Photonic crystals

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I- A general overview on photonic crystals

1. What are photonic crystals ?

Photonic

crystals

Light

Periodic structures

Periodic structures that interacts with light

Why would one be interested in the interaction of light with a periodic structure?

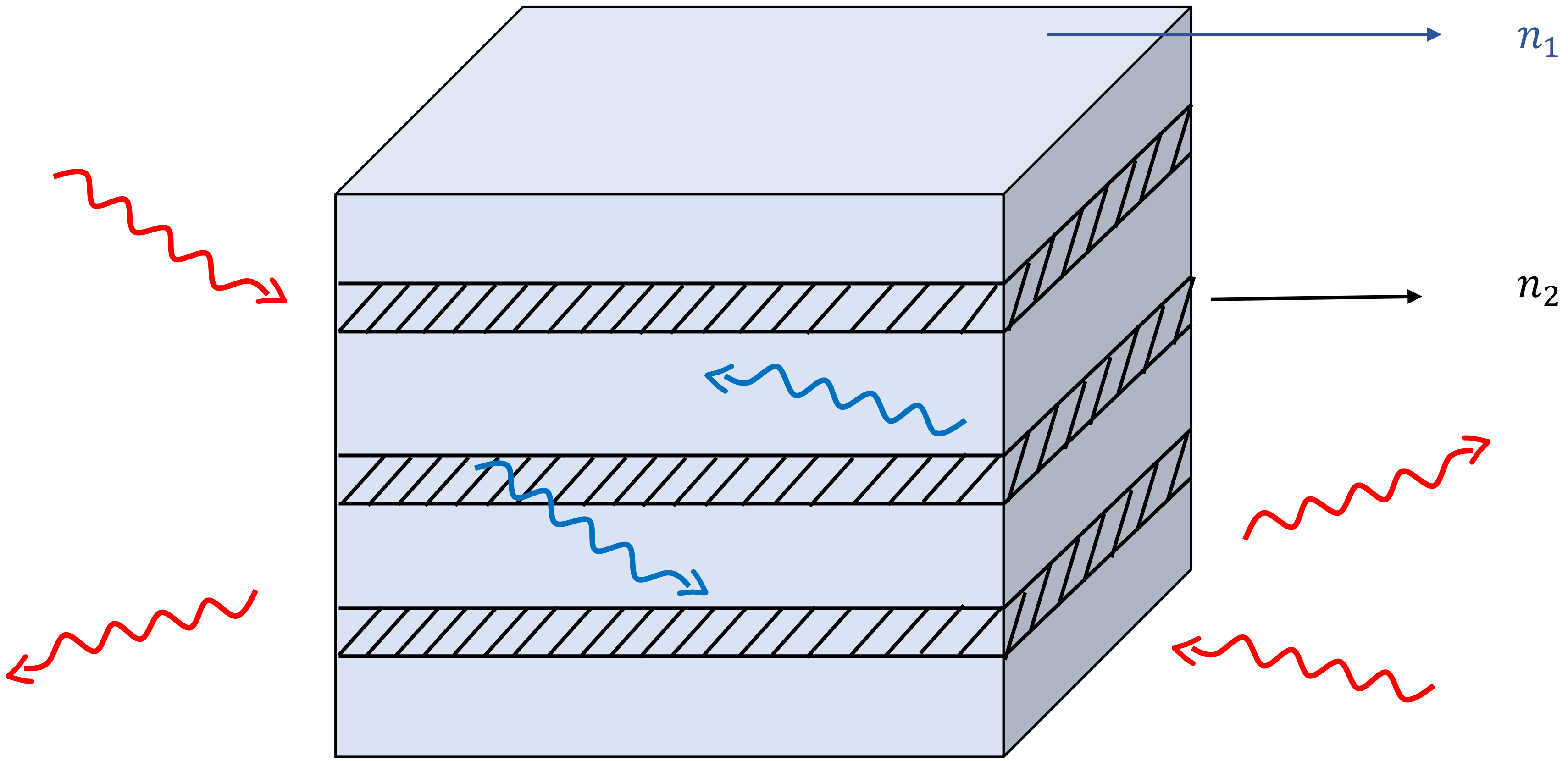
→ Because it gives complete control over light propagation

What are the advantages over other technologies that also control light propagation like the usual waveguides made up off metals?

→ Because photonic crystals control and guide light with lower loss then other technologies that are made up off metals.

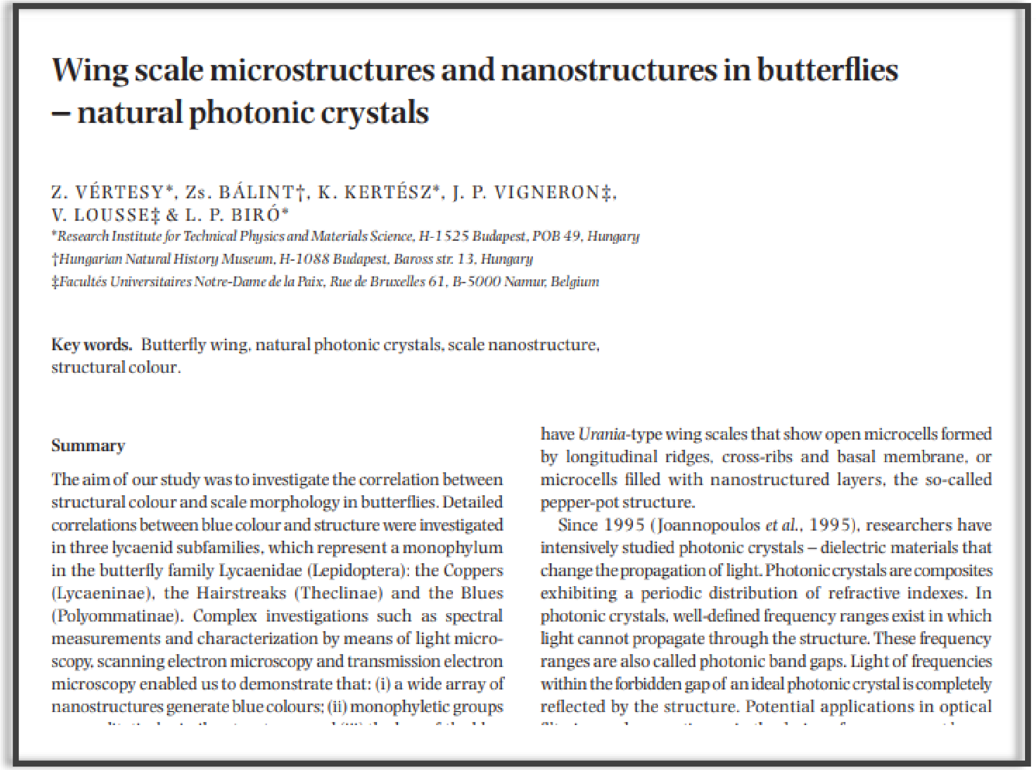
From where does the periodicity come from ?

