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Probing entanglement via $t\bar{t}$ production at LHC

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- Entanglement: inseparable nature of states describing physical systems yielding nonlocal yet relativistically consistent correlations
- For bi-partite systems, separable states can always be written as

$$\rho = \sum_{i,j} p_{ij} |\psi_i^{(A)}\rangle\langle\psi_i^{(A)}| \otimes |\psi_j^{(B)}\rangle\langle\psi_j^{(B)}|$$

  with $p_{ij} \geq 0$ and $\sum_{ij} p_{ij} = 1$, providing "classical" interpretation
- States not in this form are called entangled or non-separable
- The structure of these correlations allows distinguishing local from nonlocal, as stated by Bell's theorem[1]

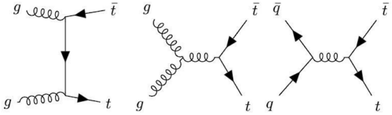
- For bi-partite systems, such as $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2$ or $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^3$, Peres-Horodecki criterion gives necessary and sufficient conditions such that a state is entangled[2, 3]
- For multi-partite systems, there are no general criteria yet but one can make use of entanglement witness that quantify the amount of entanglement
- In general, for bi-partite systems one can use
  1. von Neumann entropy $S[\rho] = -\text{Tr}(\rho_A \log(\rho_A)) = -\text{Tr}(\rho_B \log(\rho_B))$ where $\rho_{A,B} = \text{Tr}_{B,A}\rho$
  2. concurrence $C[\rho] = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) \in [0, 1]$, where λ_i are the eigenvalues, in decreasing order, of the hermitian matrix $R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ with $\tilde{\rho} = \sigma_2^{(A)} \otimes \sigma_2^{(B)} \rho^* \sigma_2^{(A)} \otimes \sigma_2^{(B)}$
- ...what is the fate of entanglement under the action of a Lorentz transformation?

- The general form of $\rho \in \mathcal{H} = \mathcal{H}_2^{(A)} \otimes \mathcal{H}_2^{(B)}$, with $\mathcal{H}_2 \equiv \mathbb{C}^2$ representing a single spin $1/2$ particle, is

$$\rho = \frac{1}{4}\left(I_4 + \sum_i B_i^+ \sigma_i^{(A)} \otimes I_2 + I_2 \otimes \sum_i B_i^- \sigma_i^{(B)} + \sum_{i,j} C_{ij} \sigma_i^{(A)} \otimes \sigma_j^{(B)}\right)$$

  where $i, j = 1, 2, 3$ and I_n representing the $n \times n$ identity matrix
- Given an observable $\hat{O}$, its expectation value is $\langle \hat{O} \rangle = \mathrm{Tr}[\rho \hat{O}]$
- $\boldsymbol{B}^{\pm}$ and C_{ij} will be functions of the parameters describing the kinematics, the Mandelstam variable $\sqrt{s}$ of the process and the scattering angle θ
- By measuring these 15 (or less, by using symmetries) expectation values, one can experimentally reconstruct the state $\longrightarrow$ quantum tomography

- At the LHC energies, $t\bar{t}$ pair arises from pp collisions, summarized by the processes $gg \to t\bar{t}$ ($\approx 90\%$ at $\sqrt{s} = 13$ TeV) and $q\bar{q} \to t\bar{t}$. At the leading order



- Why $t\bar{t}$? Top quarks have a lifetime ($\sim 10^{-25} s$) shorter than the time scale for hadronisation ($\sim 10^{-23} s$) and spin decorrelation ($\sim 10^{-21} s$)[4]
- $t\bar{t}$ pairs follow the chains[4] $t\bar{t} \to W^+ b\, W^- \bar{b} \to$
  - A) $q\bar{q}' b\, q'' \bar{q}''' \bar{b}$ (45.7%)
  - B) $q\bar{q}' b\, l^- \bar{\nu}_l \bar{b} + l^+ \nu_l b q'' \bar{q}''' \bar{b}$ (43.8%)
  - C) $l^+ \nu_l b\, l^- \bar{\nu}_l \bar{b}$ (10.5%)

- Leptons carry almost all the spin information of their parent top quarks $\longrightarrow$ this is the only channel considered
- The normalized differential cross-section of the process is [5]

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^+ d\Omega^-} = \frac{1 + \boldsymbol{B}^+ \cdot \boldsymbol{q}^+ - \boldsymbol{B}^- \cdot \boldsymbol{q}^- - \boldsymbol{q}^+ \cdot \boldsymbol{C} \cdot \boldsymbol{q}^-}{(4\pi)^2}$$

- From the kinematic reconstruction of each event, one can obtain the quantities $\boldsymbol{B}^\pm$ and C_{ij}, reconstruct the state and then calculate, for e.g., the concurrence $C[\rho]$
- Or, equivalently, one can experimentally measure the quantity D defined as [6] $D = -3\langle\cos\varphi\rangle$, with

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}\left(1 - D\cos\varphi\right)$$

where φ is the angle between the lepton directions in each one of the parent t and $\bar{t}$ rest frames

