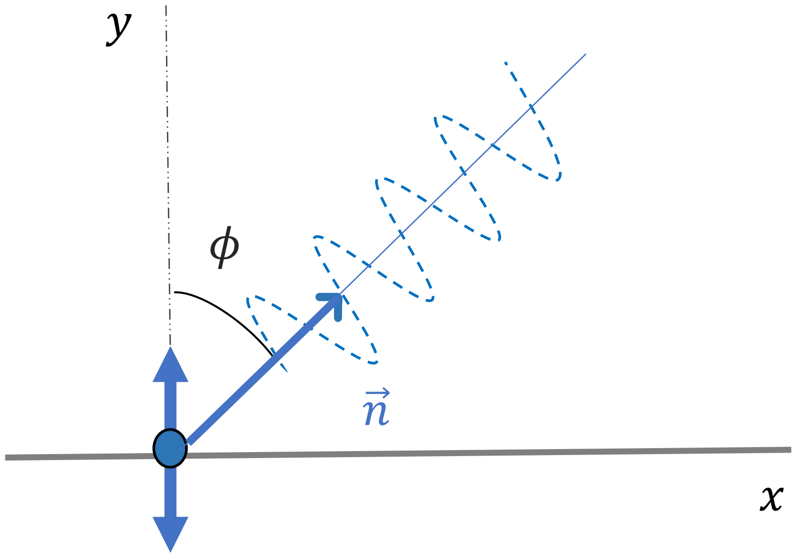
$< S >$= The incident flux (the time-average Poynting vector)

Differential cross section for Thomson's scattering

$$\frac{d\sigma}{d\Omega} = \frac{dP}{d\Omega}\frac{1}{< S >}$$

$$\Rightarrow \quad \frac{dP}{d\Omega} = \frac{dP}{dA_n}r^2 = r^2(\vec{n}.\vec{S})$$

$$\Rightarrow \quad \frac{dP}{d\Omega} = \frac{e^4\ E_0^2}{4m_0^2c^3}\sin^2\phi$$



$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_0^2c^4}\ \sin^2\phi$$

→ Thomson Cross section (independent from the frequency)

IV- Photon-electron scattering

Compton scattering

- Compton scattering, is the <u>decrease in energy</u> (increase in wavelength) of an X-ray or gamma ray photon, when it interacts with matter. It is the high frequency limit of Thomson scattering.

- Thomson scattering, the classical theory of charged particles scattered by an electromagnetic wave, cannot explain any shift in wavelength.