→ From the construction of a media with periodic dielectric functions of the order of the wavelength of light being localized



2. Examples of photonic crystals in nature

Biologically, it is believed that species have evolved color through photonic crystals as a means of thermal regulation and signaling



Wing scale microstructures and nanostructures in butterflies – natural photonic crystals

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Key words. Butterfly wing, natural photonic crystals, scale nanostructure, structural colour.

Summary

The aim of our study was to investigate the correlation between structural colour and scale morphology in butterflies. Detailed correlations between blue colour and structure were investigated in three lycaenid subfamilies, which represent a monophylum in the butterfly family Lycaenidae (Lepidoptera): the Coppers (Lycaeninae), the Hairstreaks (Theclinae) and the Blues (Polyommatinae). Complex investigations such as spectral measurements and characterization by means of light microscopy, scanning electron microscopy and transmission electron microscopy enabled us to demonstrate that: (i) a wide array of nanostructures generate blue colours; (ii) monophyletic groups

have *Urania*-type wing scales that show open microcells formed by longitudinal ridges, cross-ribs and basal membrane, or microcells filled with nanostructured layers, the so-called pepper-pot structure.

Since 1995 (Joannopoulos *et al.*, 1995), researchers have intensively studied photonic crystals – dielectric materials that change the propagation of light. Photonic crystals are composites exhibiting a periodic distribution of refractive indexes. In photonic crystals, well-defined frequency ranges exist in which light cannot propagate through the structure. These frequency ranges are also called photonic band gaps. Light of frequencies within the forbidden gap of an ideal photonic crystal is completely reflected by the structure. Potential applications in optical

Beetle

Peacock

Comb-jellyfish

Morpho butterfly

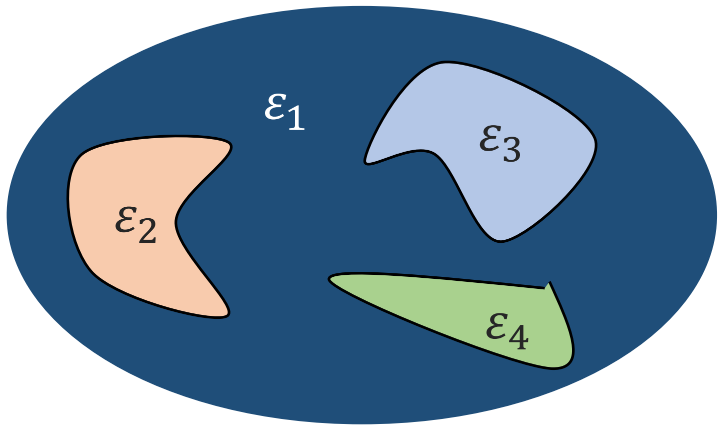
II- Electromagnetism in dielectric media

1. Maxwell equations in dielectric media

Maxwell equations for a linear and lossless materials composed of regions of homogeneous dielectric media are

$$\nabla . H(r,t) = 0 \qquad \nabla \times E(r,t) + \mu_0 \frac{\partial H(r,t)}{\partial t} = 0$$

$$\nabla . [\, \varepsilon(r) E(r,t) \,] = 0 \qquad \nabla \times H(r,t) - \varepsilon_0 \varepsilon(r) \frac{\partial E(r,t)}{\partial t} = 0$$



Maxwell equations are linear so we can separate the time dependence from the spatial dependence by expanding the fields into a set of harmonic modes

$$H(r,t) = H(r) e^{-i\omega t}$$

$$E(r,t) = E(r) e^{-i\omega t}$$

Maxwell equations describing the mode profiles for a given frequency in a linear and lossless material

Divergence equation

$$\boxed{\nabla . H(r) = 0}$$

$$\nabla . [\, \varepsilon(r) E(r)\,] = 0$$

$$\nabla \times E(r) - i\omega\mu_0 H(r) = 0$$

$$\nabla \times H(r) + i\omega\varepsilon_0 \varepsilon(r)\, H(r) = 0$$

$$\Rightarrow \quad \boxed{\nabla \times \left(\frac{1}{\varepsilon(r)} \nabla \times H(r) \right) = \left(\frac{\omega}{c}\right)^2 H(r)}$$

Master equation

2. Electromagnetism as an eigenvalue problem

The master equation can be seen as an eigenvalue problem

$$\Theta\, \mathrm{H(r)} = \left(\frac{\omega}{\mathrm{c}}\right)^2 H(r)\ ,$$

where $\Theta = \nabla \times \left(\frac{1}{\varepsilon(r)}\, \nabla \times H(r)\right)$ is a Hermitian operator.

The inner product is defined as follows

$$(A, B) \coloneqq \int d^3r A^*(r). B(r)$$

3. The energy functional

The energy functional is the total energy of the system as a functional of its states

$$\Theta\,\mathrm{H(r)} = \left(\frac{\omega}{c}\right)^2 H(r) \qquad \Rightarrow \qquad \boxed{U(H) = \frac{(H,\Theta H)}{(H,H)}}$$

Expressed in term of the electric field gives, it reads

$$\boxed{U(H) = \frac{\int d^3r\,|\nabla \times E(r)|^2}{\int d^3r\,\varepsilon(r)[E(r)|^2}}$$

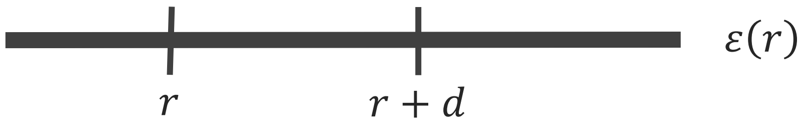
$\downarrow$

The lowest frequency mode is localized in the medium with the highest dielectric constant.

III- Light guidance in dielectric media

1. Continuous translational symmetry

For a translation d we can define a translation operator T_d. If the system is translationally invariant, then



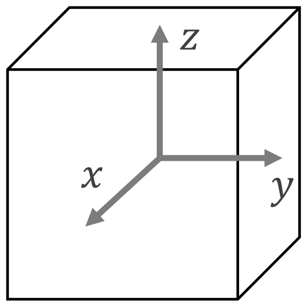
$$T_d \varepsilon(r) = \varepsilon(r - d) = \varepsilon(r)$$

$$[\, T_d, \Theta] = 0$$

- Translational symmetry in the three directions

$\varepsilon(\vec{r})$ is a constant

$$T_{\vec{d}}e^{i\,\vec{k}\,\vec{r}} = e^{i[\,k_x(x-d_x)+k_y(y-d_y)+k_z(z-d_z)\,]} = \left(e^{i\vec{k}\,\vec{d}}\right)e^{i\vec{k}\,\vec{r}}$$

