UNIVERSITÀ DI SIENA 1240

Istituto Nazionale di Fisica Nucleare

Statistical methods in the search for cosmological axions

Statistical Data Analysis

Raffaele Di Vora

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(Extra-brief) Introduction to axion searches

What is an axion

- Introduced to fix the strong CP problem of QCD
- Added term to SM Lagrangian:

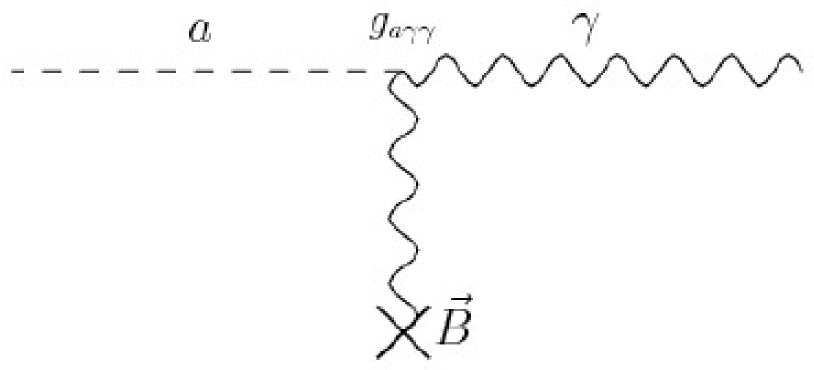
$$\mathcal{L}_{a\gamma\gamma} = \frac{1}{4} g_{a\gamma\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu}$$

Why search for axions

- **Axion: Cold Dark Matter candidate!!**

How to search for CDM axion

- We exploit the interaction with virtual photons: inverse Primakoff effect



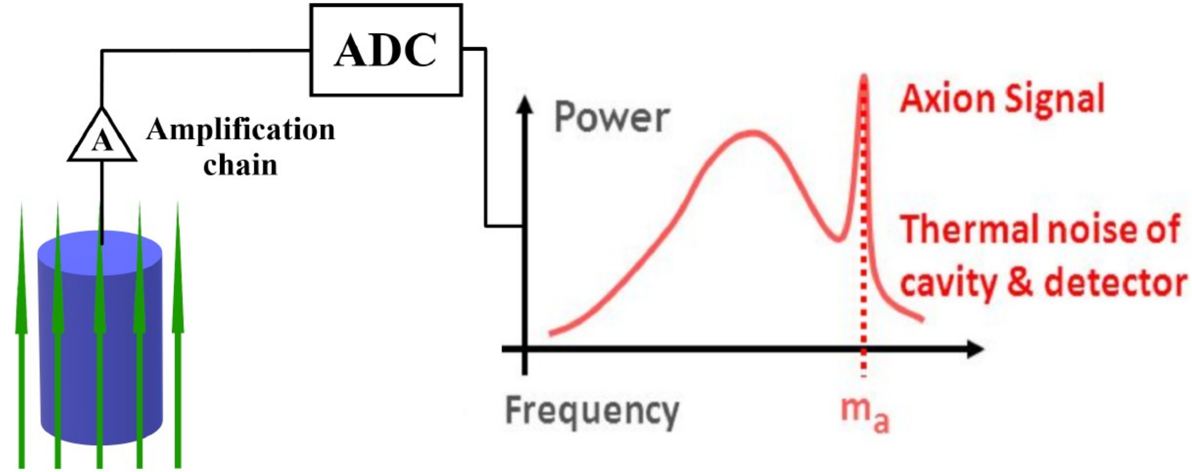
$$g_{a\gamma\gamma} = \alpha g_\gamma / \pi f_a$$

- $g_\gamma \approx 0.36$ for DFSZ model
- $f_a \approx 10^{-8}$-10^{-11} Hz
- CDM → $\beta \approx 10^{-3}$ → $Q_a \approx 10^6$

(Extra-brief) Introduction to axion searches

The Sikivie Haloscope scheme

- We provide virtual photons to scatter on with a high magnetic field
- Signal is boosted integrating inside high-Q factor microwave cavity
- From there, collected with standard MW electronics



