Experimental applications of interaction-free measurements

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Brief introduction to interaction-free measurements

There are many examples of measurements that exploit the non-locality of quantum mechanics:

- if object with charge or electric/magnetic moment → Aharonov–Bohm effect
- measurement on entangled states → spatial case: EPR state

The latter is an example of what we call an interaction-free measurement:

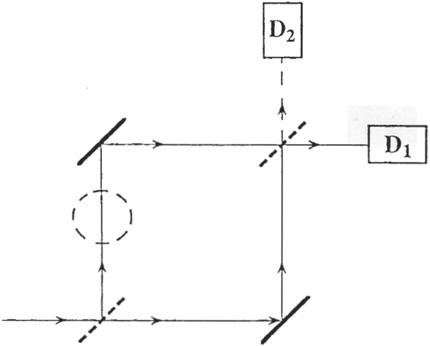
- a measurement of position performed on one of the two particles gives information on the position of the second without interacting with the second one

However, in the case of entanglement, we had information on the object prior to the measurement: can we do without?

- Answer: yes! → "Quantum mechanical interaction-free measurements", A. Elitzur, L. Vaidman (1993)

The Elitzur-Vaidman thought experiment

We employ a Mach-Zehnder interferometer with a single-particle state (i.e. single photon)



A measurement can have 3 outcomes:

- no detector clicks
- D1 clicks
- D2 clicks

Let $|1\rangle$ be the state of a photon moving to the right, $|2\rangle$ the one moving upwards and $|\textit{scattered}\rangle$ the one of a photon absorbed or scattered by the object.

The action of the beamsplitters is:

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}[|1\rangle + i\,|2\rangle]$$

$$|2\rangle \rightarrow \frac{1}{\sqrt{2}}[|2\rangle + i\,|1\rangle]$$

and the one of the mirrors is:

$$|1\rangle \rightarrow i\,|2\rangle$$

$$|2\rangle \rightarrow i\,|1\rangle$$

The Elitzur-Vaidman thought experiment

When there is no object, we have:

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}[|1\rangle + i\,|2\rangle] \rightarrow \frac{1}{\sqrt{2}}[i\,|2\rangle - |1\rangle] \rightarrow \frac{1}{2}[i\,|2\rangle - |1\rangle] - \frac{1}{2}[|1\rangle + i\,|2\rangle] = -\,|1\rangle$$

$\rightarrow$ with no object, D_2 never clicks!

With an object:

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}[|1\rangle + i\,|2\rangle] \rightarrow \frac{1}{\sqrt{2}}[i\,|2\rangle + i\,|scattered\rangle] \rightarrow \frac{1}{2}[i\,|2\rangle - |1\rangle] + \frac{i}{\sqrt{2}}\,|scattered\rangle$$

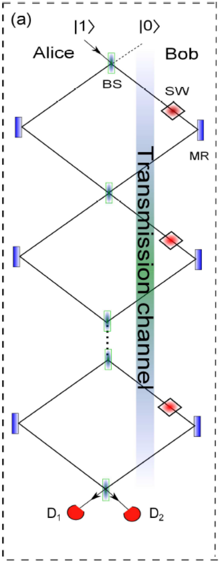
Once the state is measured at the detectors, the probabilities of detection are:

$$\frac{1}{2}[i\,|2\rangle - |1\rangle] + \frac{i}{\sqrt{2}}\,|scattered\rangle \rightarrow \begin{cases} |2\rangle \rightarrow D_2\ clicks,\ probability\ 1/4 \\ |1\rangle \rightarrow D_1\ clicks,\ probability\ 1/4 \\ |scattered\rangle \rightarrow no\ clicks,\ probability\ 1/2 \end{cases}$$

More generally, with unbalanced beamsplitters of reflectivity a and b, $a^2 + b^2 = 1$:

$$ia[b\,|2\rangle + ia\,|1\rangle] + ib\,|scattered\rangle \rightarrow \begin{cases} |2\rangle \rightarrow D_2\ clicks,\ probability\ a^2 b^2 \\ |1\rangle \rightarrow D_1\ clicks,\ probability\ a^4 \\ |scattered\rangle \rightarrow no\ clicks,\ probability\ b^2 \end{cases}$$

Can we do better than $P_2/P_{scattered} = a^2 \rightarrow 1$ for $a \rightarrow 1$?



