Micro-and Nano-Structured Materials

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DSFTA

- Materials engineered for total reflection, guided propagation, volume confinement in a defined range of frequencies

- Application fields: ultra compact laser system, high speed computing in purely optical circuits, advanced spectroscopy, enanced sensors working on different energy scales or more efficient solar cells systems

- Why micro and nano materials?

Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

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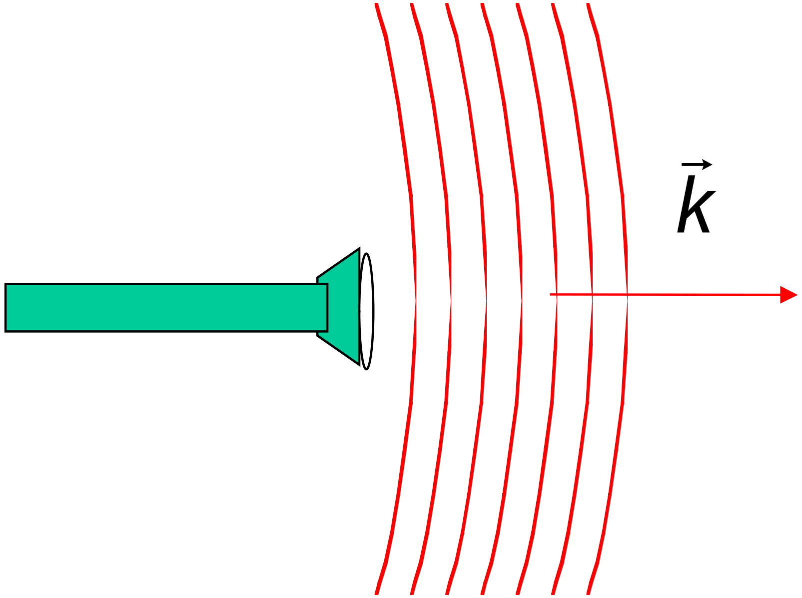
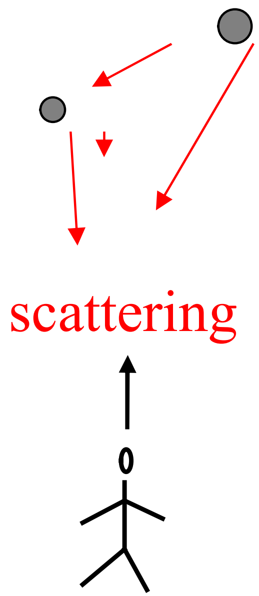
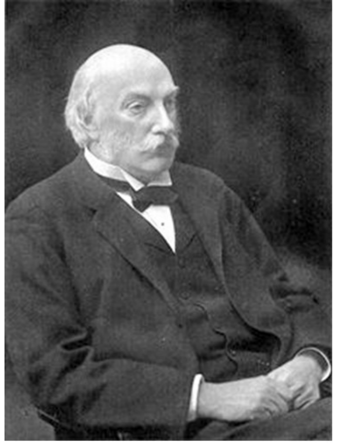