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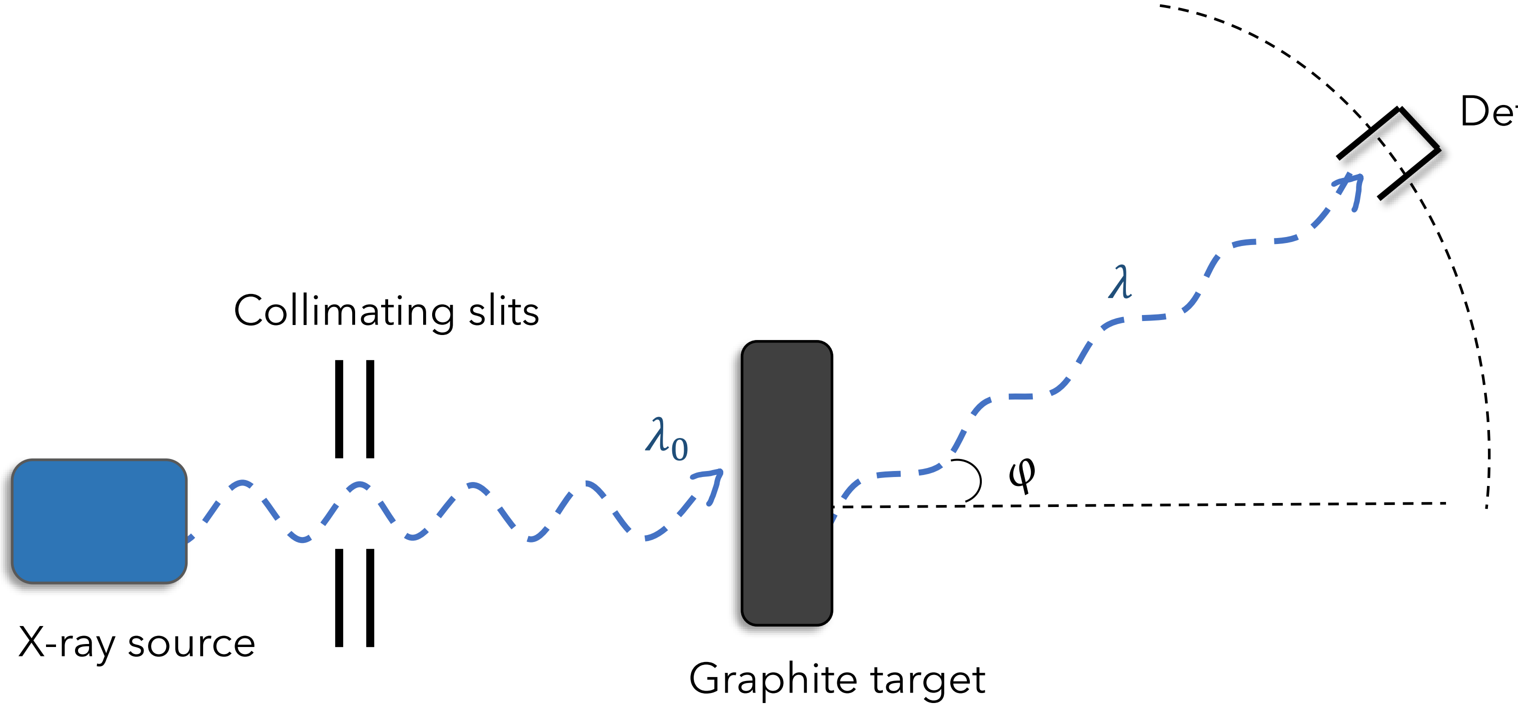
Compton embarked upon a more radical explanation based on the (at the time) new quantum theory.