Yes! $\rightarrow$ "Protocol for Direct Counterfactual Quantum Communication", H. Salih, Z. Li, M. Al-Amri, and M. S. Zubairy (2013)

The trick is exploiting a chained version of the quantum Zeno effect

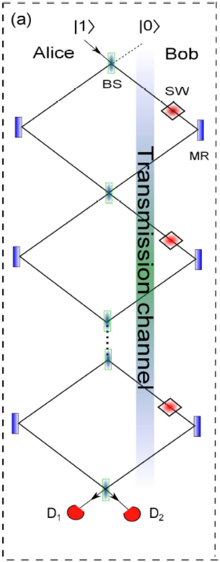
Let us suppose to use a large number $N >> 1$ of beamsplitters with very small transmittivity, $cos(\theta) = \sqrt{R}$ with R being the reflectivity and $\theta = \pi/2N$.

The action of the beamsplitters is:

$$|10\rangle \rightarrow cos(\theta)\,|10\rangle + sin(\theta)\,|01\rangle$$

$$|01\rangle \rightarrow cos(\theta)\,|01\rangle - sin(\theta)\,|10\rangle$$

Can we do better than $P_2/P_{scattered} = a^2 \to 1$ for $a \to 1$?



If the switches are unblocked, the state evolves coherently. After n cycles, the action on the initial state $|10\rangle$ is then:

$$|10\rangle \to cos(n\theta)\,|10\rangle + sin(n\theta)\,|01\rangle$$

$\to$ after N cycles, the state is $|01\rangle \to$ *D_2 clicks*

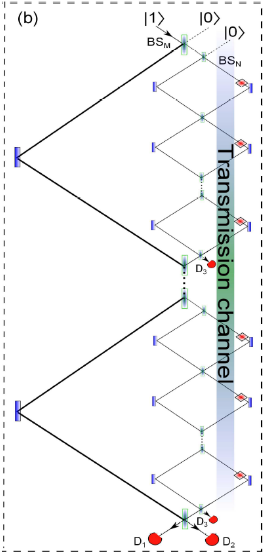
If instead the switches are activated, they continuously measure the state (each time with extremely low probability $sin^2(\theta)$). Then, after n cycles:

$$|10\rangle \to cos^{n-1}(\theta)[cos(\theta)\,|10\rangle + sin(\theta)\,|01\rangle]$$

$cos^N(\theta) \approx 1 \to$ after N cycles, the state is $\approx |01\rangle$

$\to$ *D_1 clicks*

This scheme then allows to measure the state of the switch with arbitrary efficiency.



A modified version of the previous setup can be used to perform fully counterfactual communication (i.e., to make it so that Alice can know the state of the switch set by Bob without having any photon passing through Bob's branch of the setup (which still happened for open switch).

The scheme exploits a nested version of the previous one, with M "big cycles" of the N cycles from before. As previously, $\theta_N = \pi/2N$ and $\theta_M = \pi/2M$, $N, M >> 1$. If the switches are off, for every m-th cycle:

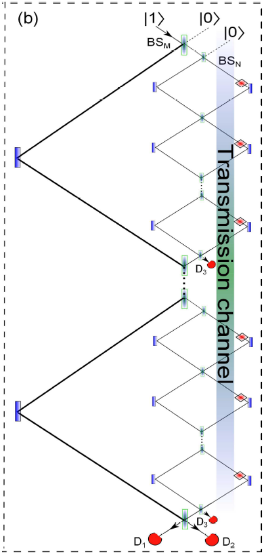
$$|010\rangle \rightarrow cos(n\theta_N)\,|010\rangle + sin(n\theta_N)\,|001\rangle \xrightarrow{n=N} |001\rangle$$

The detectors D_3 perform a measurement in the same way as a closed switch does, so for open switch after the m-th cycle the initial state $|100\rangle$ goes to:

$$|100\rangle \rightarrow cos^{m-1}(\theta_M)[cos(\theta_M)\,|100\rangle + sin(\theta_M)\,|010\rangle] \xrightarrow{m=M} |100\rangle$$

$\rightarrow$ at the end, *D_1 clicks*. No photons have passed through Bob's branch (D_3 didn't click!)