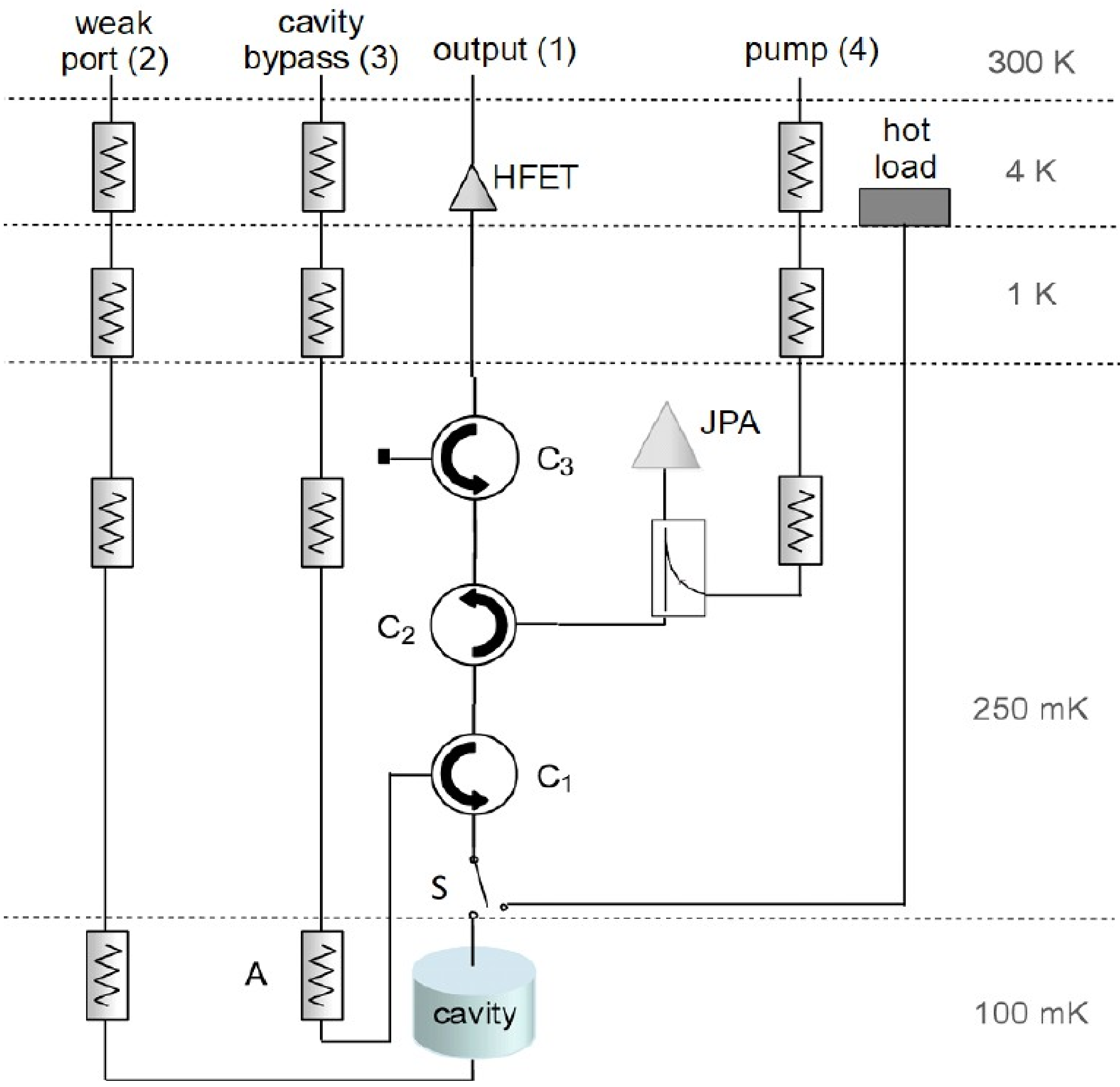
$$P_{\rm sig} = \left(g_\gamma^2 \frac{\alpha^2}{\pi^2} \frac{\hbar^3 c^3 \rho_a}{\chi}\right) \left(\frac{\beta}{1+\beta} \omega_c \frac{1}{\mu_0} B_0^2 V C_{mn\ell} Q_L \frac{1}{1+(2\delta\nu_a/\Delta\nu_c)^2}\right)$$

$$P_{\rm axion} = 2.2\times10^{-23}{\rm W} \left(\frac{\beta}{1+\beta}\right) \left(\frac{V}{136\ \ell}\right) \left(\frac{B}{7.6\ {\rm T}}\right)^2 \left(\frac{C_{010}}{0.4}\right) \left(\frac{g_\gamma}{0.36}\right)^2 \left(\frac{\rho}{0.45\ {\rm GeVcm^{-3}}}\right) \left(\frac{Q_{\rm axion}}{10^6}\right) \left(\frac{f}{740\ {\rm MHz}}\right) \left(\frac{Q_{\rm L}}{30{,}000}\right) \left(\frac{1}{1+(2\delta f_a/\Delta f)^2}\right)$$

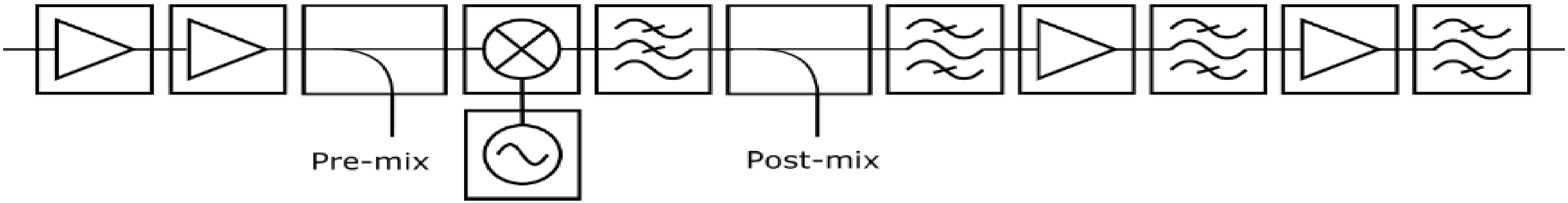
ADMX Run 1B: Experimental Setup

Receiver Chain

- Switch → two modes of operation:
    - cavity readout
    - Noise characterization
- Amplifiers:
    - JPA (Josephson Parametric Amplifier)
    - HFET (Heterostructure Field-Effect Transistor)
- Mixer and bandpass filters at room temperature



ADMX Run 1B: Hardware signal treatment



Signal digitization

- Signal bandwidth after last filter 150 kHz
- ADC sampling rate 25 MS/s
- Single sample total sampling time 10 ms

→ by Nyquist-Shannon Theorem we can reconstruct the signal

FFT (Fast Fourier Transform)

- 48.8 kHz-wide window
- 512 x 95 Hz-wide bins
- Total integration time: 100 s → 10^4 samples/bin

Analysis strategy

Signal low → search for power fluctuations above average noise background

We need:

- **Threshold** for axion detection

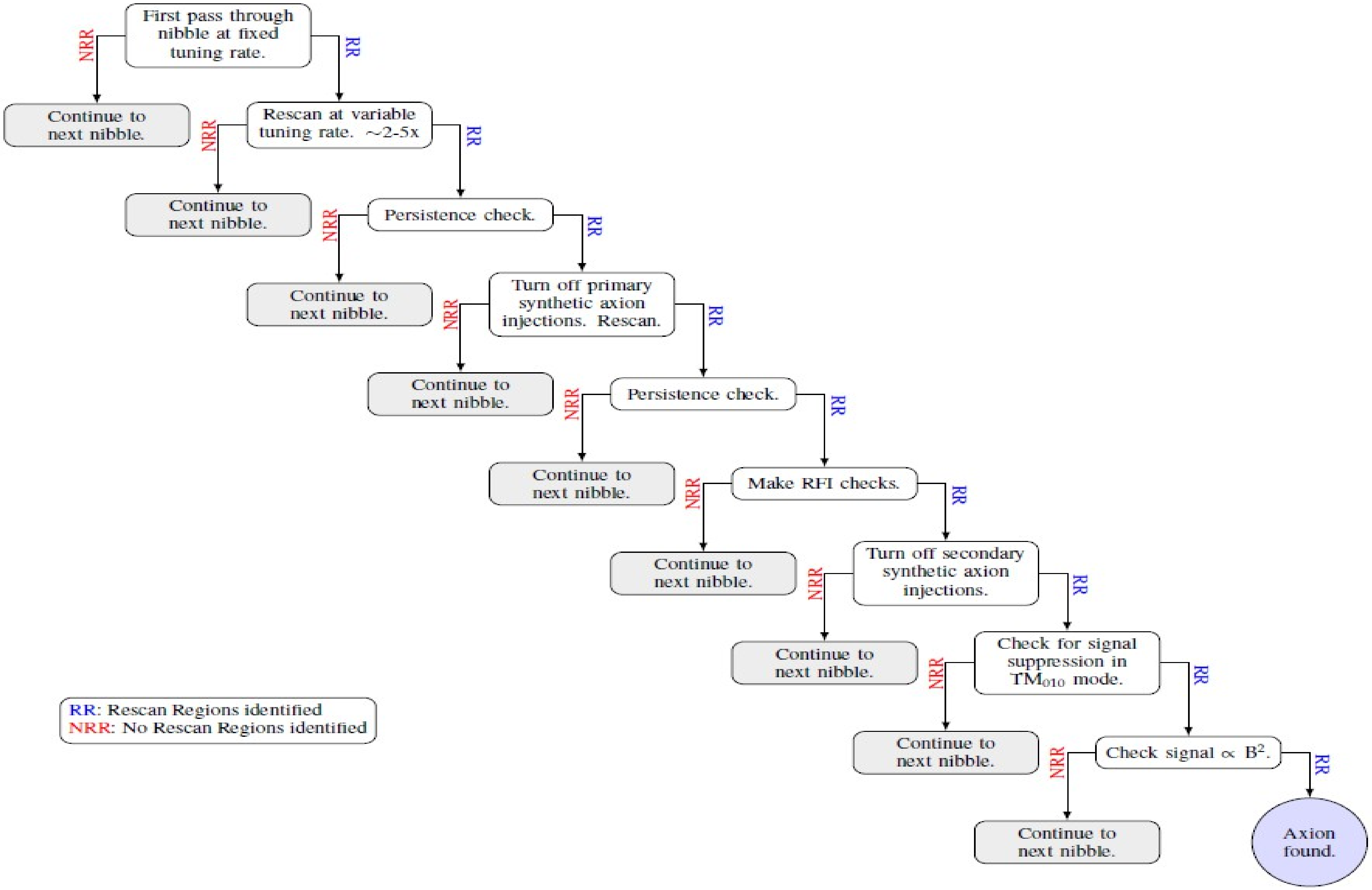
  → Needs to be high to limit number of candidates!

- A way to identify **false signals**
- A procedure to estimate the **noise background**

Run cadence

Process	Frequency	Fraction of Time per Iteration
Transmission Measurement	Every Iteration	$< 1\%$
Reflection Measurement	Every Iteration	$< 1\%$
JPA Rebias	Every 5-7 Iterations	25%
Check for SAG Injection	Every Iteration	$< 1\%$
Digitize	Every Iteration	98%[a]
Move Rods	Every Iteration	$< 1\%$

Data-taking decision tree



System noise characterization

We define the overall noise temperature of the system $\mathbf{T_{sys}}$:

$$T_{\rm sys} = T_{\rm cav} + T_{\rm amp} \;\rightarrow\; P_n = k_{\rm B} T_{\rm sys} b$$

with $\mathbf{P_n}$ the noise power in the measured bandwidth **b**

0.03 K ≈ $\mathbf{hf/k_B \ll T_{sys}}$ → Quantum fluctuations negligible!

Then, the Signal to Noise Ratio at integration time t is:

$$\mathrm{SNR} = \frac{P_{\rm axion}}{k_{\rm B} T_{\rm sys}} \sqrt{\frac{t}{b}}$$

But how to measure T_{sys}?

System noise characterization

We use Y-factor measurements:

1) **Hot Load method (Feb., Sept.):**

- JPA off (saturation, unstable gain with thermal load) → acts as mirror, can be used for wide bandwidth (5MHz) power measurement

$$P = G_{\mathrm{HFET}} k_{\mathrm{B}} \left[T_{\mathrm{JPA}}(1-\epsilon) + T_{\mathrm{load}}\epsilon + T_{\mathrm{HFET}} \right]$$