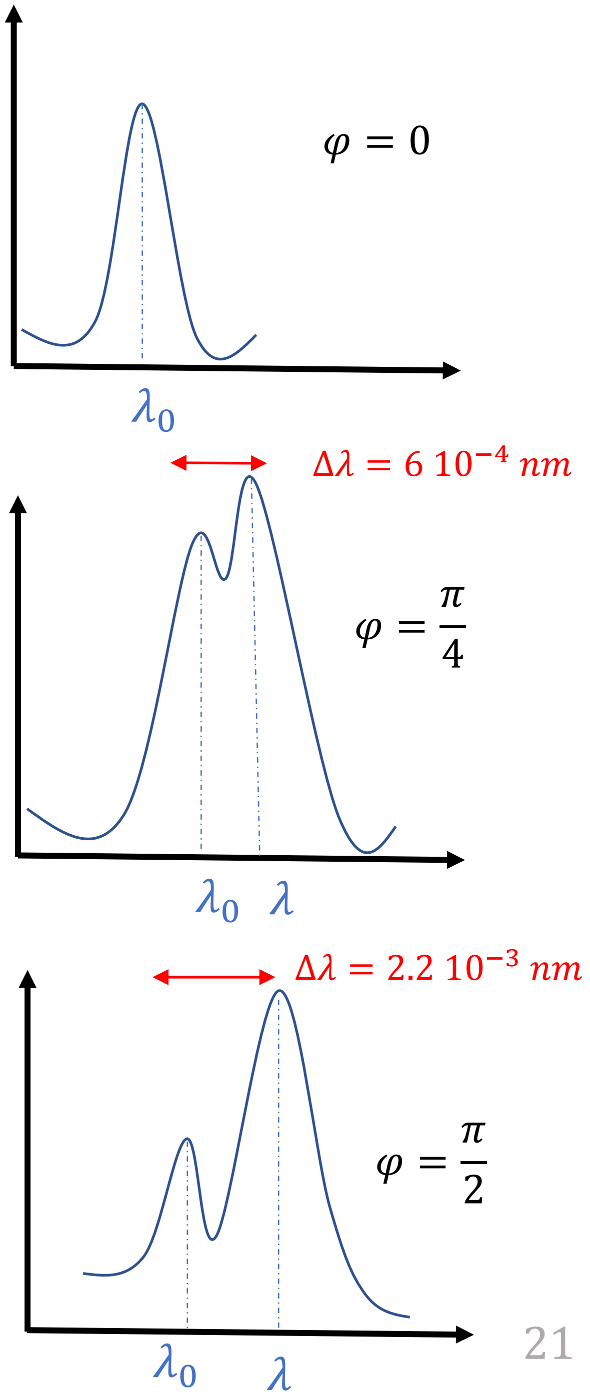
Arthur Compton
1923

Experimental set-up of Compton scattering **1923**



Compton found that:

- One component had a wavelength λ_0 equal to that of the incident radiation and second component had a wavelength $\lambda = \lambda_0 + \Delta\lambda$.
- $\Delta\lambda$ varies with the scattering angle
- $\Delta\lambda$ increases rapidly at large scattering angles
- $\Delta\lambda$ is independent of the incident wavelength λ_0
- $\Delta\lambda$ is independent of the scattering material



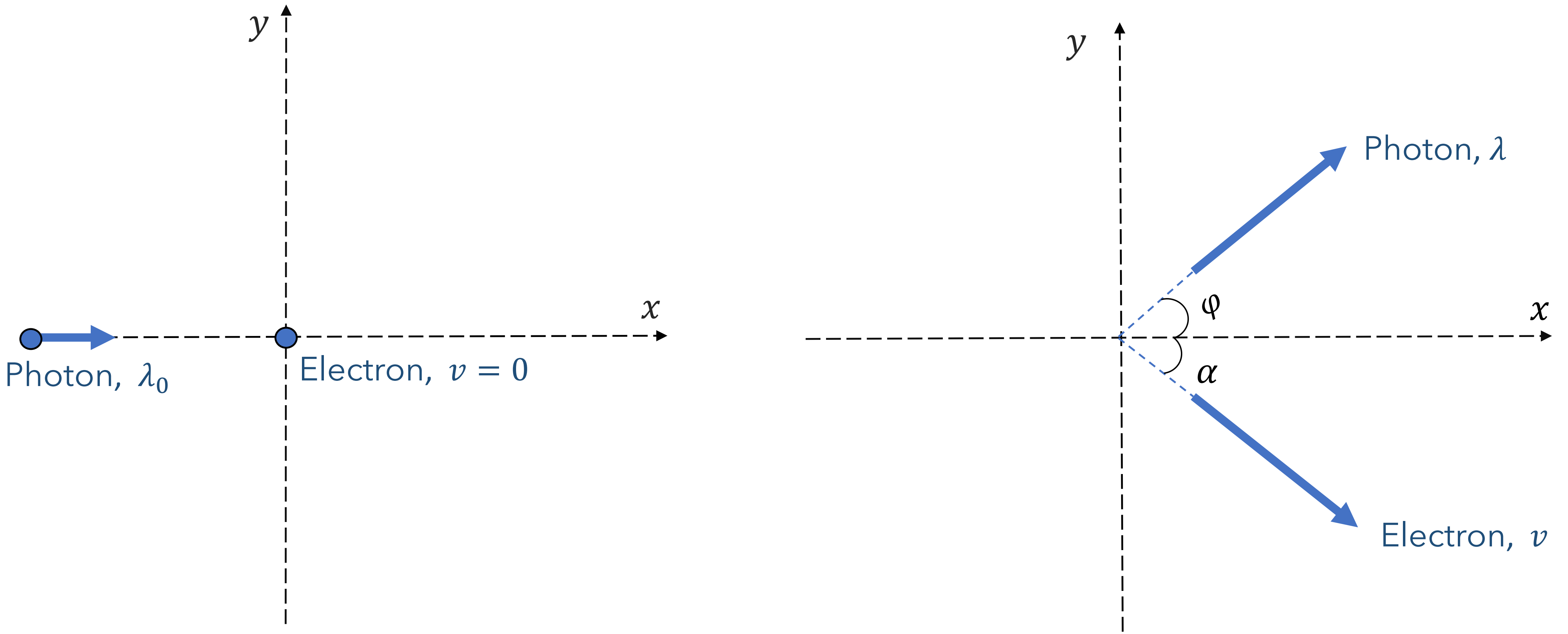
Compton explained and modeled the data by :