Applications: direct counterfactual communication



If instead Bob activates the switches, for each m-th cycle:

$$|010\rangle \rightarrow cos^{n-1}(\theta_N)[cos(\theta_n)\,|010\rangle + sin(\theta_N)\,|001\rangle \xrightarrow{n=N} |010\rangle$$

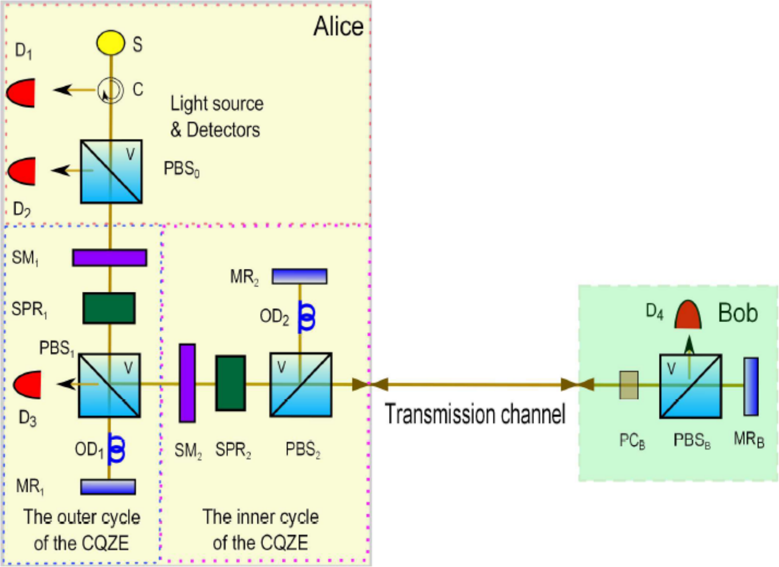
After the m-th cycle, the state is:

$$|100\rangle \rightarrow cos(m\theta_M)\,|100\rangle + sin(m\theta_M)\,|010\rangle \xrightarrow{m=M} |010\rangle$$

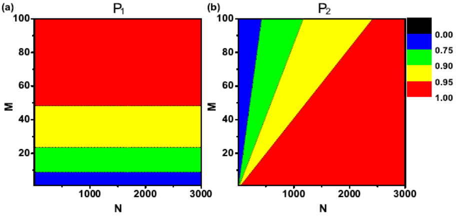
$\rightarrow$ at the end, *D_2 clicks*.

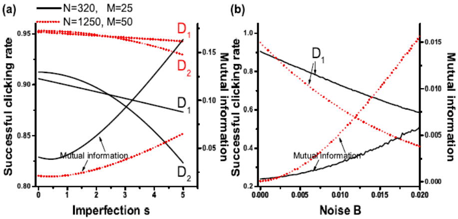
Again, no photons have passed through Bob's branch (they would have been blocked by the switches)

Michelson version of Salih et al's protocol



Efficiency and imperfections in Salih et al.'s protocol





Detection probability estimated via a recursive calculation using setup parameters

Efficiency impacted by two classes of phenomena:

- Ones that don't cause errors but only missed detections, i.e. efficiency $\eta < 1$ of the detectors $\rightarrow$ reduced efficiency of communication
- Ones that cause measurement errors:
  - For alternative Michelson-type interferometer setup, mainly due to imperfect polarization rotators $\Delta\theta_{N(M)} = s_{N(M)}[\theta_{N(M)}/N(M)]$
  - Loss of photons in the transmission channel when Bob is not blocking
- efficiency and error rate (measured as *mutual information*) quantified in simulations