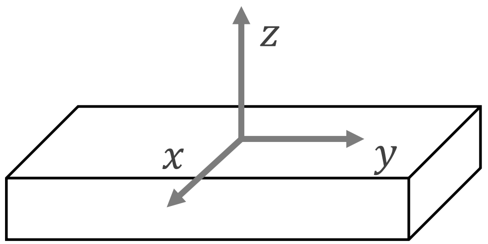


$$\Rightarrow \qquad H_k(\vec{r}) = H_0 e^{i\vec{k}\,\vec{r}}$$

$$\Rightarrow \qquad \boxed{\omega = \frac{c|\vec{k}|}{\sqrt{\varepsilon}}}$$

Dispersion relation

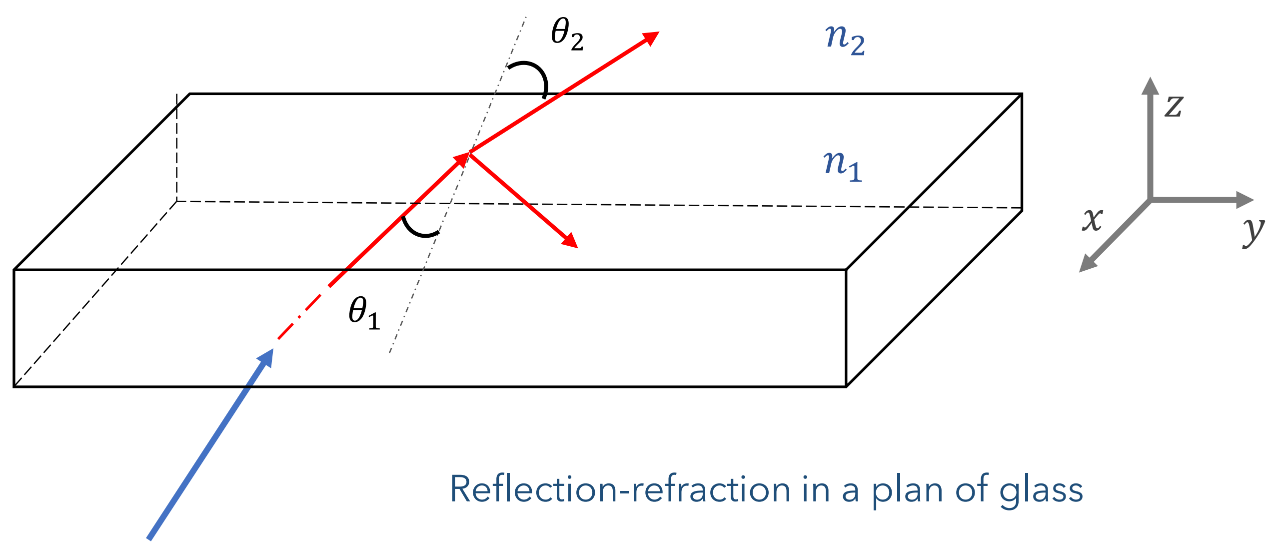
- Translational symmetry in an infinite plane



$$\varepsilon(\vec{r}) = \varepsilon(z)$$

$$\Rightarrow \qquad H_k(\vec{r}) = h(z)e^{i\vec{k}_{||}\vec{r}}$$

- Snell's law



$n_2 < n_1 \quad n_i = \sqrt{\varepsilon_i}$

Reflection-refraction in a plan of glass

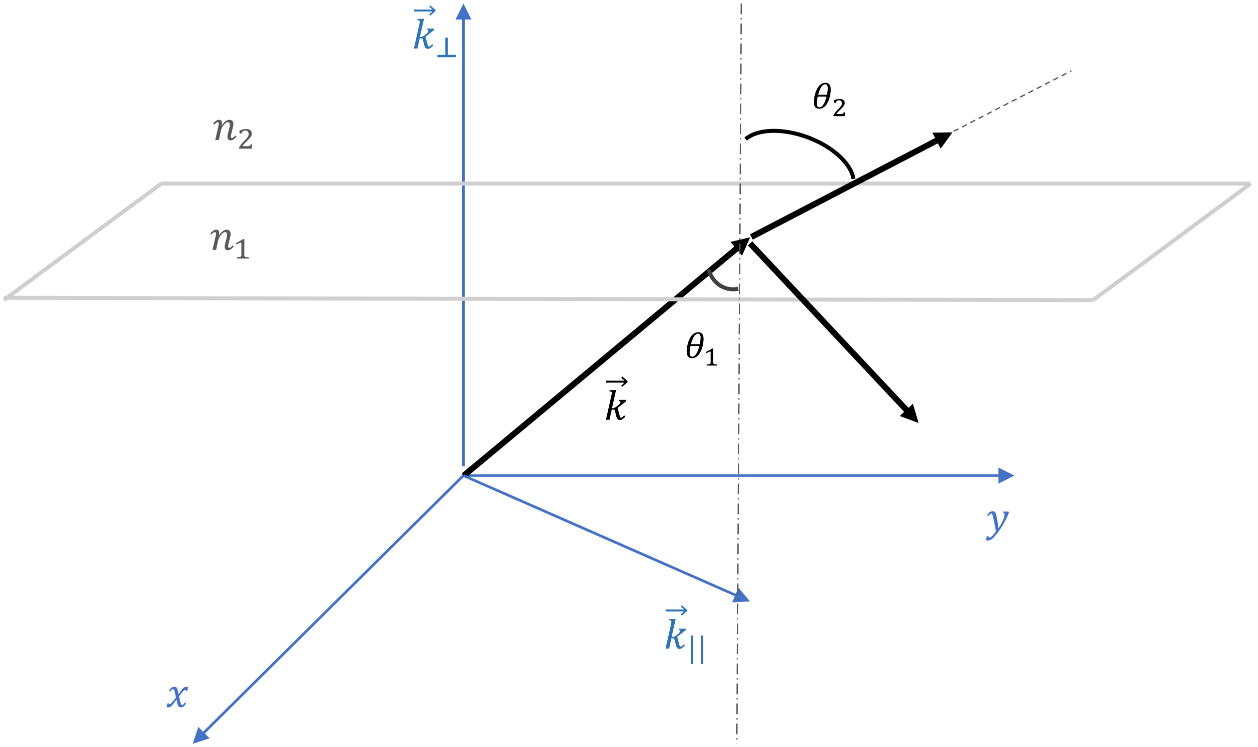
Snell's law of refraction: $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$

The critical angle for which a total internal reflection happens is given by setting $\theta_2 = \frac{\pi}{2}$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$\exists$ only for $n_2 < n_1$