1) Assuming that the electron is free or loosely bound to the valence shell of atoms.

2) Assuming a particle (photon) nature for light.

3) Applying conservation of energy and conservation of momentum to the collision between the photon and the electron.

Derivation of the Compton scattering calculations



Pre-collision

Post-collision

Conservation of energy

$$h\,\nu_0 + m_0c^2 = h\nu + mc^2$$

$c = \nu\lambda$

1 $$\frac{h}{\lambda_0} - \frac{h}{\lambda} + m_0c = mc$$

Conservation of momentum

2

x −component $$\frac{h}{\lambda_0} = \frac{h}{\lambda}cos\varphi + mv\,cos\alpha$$

y −component $$0 = \frac{h}{\lambda}sin\varphi - mv\,sin\alpha$$

$$\textcircled{1} \Rightarrow \frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda_0 \lambda} - 2m_0 c \left(\frac{h}{\lambda_0} - \frac{h}{\lambda}\right) + m_0^2 c^2 = m^2 c^2 \quad \textcircled{3}$$

$$\textcircled{2} \Rightarrow \frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda^2} - \frac{2h^2}{\lambda \lambda_0} cos\varphi = m^2 v^2 \quad \textcircled{4}$$

Compton wavelength

$$\textcircled{3} - \textcircled{4} \Rightarrow \Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_0 c}(1 - \cos\varphi)$$