2) **Heating Switch (Jul., Sept.):**

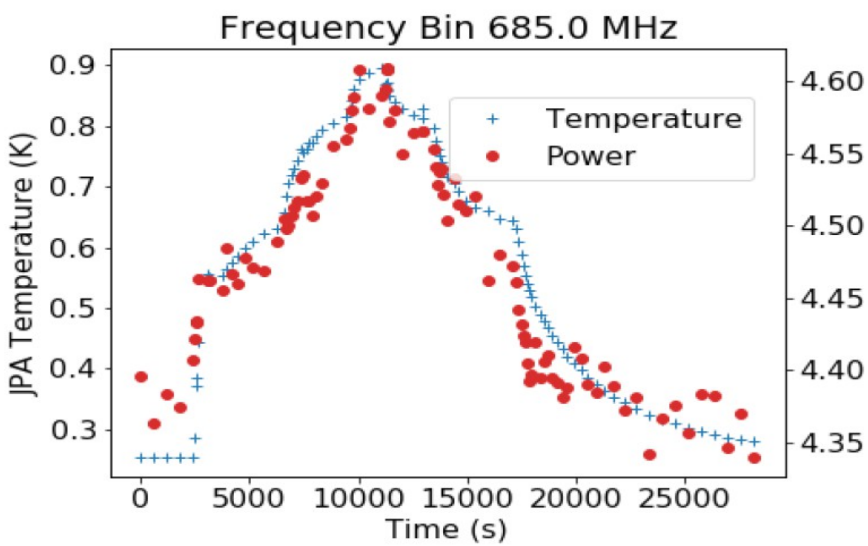
- Small ΔV applied to switch

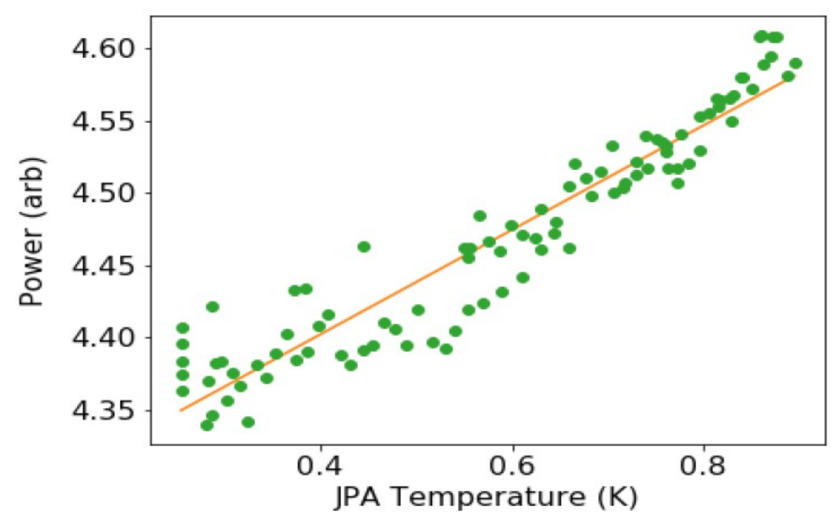
- all quantum amplifier package heats

$$P = G_{\mathrm{HFET}} k_{\mathrm{B}} \left[T_{\mathrm{JPA}}(1-\epsilon) + T_{\mathrm{cav}}\epsilon + T_{\mathrm{HFET}} \right]$$

- If $\mathbf{T_{JPA} \approx T_{cav}}$:

$$P = G_{\mathrm{HFET}} k_{\mathrm{B}} \left[T_{\mathrm{JPA}} + T_{\mathrm{HFET}} \right]$$





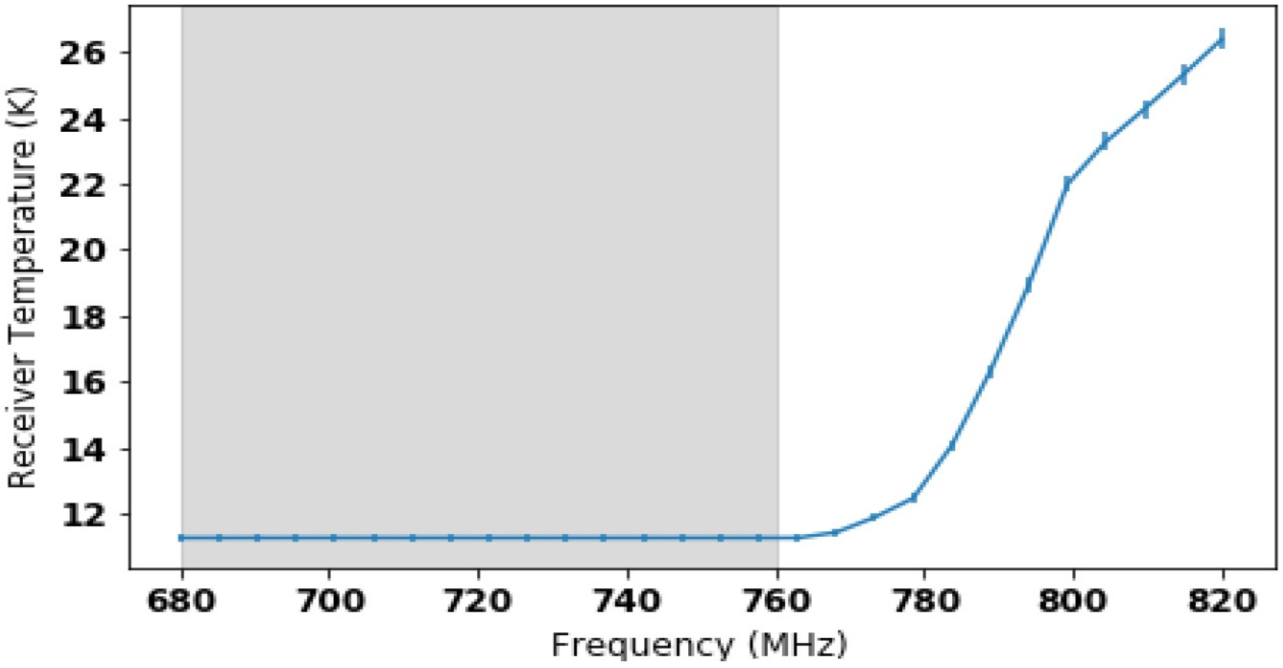
System noise characterization: results

No changes observed → **assumption: T_{HFET}, G_{HFET} stable**

- All 4 measurements put together, **Least squares fit** on combined results:

Receiver noise temperature T_{HFET} = (11.3 ± 0.1) K

(mostly from square root of covariance)

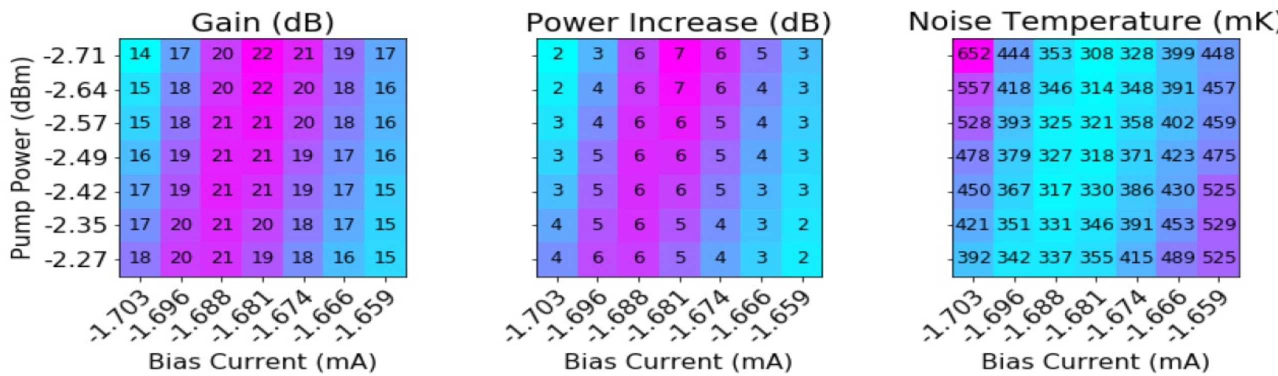


System noise characterization: results

Signal to Noise Ratio Improvement (SNRI):