Symmetries and conservations law for the plan of glass



Far away from the glass

$$\omega = c|\vec{k}\,| = c\sqrt{\vec{k}_\perp^2 + \vec{k}_{||}{}^2}$$

Near the surface

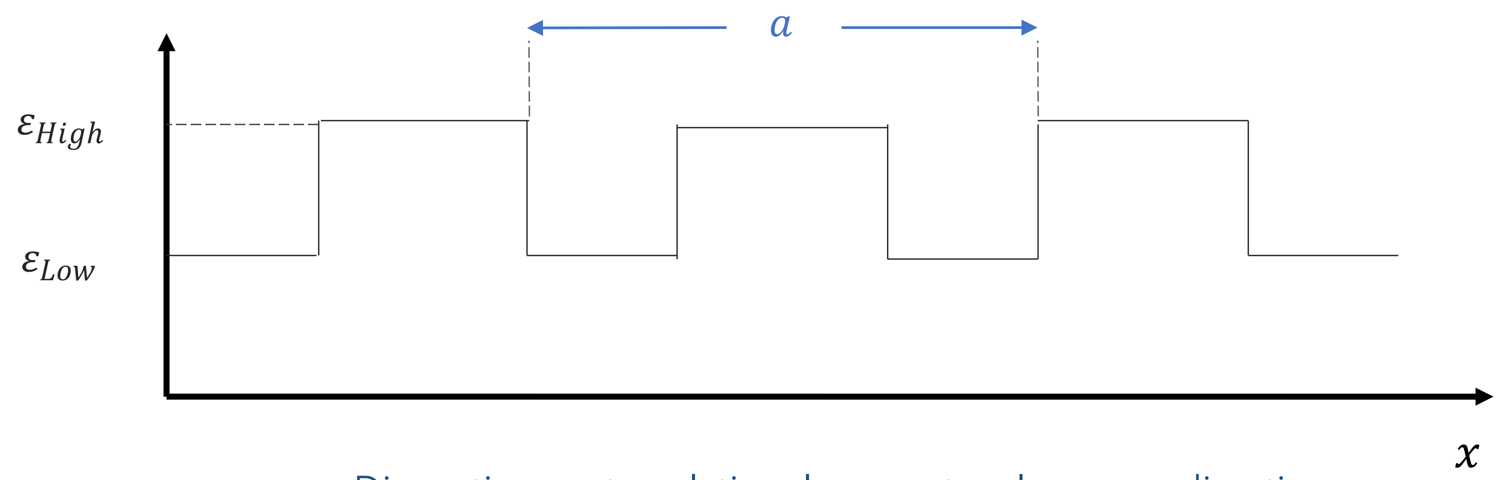
$$|\overrightarrow{k_{||}}| = |\vec{k}|\sin(\theta_1) = |\vec{k}|$$

$$\Rightarrow \quad \omega = c|\overrightarrow{k_{||}}|$$

Inside the glass plane

$\omega < c|\overrightarrow{k_{||}}|$ $\longrightarrow$ The only solutions in air are those with imaginary $\vec{k}_\perp = \mp i\sqrt{-\frac{\omega^2}{c^2} + \vec{k}_{||}{}^2}$ corresponding to evanescent waves.

3. Discontinuous translational symmetry



Discontinuous translational symmetry along one direction

$$\varepsilon(\vec{r}) = \varepsilon(\vec{r} \mp \vec{a}).$$

$\vec{a} = a\,\vec{x}$ is the primitive lattice vector

$$T_{\vec{R}}\, e^{i\,k_x x} = e^{i\,k_x\,(x-l\,a)} = \left(e^{-ik_x\,l\,a}\right) e^{ik_x\,x}$$

$$T_{\vec{R}} e^{i\left(k_x+\frac{2\pi}{a}\right)x} = \left(e^{-ik_x l\,a}\right) e^{i\left(k_x+\frac{2\pi}{a}\right)x}$$

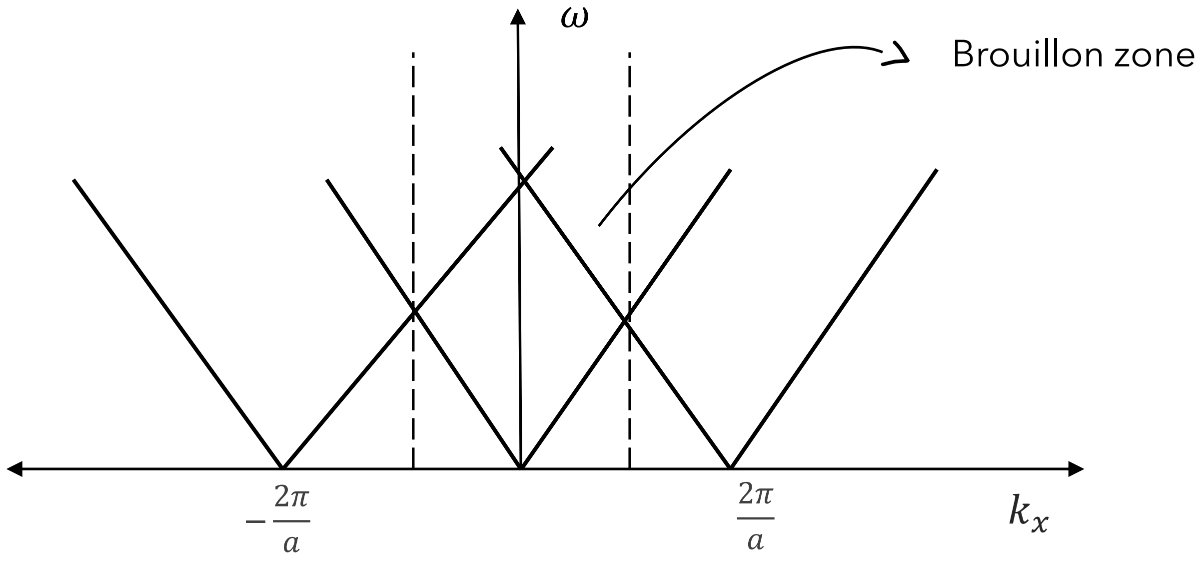
$\vec{b} = \frac{2\pi}{a}\,\vec{x}$ is the primitive reciprocal lattice vector

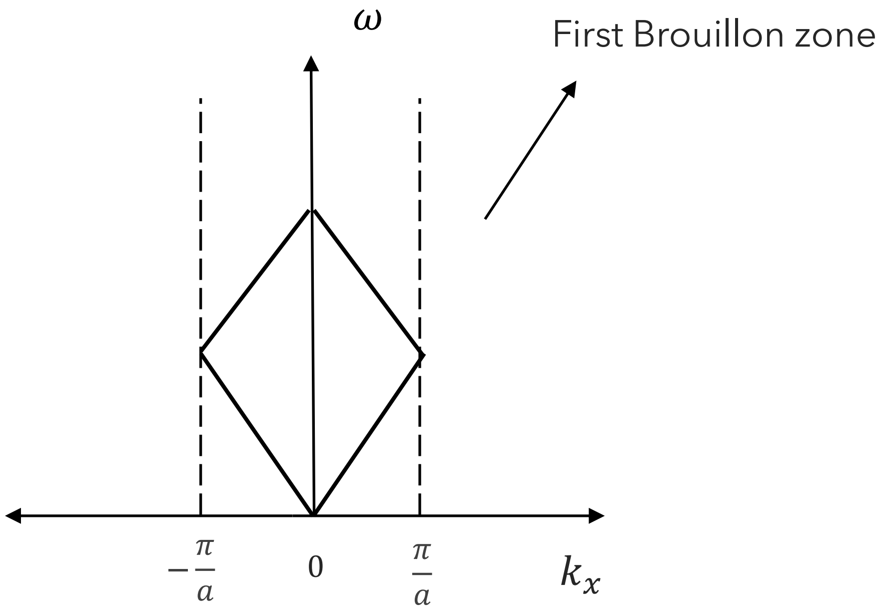
$$\boxed{H_{k_x}(r) = \sum_m C_{k_x,m}\, e^{i\,(k_x+mb)x} = e^{i\,k_x x}\, U_{k_x}(x)}$$