- There is a simple relation between D and C_{ij}...that is,

$$D = \frac{\text{Tr}[C_{ij}]}{3}$$

- From the Peres - Horodecki criterion, one derives the condition

$$\text{Tr}[C_{ij}] < -1,$$

  that is equivalent to $D < -\frac{1}{3}$

- The concurrence $C[\rho]$ can also be computed and it is given by

$$C[\rho] = \frac{1}{2}\text{max}(-1 - 3D, 0)$$

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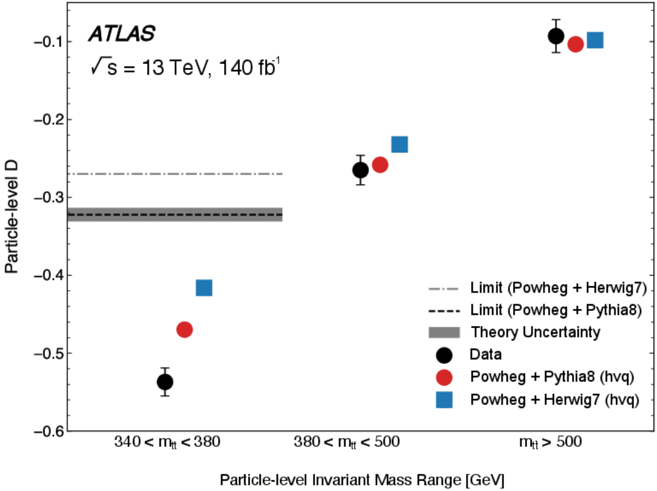
Using the data recorded during Run 2 at the LHC, with a centre of mass energy of $\sqrt{s} = 13$ TeV with an integrated luminosity of 140 fb^{-1}



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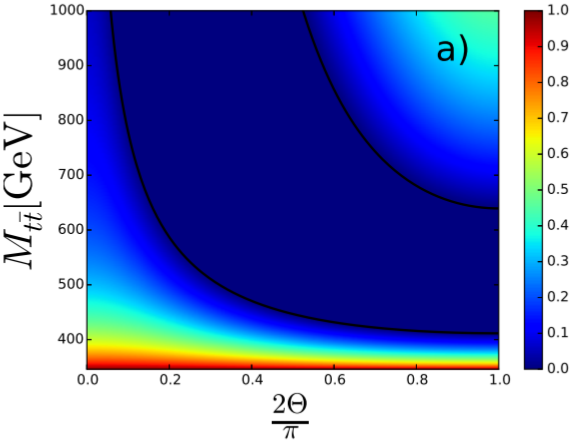
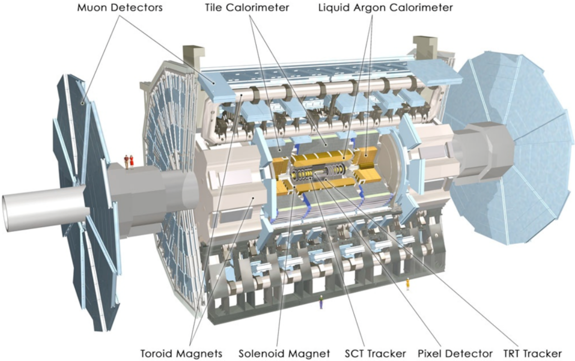
By reconstructing the state with experimental data, one reproduces the concurrence [7]



Atlas detector is made of 4 main parts (see the figure below): *Inner detector*, *Calorimeter*, *Magnet System* and *Muon Spectrometer*



- The Inner Detector measures the direction, momentum, and charge of electrically-charged particles
- It reconstructs the tracks of charged particles from hits in the silicon pixel, microstrip, and straw-tube layers. The curvature of their tracks in the 2 T magnetic field is used to find the transverse momentum and charge
- Trivially... it takes the hits (r_i, ϕ_i, z_i), reconstructs $r(\phi) = R$ and $z(\phi) = R \cot\theta \cdot \phi$ where θ is the angle with the z-axis. From R, we estimate p_T (transverse momentum), while from θ we estimate $p_z = p_T \cot\theta$. Usually, experimentalists use the pseudo-rapidity $\eta = -\ln\tan(\theta/2)$

- The calorimeter measures particle energy through ionization and radiative processes such as bremsstrahlung and pair production, which together produce the characteristic electromagnetic or hadronic showers in dense absorber materials
- Typically, electrons, photons, and hadrons lose all their energy inside the calorimeters
- For muons, bremsstrahlung is negligible. Radiative losses scale as $-dE/dx \propto E/m^2$, so

$$\frac{(dE/dx)^{\mu}}{(dE/dx)^{e}} \simeq \left(\frac{m_e}{m_\mu}\right)^2 \sim 10^{-5}$$

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- Located outside the calorimeters, the muon spectrometer measures the momentum of muons that escape the inner detector.
- Based on the same principles as the inner detector, it reconstructs their curved trajectories in a large toroidal magnetic field (3.5T)
- The Muon Spectrometer constitutes the outermost layer of the detector. Anyway, *neutrinos* cannot be detected directly, their presence is inferred from the missing momentum among detected particles

Conclusions and Outlook

- The study of spin correlations in $t\bar{t}$ production establishes a direct link between *quantum information* and *high energy physics*, allowing the exploration of quantum entanglement in this high energy regime
- The *top quark* is an ideal probe: it decays before hadronizing, so its spin information is preserved and transferred to its decay products
- The *quantum state tomography* is a fundamental tool, as it allows the reconstruction of the full spin density matrix ρ of the $t\bar{t}$ system. From ρ, all entanglement properties can be derived and quantified using the various criteria discussed above
- Future work may explore new directions... but let's keep it a secret for now :D

Thank you! :D

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