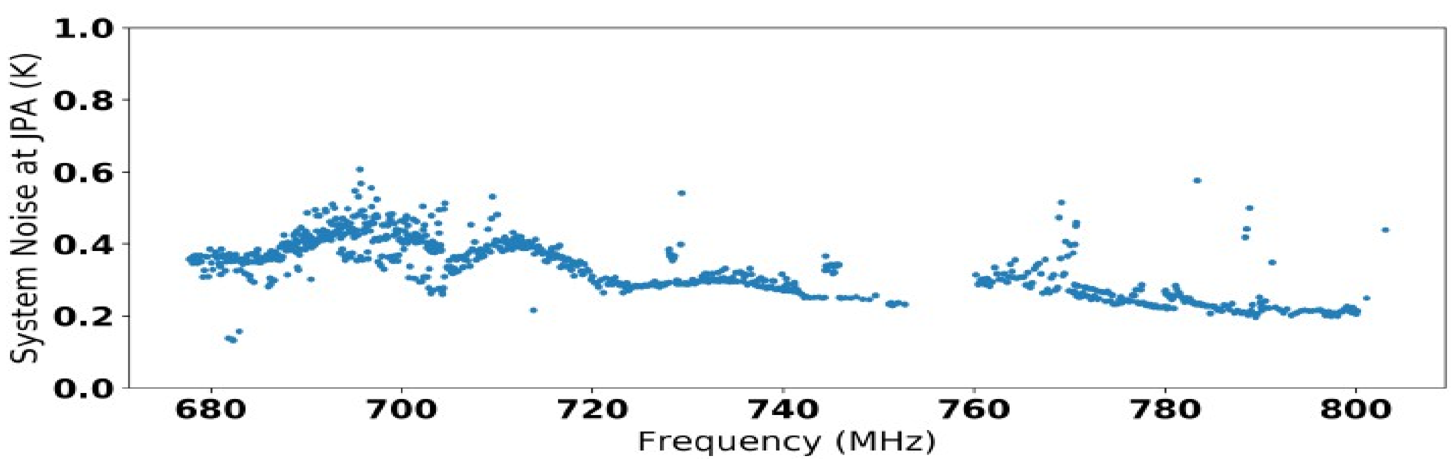
Monitored throughout the run every ≈10 minutes

$$\mathrm{SNRI} = \frac{G_{\mathrm{on}}}{G_{\mathrm{off}}}\frac{P_{\mathrm{off}}}{P_{\mathrm{on}}}$$



Changes over course of run since JPA gain is unstable for ΔT≈300/400 mK

By definition, allows to monitor T_{sys}: $T_{sys}=T_{HFET}/\mathrm{SNRI}$



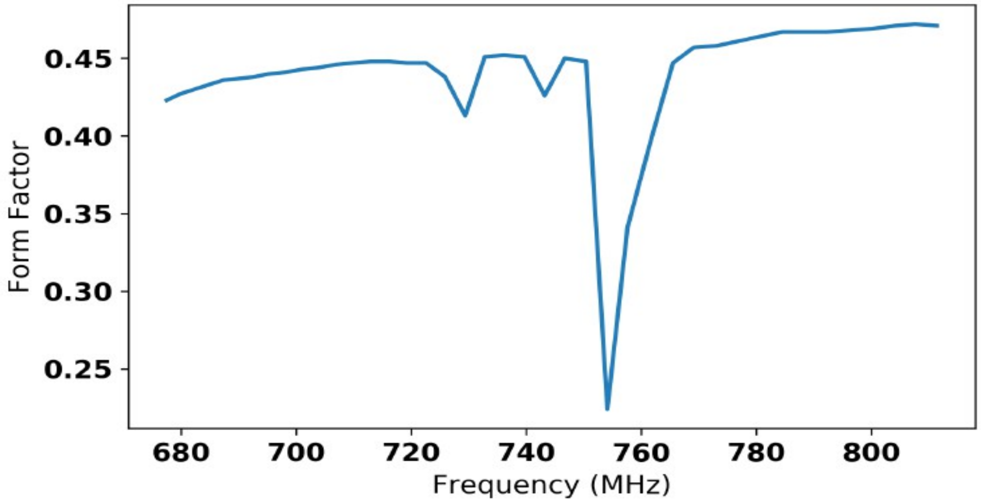
Model parameters and systematics

Other parameters to be fed to proper data analysis:

- $\boldsymbol{\omega_0}$ and $\mathbf{Q_L}$ of cavity → from Lorentzian fit to transmission measurement
- **β** coefficient → from fit to reflection meas.

$$\Gamma = \frac{\beta - 1 - (2iQ_0\delta\omega/\omega_0)}{\beta + 1 + (2iQ_0\delta\omega/\omega_0)}$$

- Form factor $\mathbf{C_{010}}$ (from finite element simulations)



Statistical and systematical errors:

Source	Fractional Uncertainty
B^2VC_{010}	0.05
Q	0.011
Coupling	0.0055
RF model fit	0.029
Temperature Sensors	0.05
SNRI measurement	0.042
Total on power	0.088