Periodic function in x

Bloch's theorem

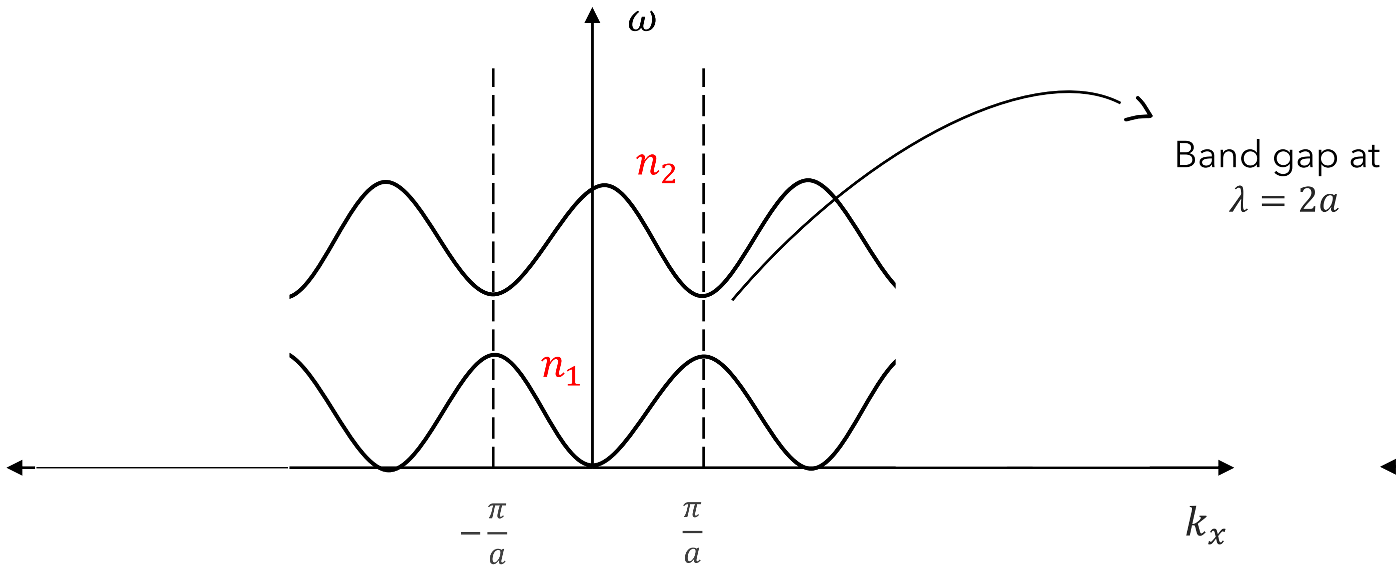
- Same dielectric constants

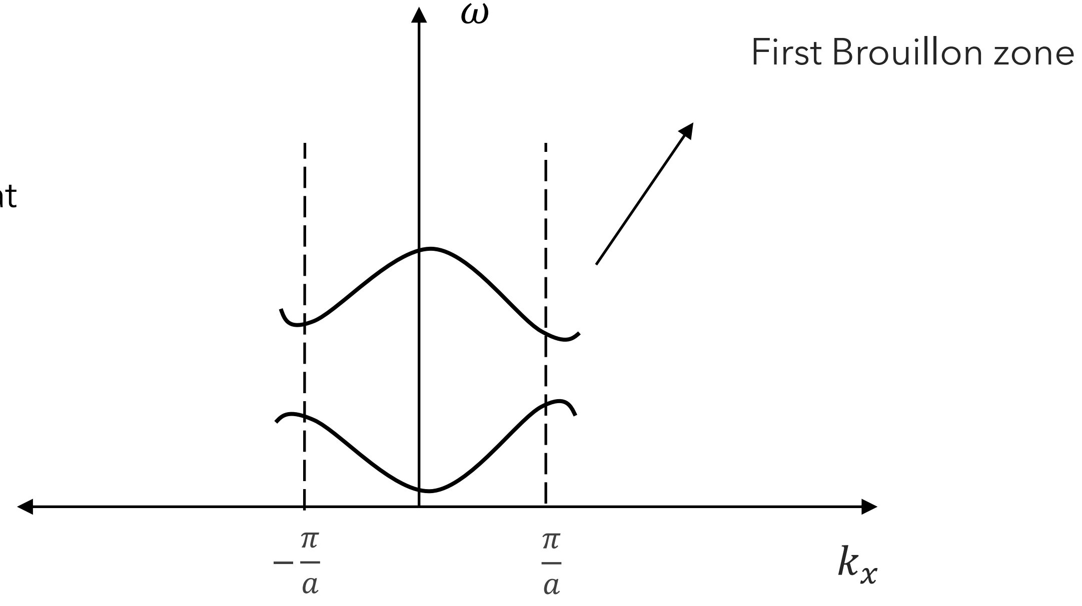




→ The periodicity allows the description of the modes with the wave vector $\vec{k}$

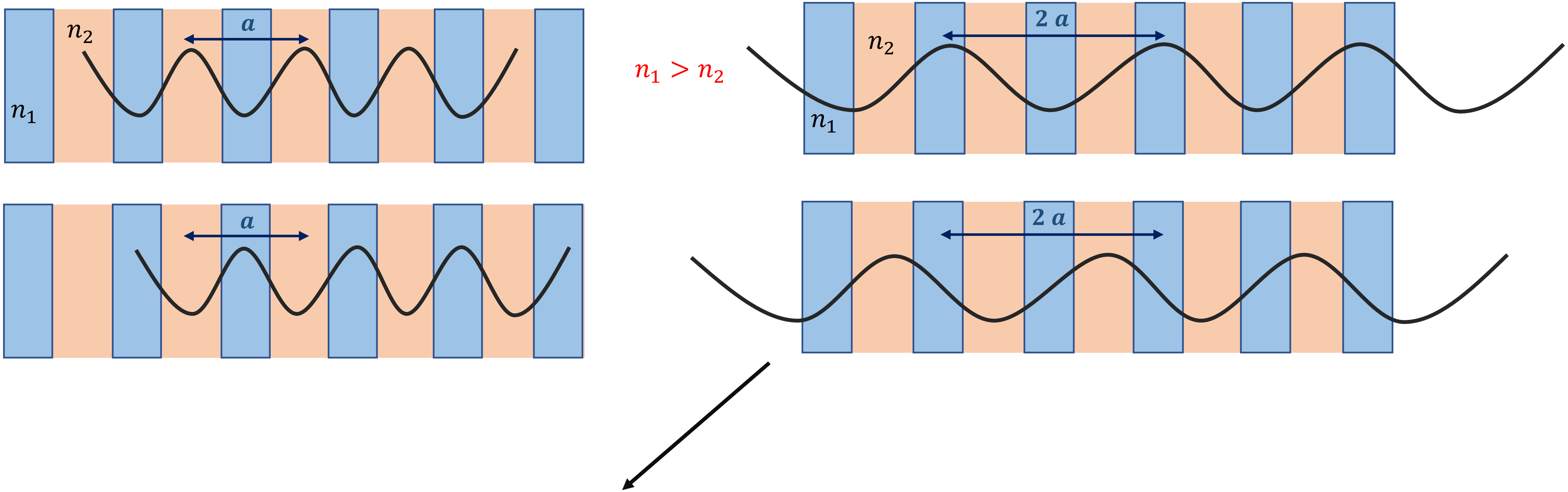
- Different dielectric constants





We obtain two different frequencies when the electric field is concentrated in two media with different dielectric functions.