planewave

$$\vec{E}, \vec{H} \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

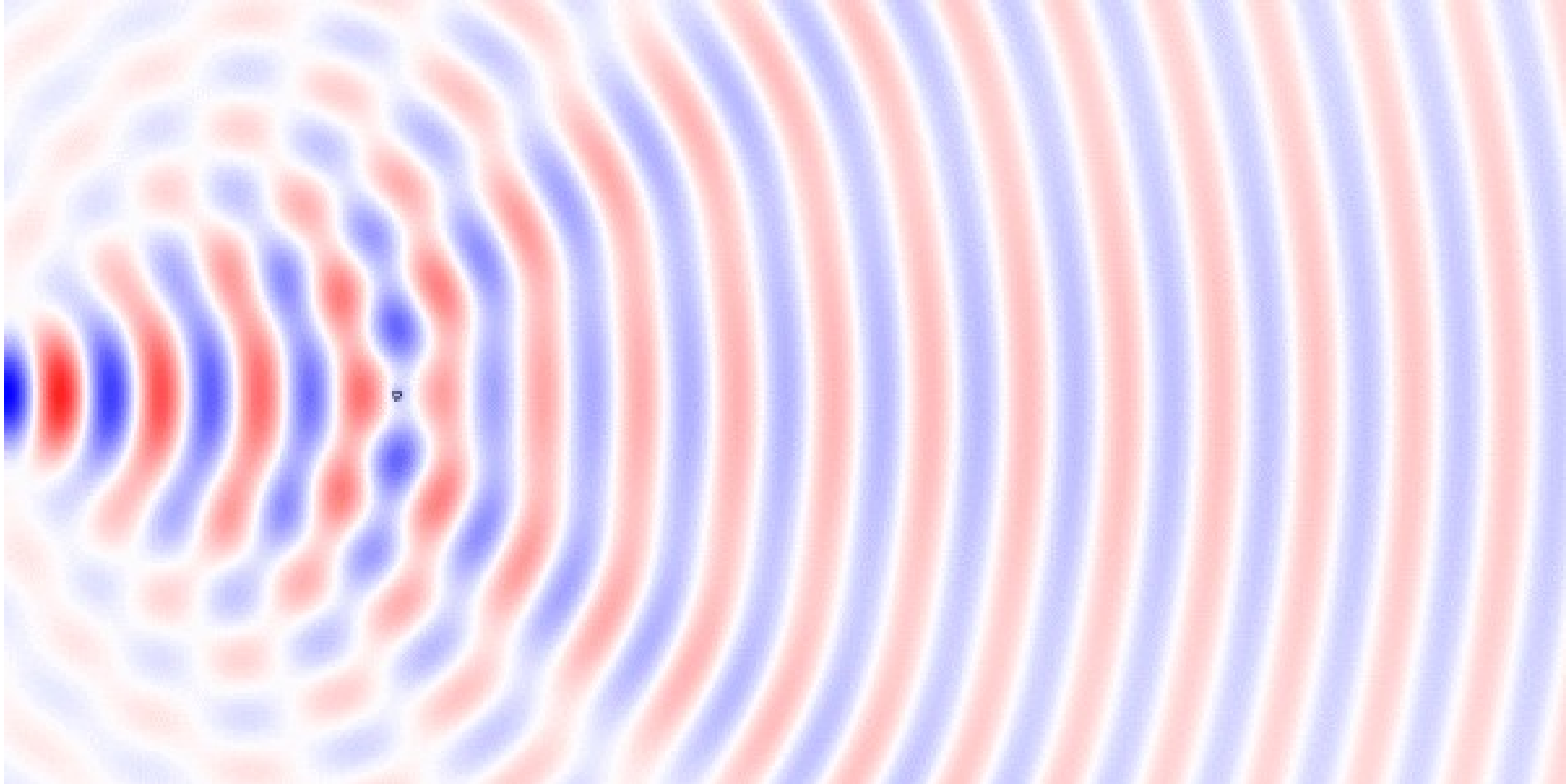




small particles:
Lord Rayleigh (1871)
why the sky is blue

…Waves Can Scatter

here: a little circular speck of silicon