Compton shift

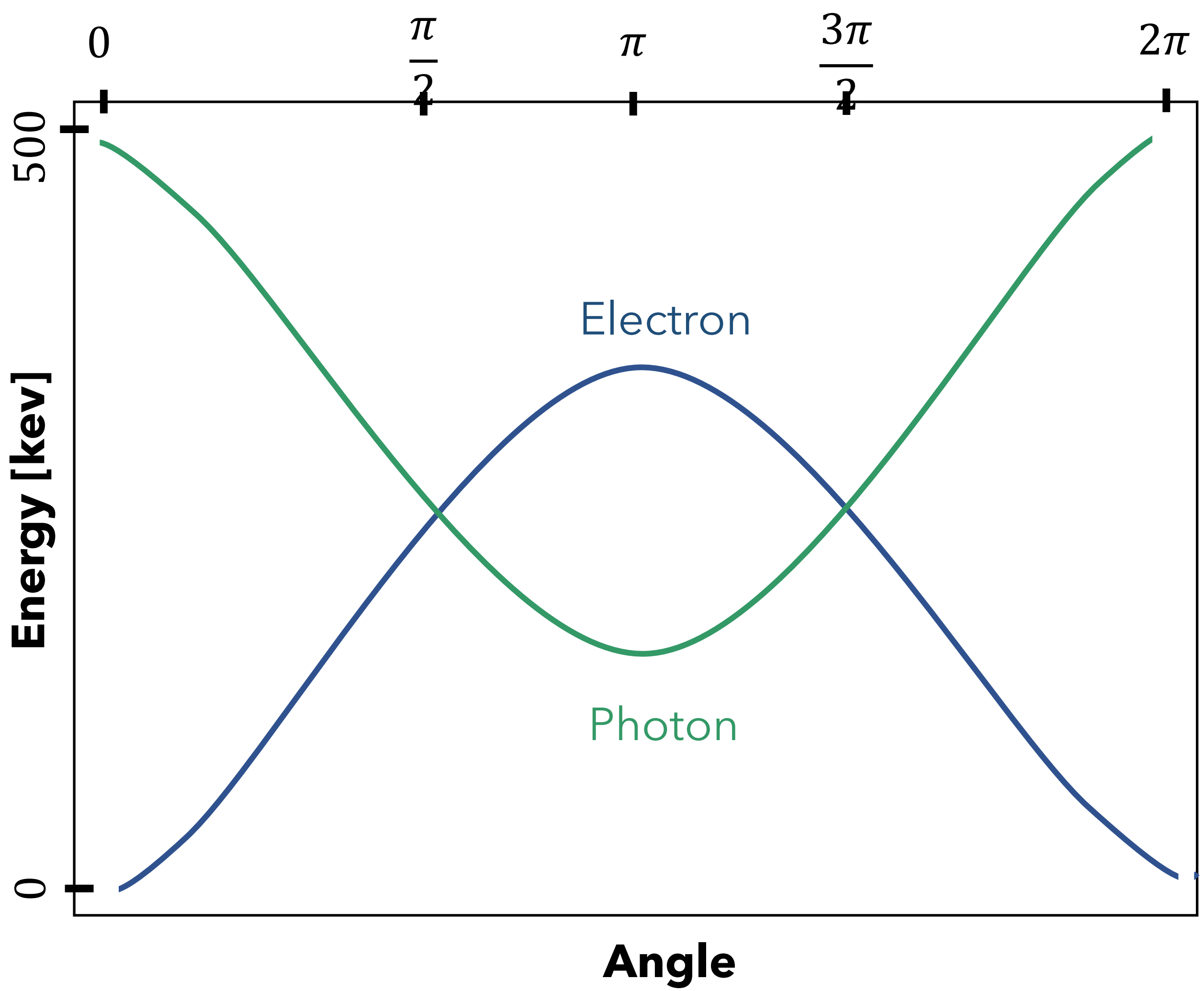
$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_0 c}\ (1 - \cos\varphi)$$

$$\lambda = \frac{hc}{E}$$

$$\Rightarrow \quad E_f = \frac{E_i}{1 + \frac{E_i\,(1-\cos\varphi)}{m_e c^2}}$$

Final energy of the scattered photon is a function of:

- Initial energy of the photon
- Scattering angle



Energies of a photon at 500 keV and an electron after Compton scattering.

Klein-Nishina cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{e^4}{m_0^2 c^4}\right)^2 \left(\frac{\lambda_0}{\lambda}\right)^2 \left(\left(\frac{\lambda_0}{\lambda}\right) + \left(\frac{\lambda}{\lambda_0}\right) - 2\sin^2\theta\cos^2\phi \right)$$