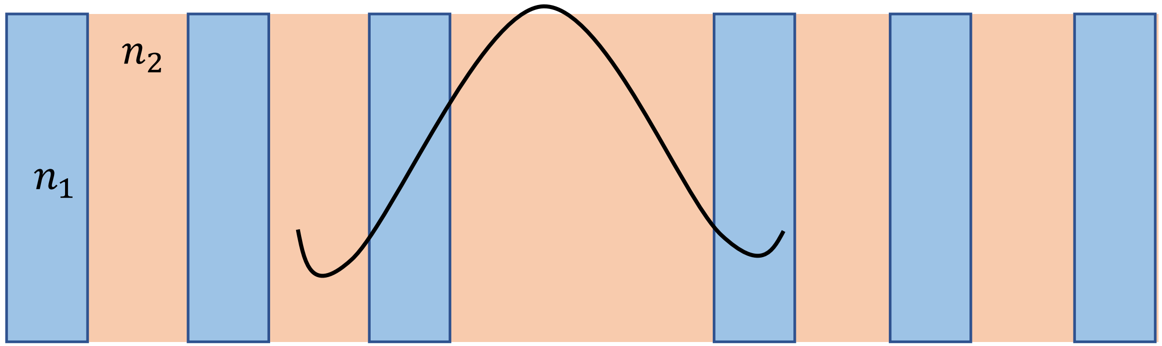
$$U(H) = \frac{\int d^3r\,|\nabla \times E(r)|^2}{\int d^3r\,\varepsilon(r)[E(r)|^2}$$



At the edge of the Brouillon zone, $\lambda = 2a$ there are two ways to locate the modes that give two different frequencies

5. Defects

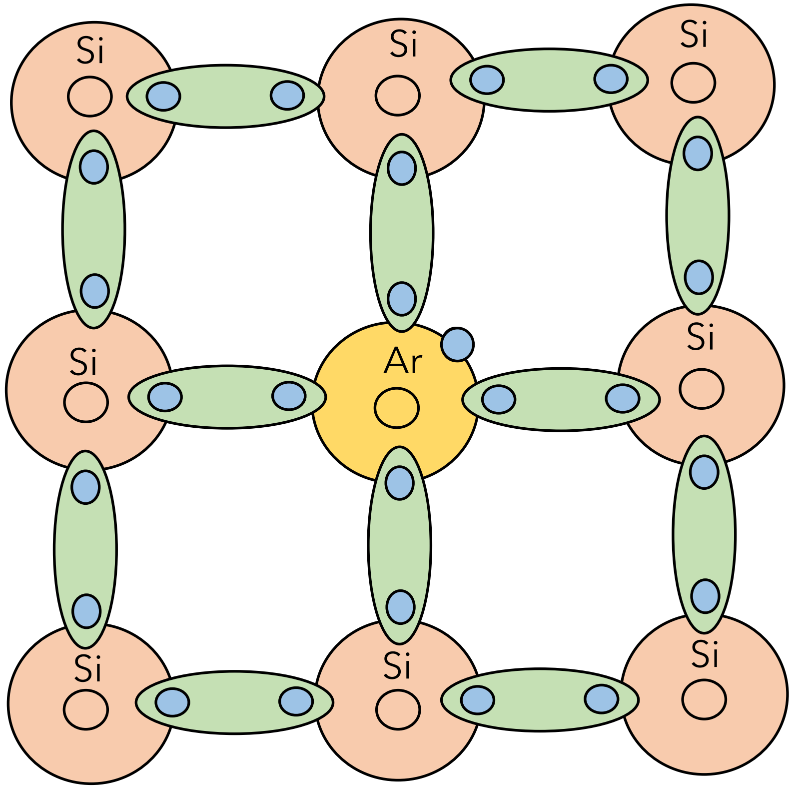
Defects break the transitional symmetry.