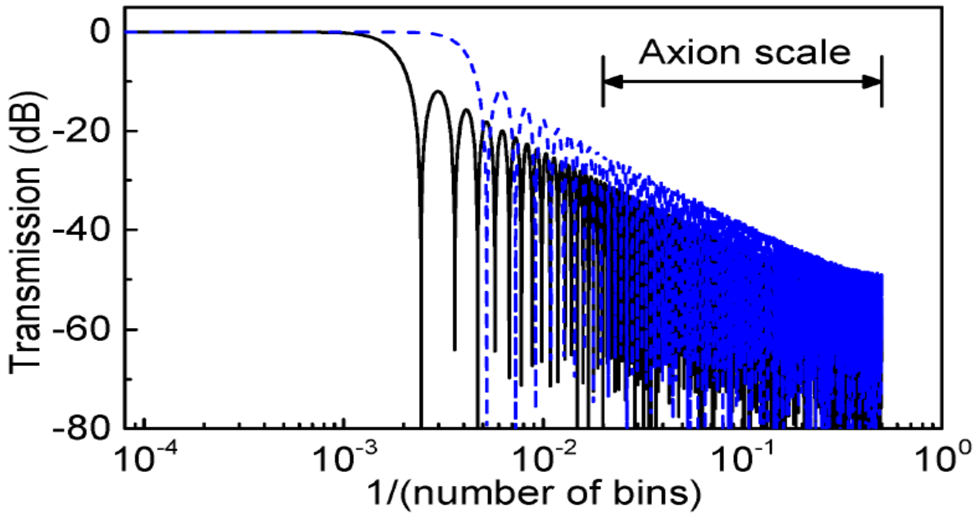
Axion search data processing: baseline removal

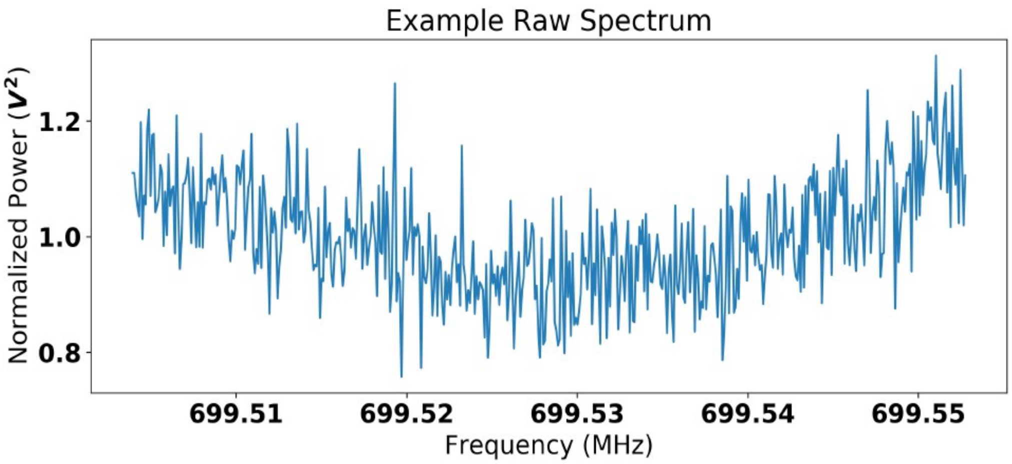
Raw data contains gain variations superimposed on the signal

- mostly from room temperature components

→ we use Savitzky-Golay filter to remove them, but:

- Passband ripple → residual unfiltered features
- Imperfect stopband attenuation → signal attenuation, introduces small negative correlations btw processed run bins





Assumption: noise statistically equivalent to thermodynamic noise at $T = T_{sys}$

→ p.d.f. of bin exp. value is Gaussian, fluctuations in subspectrum are $\chi^2(2)$

then, by the Central Limit Theorem:

after averaging over 10^4 samples, the distribution of the noise power fluctuations will be Gaussian!

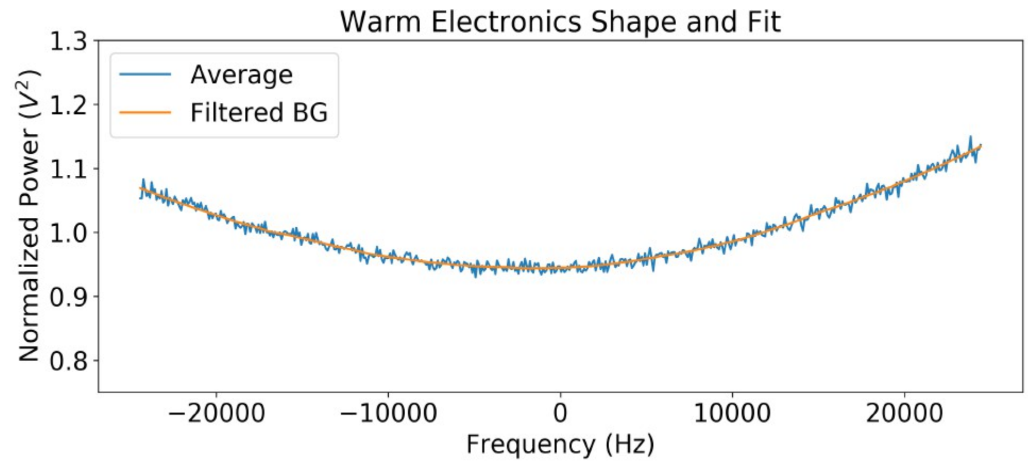
Cryogenic baseline removal and spectrum processing

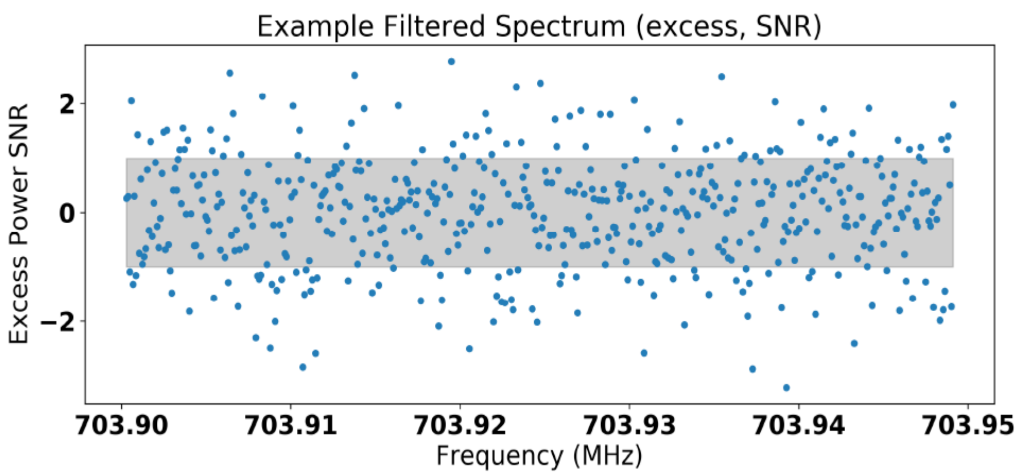
After division with S-G filter:

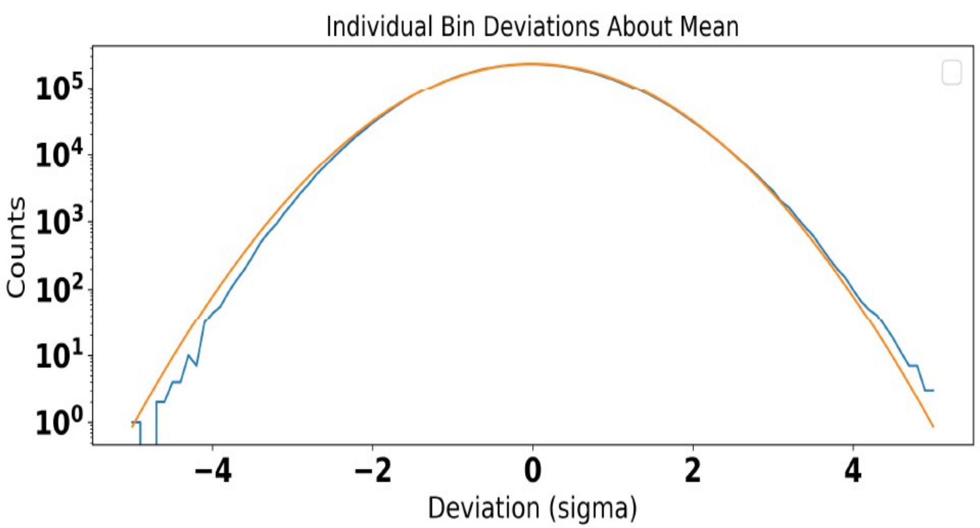
- Spectrum is convolved with **additional 6th order Padé filter** to remove residual frequency-dependent variations due to cryogenic receiver tranfer function

Power normalised by dividing every bin for overall mean, then we subtract 1

→ every bin becomes random sample from Gaussian distribution with mean μ=0







Data quality control and rescan procedure

12,492/197,680 out of raw spectra eliminated due to analysis cuts

- Timestamp cuts due to various aberrant run conditions and engineering studies
- Max. Std. Dev. Increase condition refers to normalized χ^2 of Padé filter fit

Cut Parameter	Scans Removed	Constraint
Timestamp cuts	7,189	N/A
Quality Factor	316	$10{,}000 < Q < 120{,}000$
System Noise	4,514	$0.1 < T_{sys} < 2.0$
Max Std. Dev. Increase	224	2.0
Error in filter shape	249	N/A

Rescan thresholds:

- Power excess in bin > 3σ
- Expected SNR for DFSZ axion < 2.4
- Limits set at that frequency do not meet required DFSZ sensitivity (measured power + candidate threshold power > DFSZ axion power
- Candidate power > DFSZ axion power + 0.5σ

Frequency (MHz)	Notes	Power (DFSZ)
780.255	Maximized off-resonance	1.49
730.195	Synthetic blind	1.51
686.310	Maximized off-resonance	2.36

Combining spectra into a Grand Spectrum

How to merge together data for the same frequency bin but acquired in different conditions?

- Every bin of every spectrum is divided by the relative signal power that would be generated due to the sample parameters and Lorentzian lineshape
- Discrepancies due to differing noise levels are mitigated by scaling the each bin by $k_B T_{sys}$
- Finally, the power in each bin is convolved with a filter given by the axion lineshape

Bins are combined via weighed mean, where the weights are such that they maximize the SNR . The weights are found by imposing that the Maximum Likelihood estimation of the true mean value μ of the power excesses is the same for all contributing bins

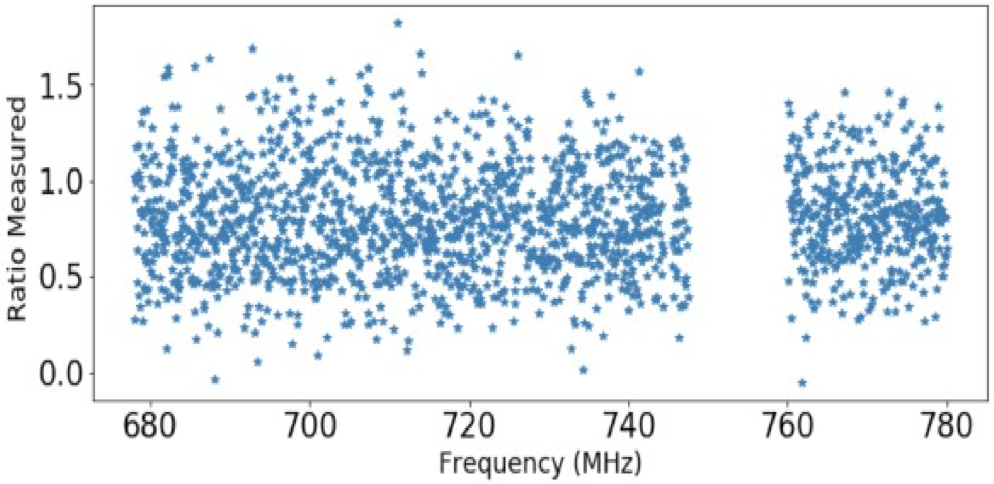
Since it is a weighted mean of indipendent Gaussian random variables with same mean, one finds:

$$P_w = \frac{\sum_{j=0}^{N} \frac{P^j_{scaled}}{\sigma j^2}}{\sum_{j=0}^{N} \frac{1}{\sigma j^2}} \qquad \sigma_w = \sqrt{\frac{1}{\sum_{j=0}^{N} \frac{1}{\sigma_j^2}}}$$