Low energy limit $\lambda_0 \approx \lambda$

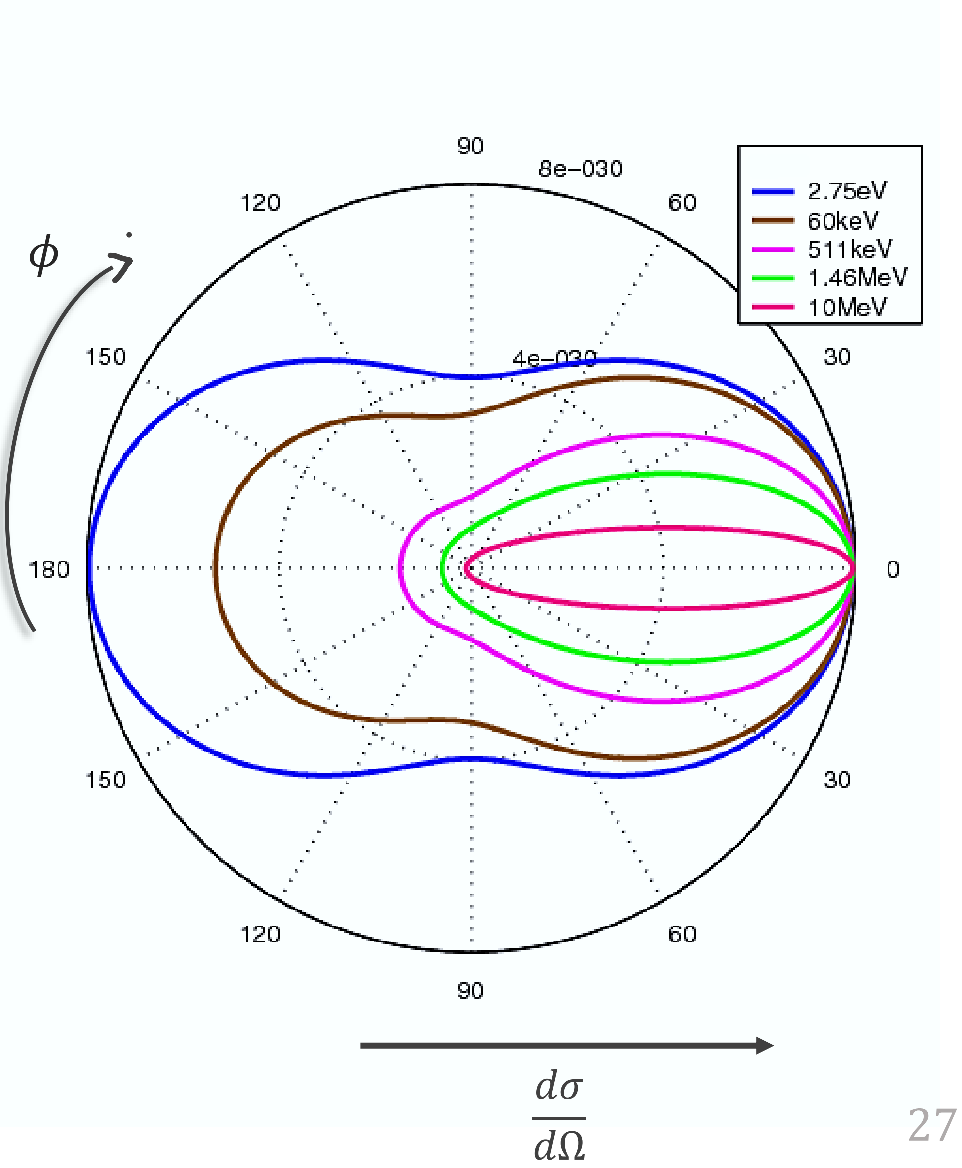
$$\frac{d\sigma}{d\Omega} = \left(\frac{e^4}{m_0^2 c^4}\right)^2 \sin^2\phi$$