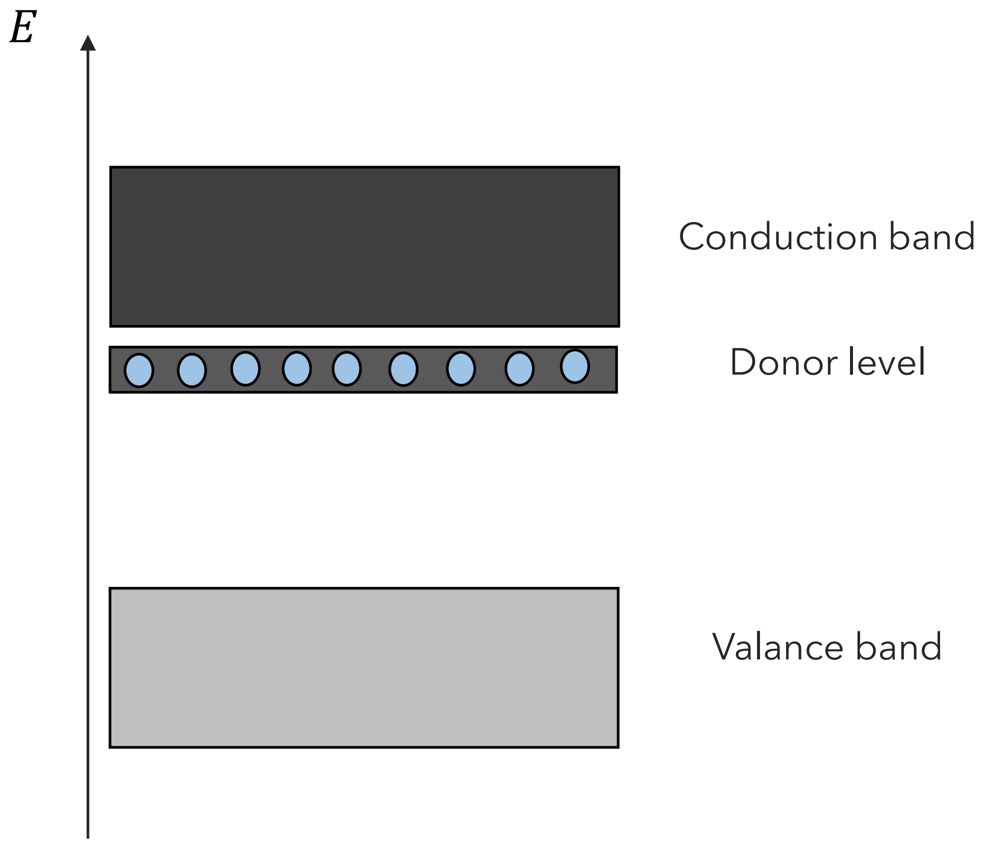


$n_1 > n_2$

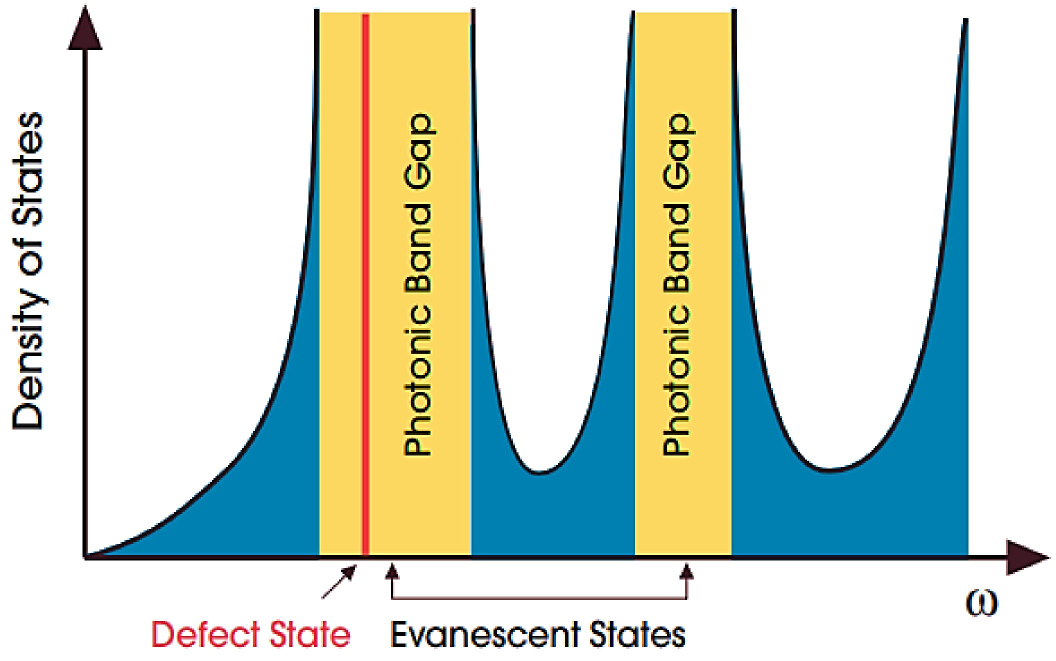
They can create localized modes.

Analogy: doping in semi-conductors





Defects permit localized modes to exist, with frequencies inside photonic band gaps.

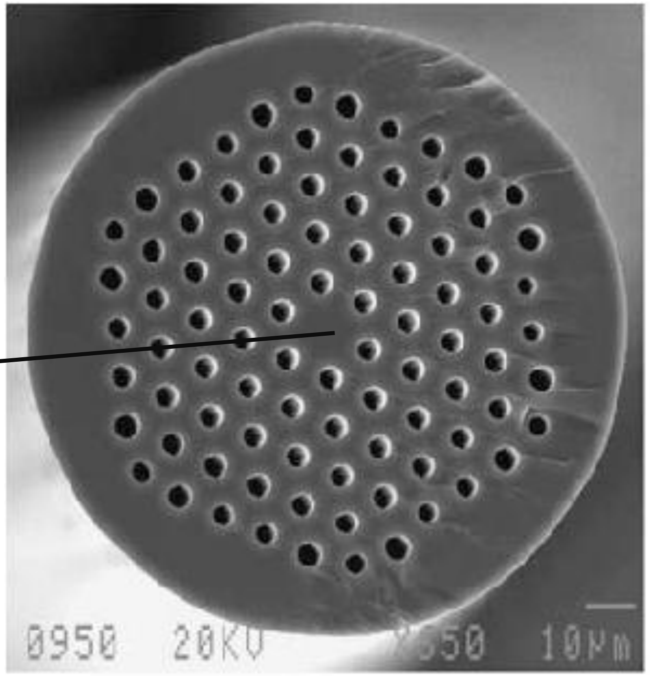


Density of state (the number of allowed modes per unit frequency) of a one-dimension periodic photonic crystal

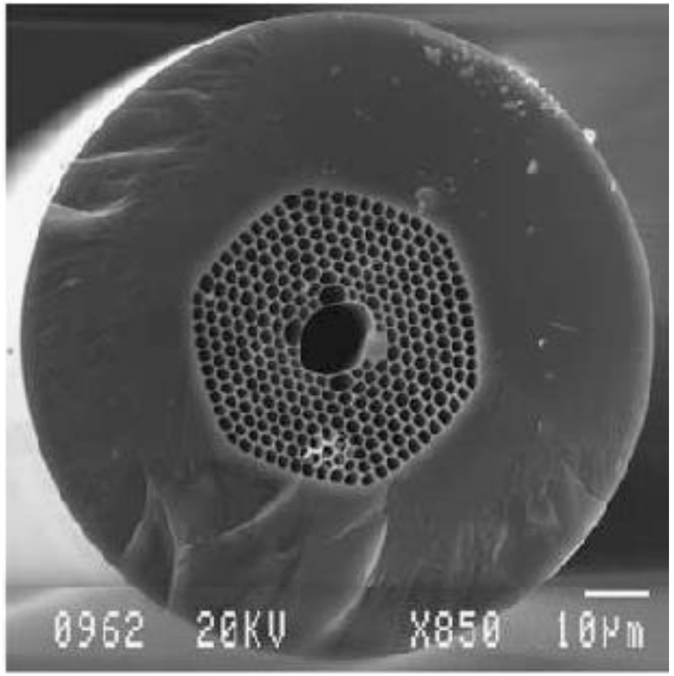
IV- Applications

1.Photonic crystal fibers

Higher index then the cladding



(a)