Synthetic hardware injections

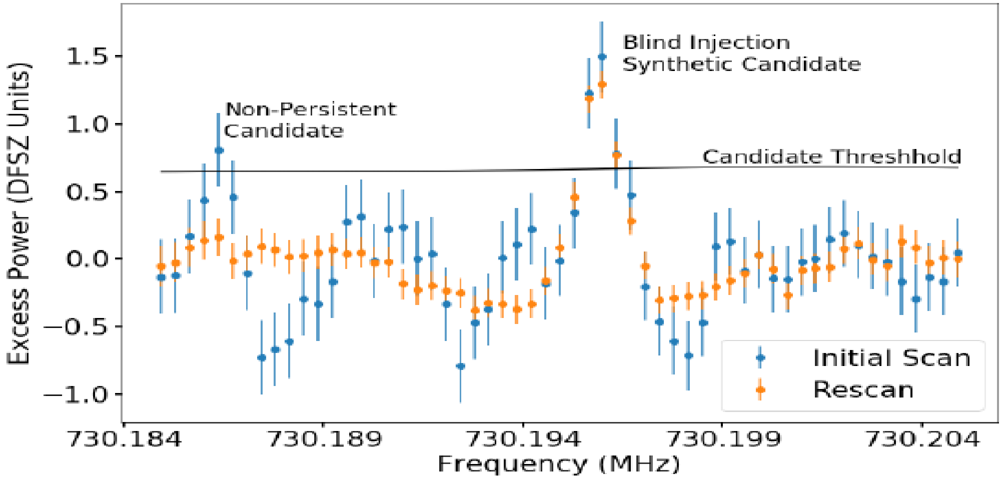
Software: 1773 evenly-spaced axion-like signals injected into real data

- Signals injected with DFSZ power levels
- Of these, **95 ± 2% detected**
- Mean power level differs from one → from ratio with expected value, we obtain estimate for efficiency of data treatment procedure: **0.818 ± 0.008**



Hardware: axion-like signals physically injected into the system

- Signals injected with DFSZ power levels to (previously calibrated) weakly coupled cavity port
- All signals detected

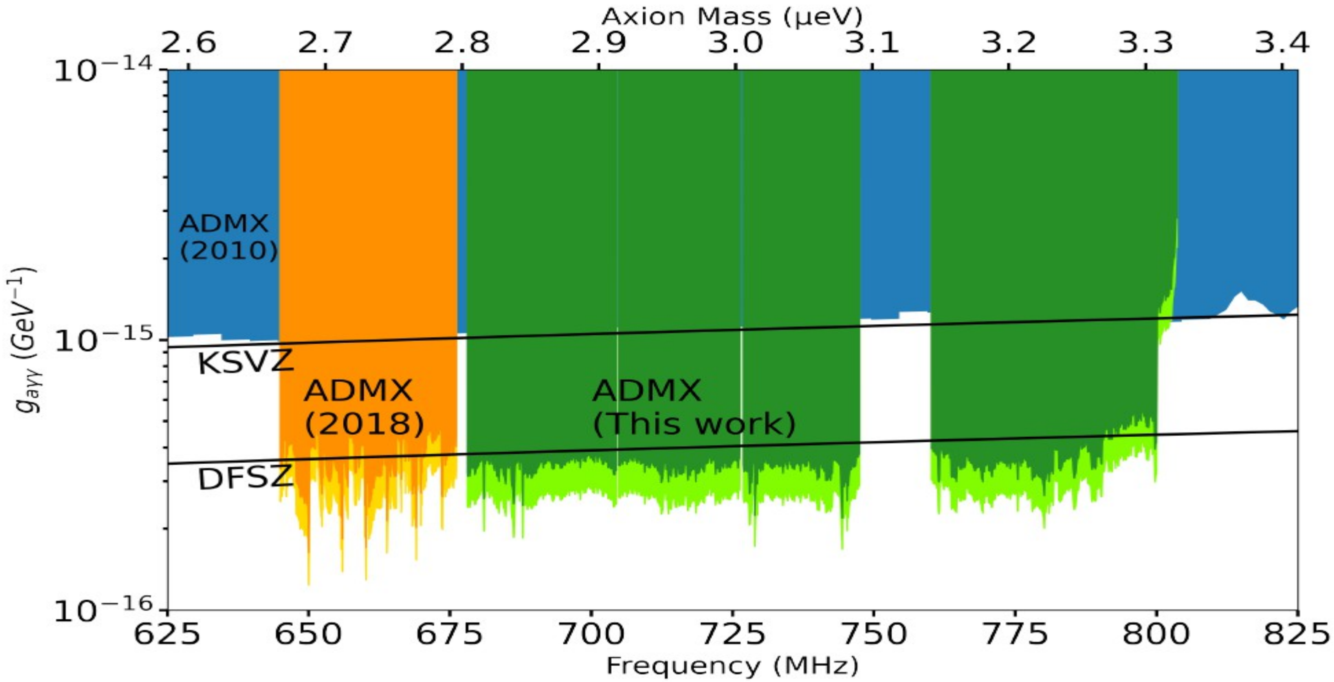


ADMX Run 1B results

Grand Spectrum bins: Gaussian variables of mean μ

- If no axion → $\mu=0$
- If axion present → $\mu=g_\gamma^2 \cdot SNR$
- A 90% C.L. single-sided exclusion limit is set by computing the value of g_γ for which $\mathbf{p(X_i<x_i|g_\gamma)<\alpha=0.1}$ (assuming X_i is gaussian random variable of mean μ and standard deviation σ_i)
- $\mathbf{g_\gamma^2<0}$ is non-physical! To take this into account, the cumulative distribution function for a truncated Gaussian was used
- Look-elsewhere effect?

  No need to correct!



References

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- "A Unified Approach to the Classical Statistical Analysis of Small Signals", G. J. Feldman and R. D. Cousins, Physical Review D 57, 3873–3889 (1998)
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Thanks for your attention!