Thomson differential cross section

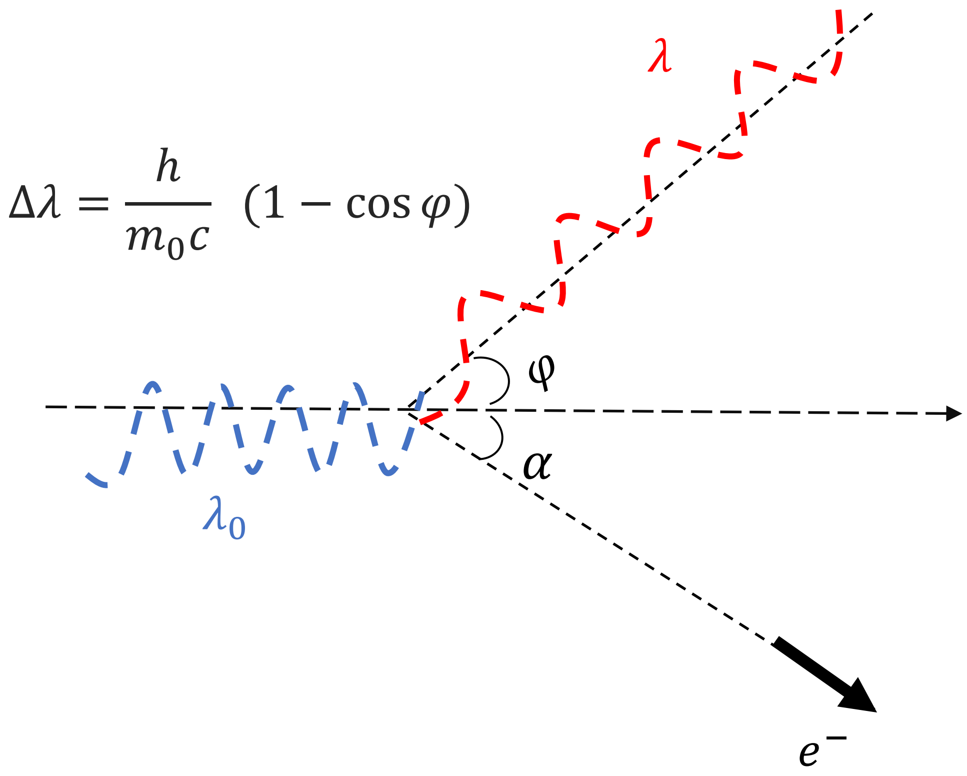
Oskar Klein

Yoshio Nishina

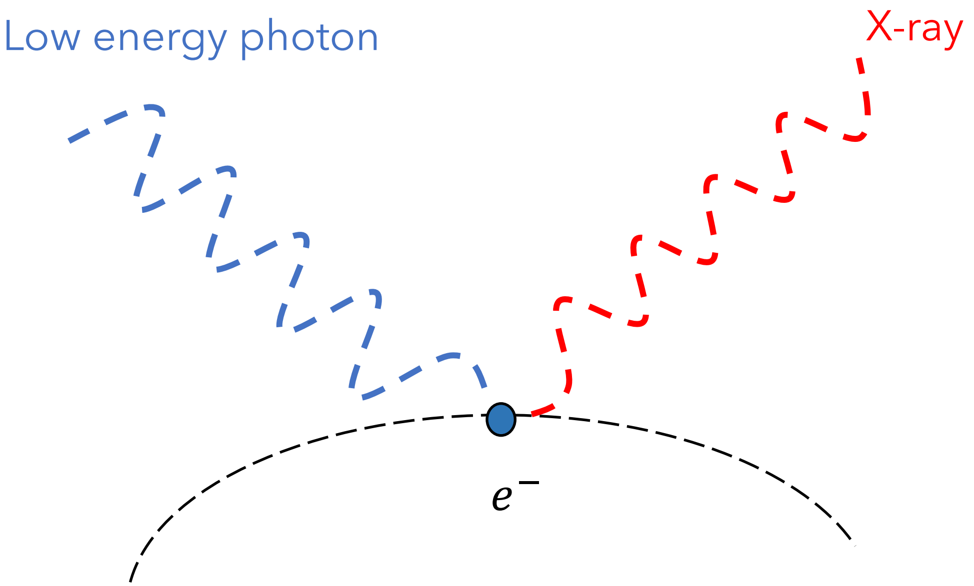
1929



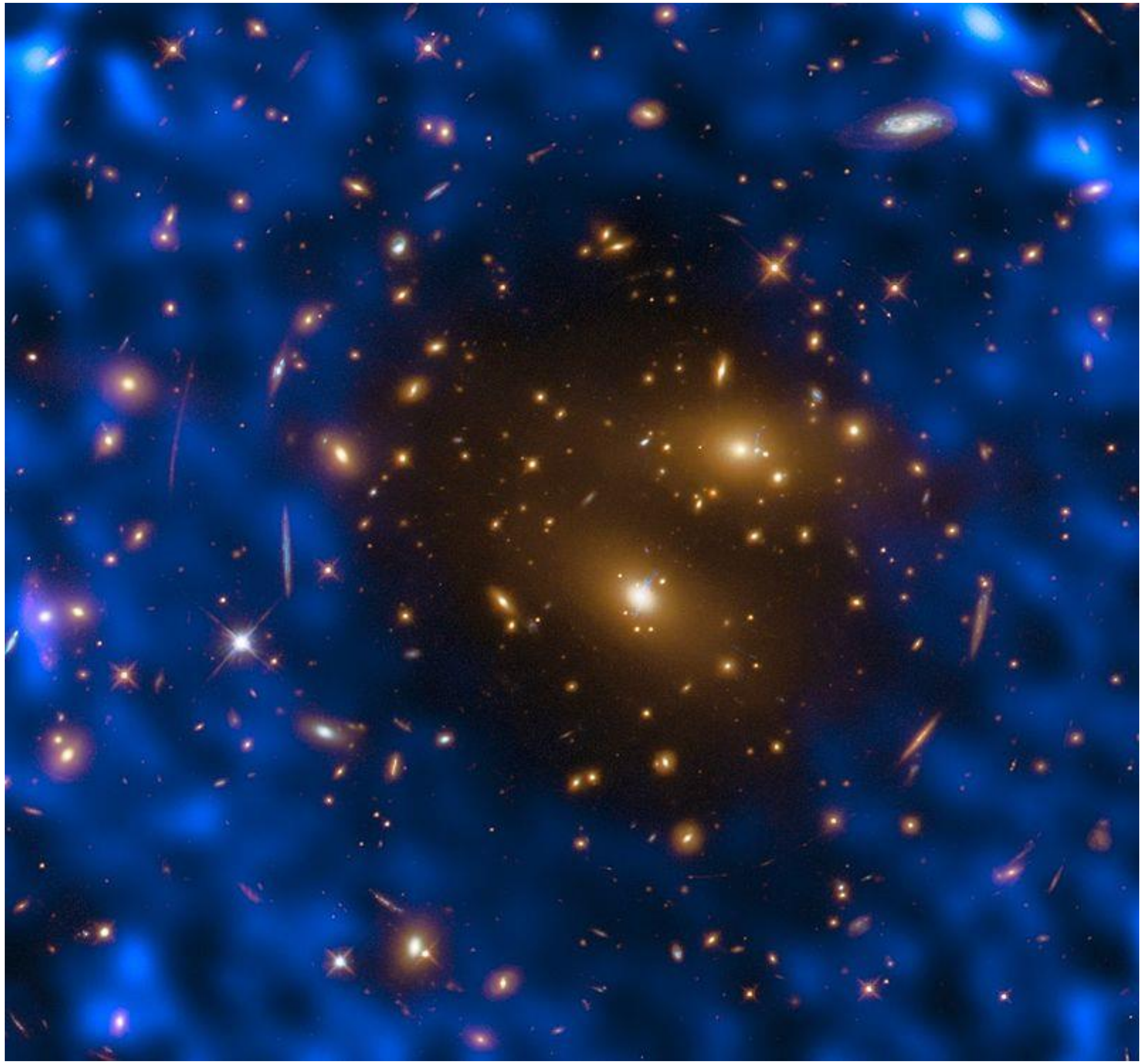
Application: Sunyaev–Zeldovich effect



Compton Scattering



Inverse Compton Scattering



Sunyaev–Zeldovich effect of the most massive galaxy clusters known, RX J1347.5-1145.

V- Conclusions

- When X-rays are scattered off some material, they have different wavelength from the wavelength of the incident X-rays, and this is classically inexplicable.

- Compton used the quantum nature of light to explain the phenomenon.

- Compton's explanation of this effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon. He earned the Nobel price for his discovery.

- The cross section of the light-electron scattering decreases with the increase of the light energy.

Thank you.