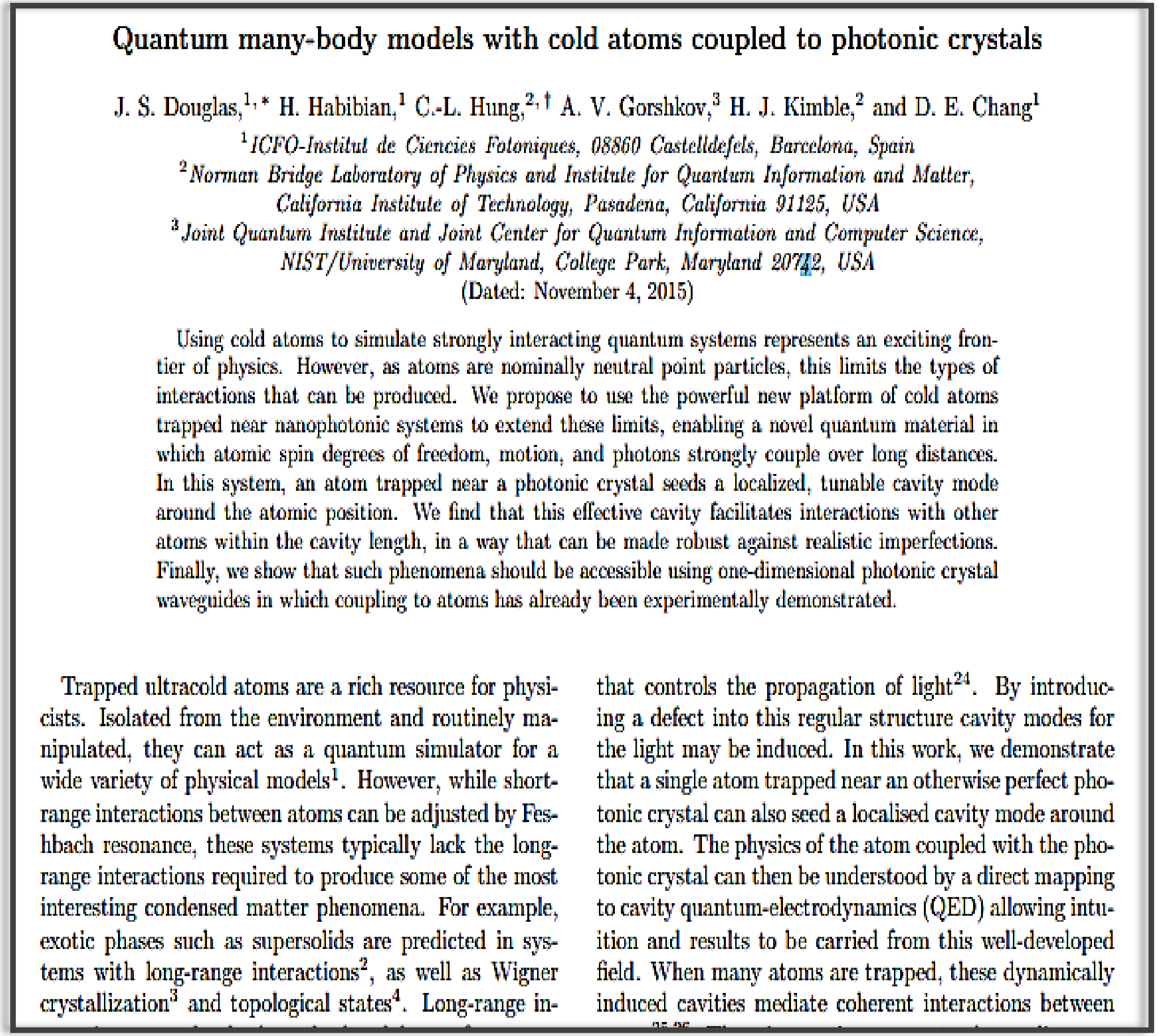


(b)

(a) Holey fiber, confining light in a solid core by index guiding

(b) Holey fiber, confining light in hallow core by a band gap.

2. Creation of coherent long-range interactions



Quantum many-body models with cold atoms coupled to photonic crystals

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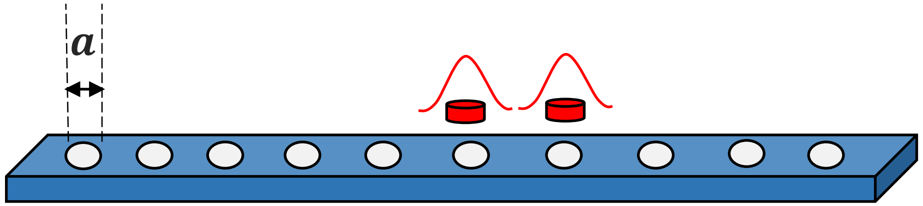
(Dated: November 4, 2015)

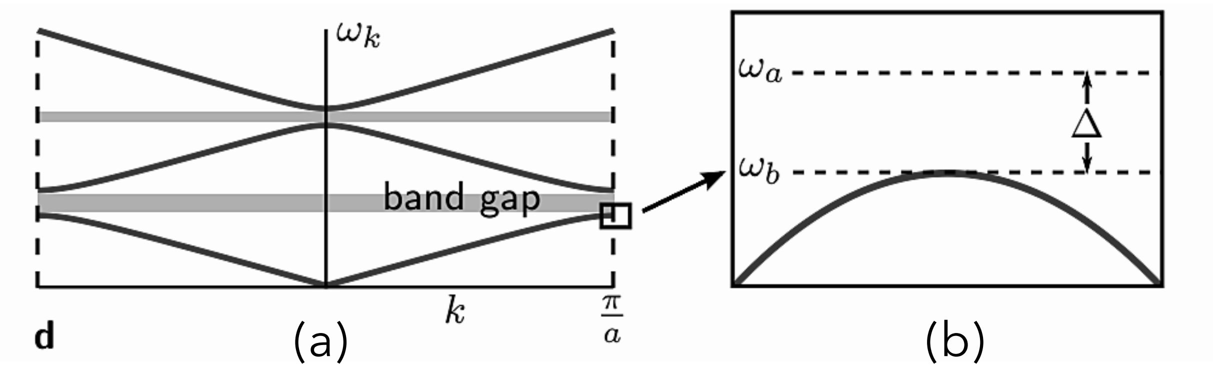
Using cold atoms to simulate strongly interacting quantum systems represents an exciting frontier of physics. However, as atoms are nominally neutral point particles, this limits the types of interactions that can be produced. We propose to use the powerful new platform of cold atoms trapped near nanophotonic systems to extend these limits, enabling a novel quantum material in which atomic spin degrees of freedom, motion, and photons strongly couple over long distances. In this system, an atom trapped near a photonic crystal seeds a localized, tunable cavity mode around the atomic position. We find that this effective cavity facilitates interactions with other atoms within the cavity length, in a way that can be made robust against realistic imperfections. Finally, we show that such phenomena should be accessible using one-dimensional photonic crystal waveguides in which coupling to atoms has already been experimentally demonstrated.

Trapped ultracold atoms are a rich resource for physicists. Isolated from the environment and routinely manipulated, they can act as a quantum simulator for a wide variety of physical models[1]. However, while short-range interactions between atoms can be adjusted by Feshbach resonance, these systems typically lack the long-range interactions required to produce some of the most interesting condensed matter phenomena. For example, exotic phases such as supersolids are predicted in systems with long-range interactions[2], as well as Wigner crystallization[3] and topological states[4]. Long-range in-

that controls the propagation of light[24]. By introducing a defect into this regular structure cavity modes for the light may be induced. In this work, we demonstrate that a single atom trapped near an otherwise perfect photonic crystal can also seed a localised cavity mode around the atom. The physics of the atom coupled with the photonic crystal can then be understood by a direct mapping to cavity quantum-electrodynamics (QED) allowing intuition and results to be carried from this well-developed field. When many atoms are trapped, these dynamically induced cavities mediate coherent interactions between



Defects in a 1D photonic crystal created by the presence of two level atoms. The atoms induce localized cavities. The atoms separated by the order of the effective cavity length can interact coherently with each other.



(a) Band structure of a 1D photonic crystal. **(b)** Two level atom coupled to the 1D photonic crystal having resonance frequency ω_a close to the band edge frequency ω_a, with $\Delta = \omega_a - \omega_b$.

V- Summary

- Photonic crystals are composed of periodic dielectric media that affect electromagnetic wave propagation in the same way that the periodic potential in a semiconductor crystal affects the propagation of electron.

- Light waves may propagate through the media or propagation may be disallowed, depending on their wavelength.

- Light can be confined either by index guiding or by introducing defects, because they allow the existence of frequencies inside the band gap.

Thank you