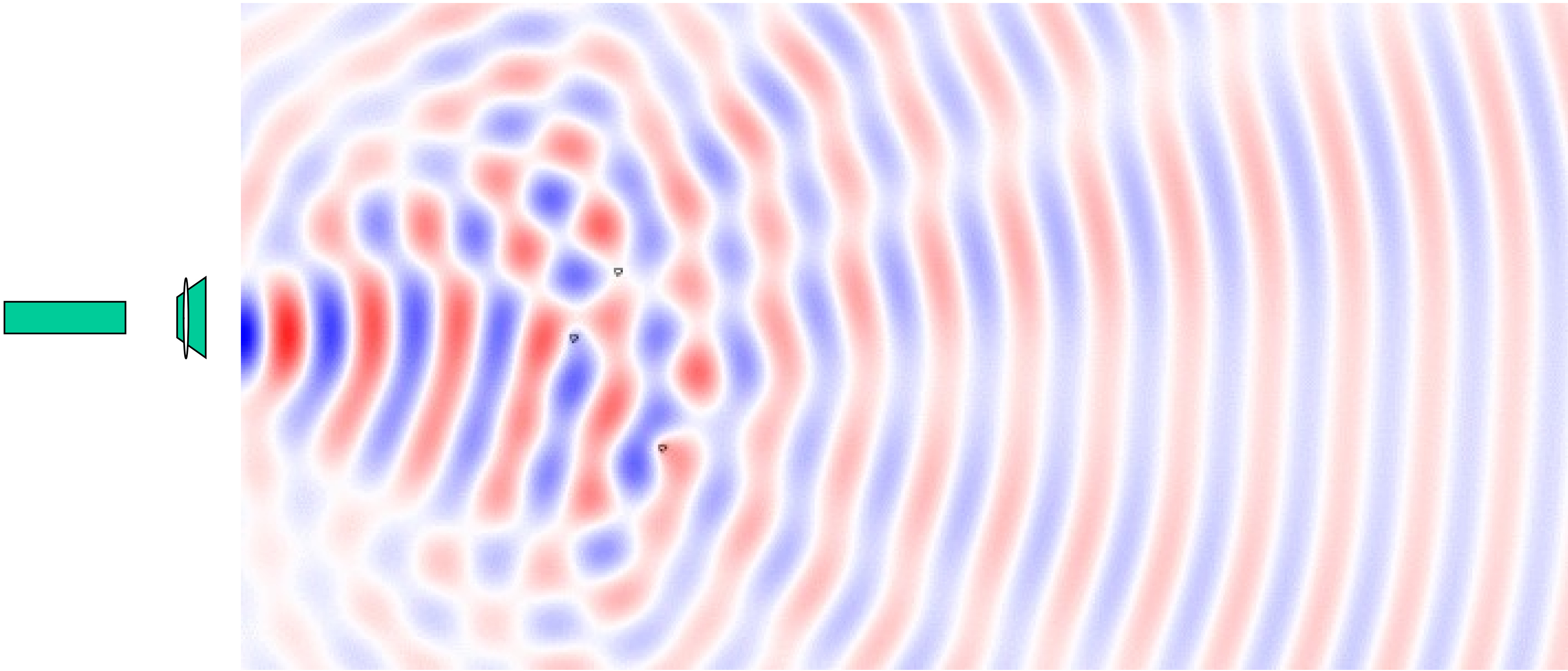


checkerboard pattern: interference of waves traveling in different directions

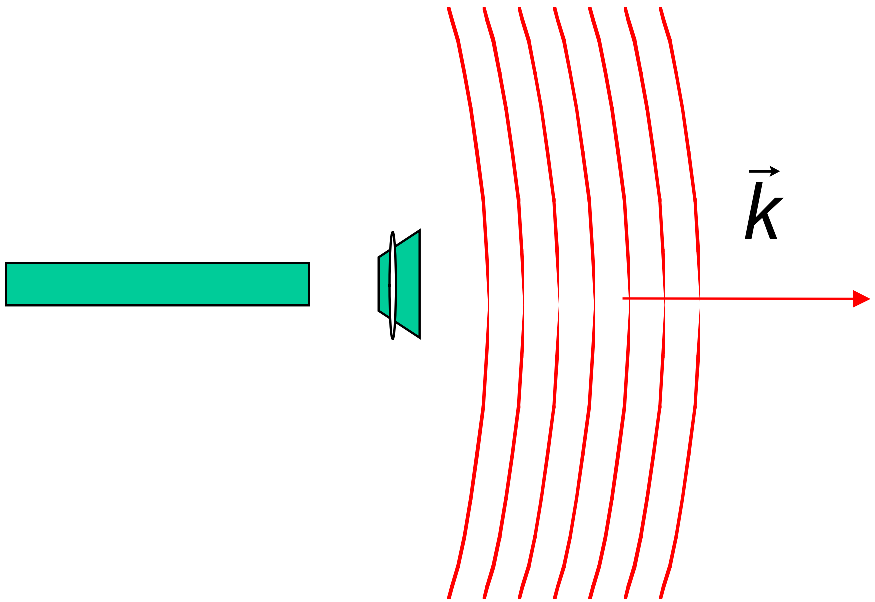
scattering by spheres:
solved by Gustave Mie (1908)

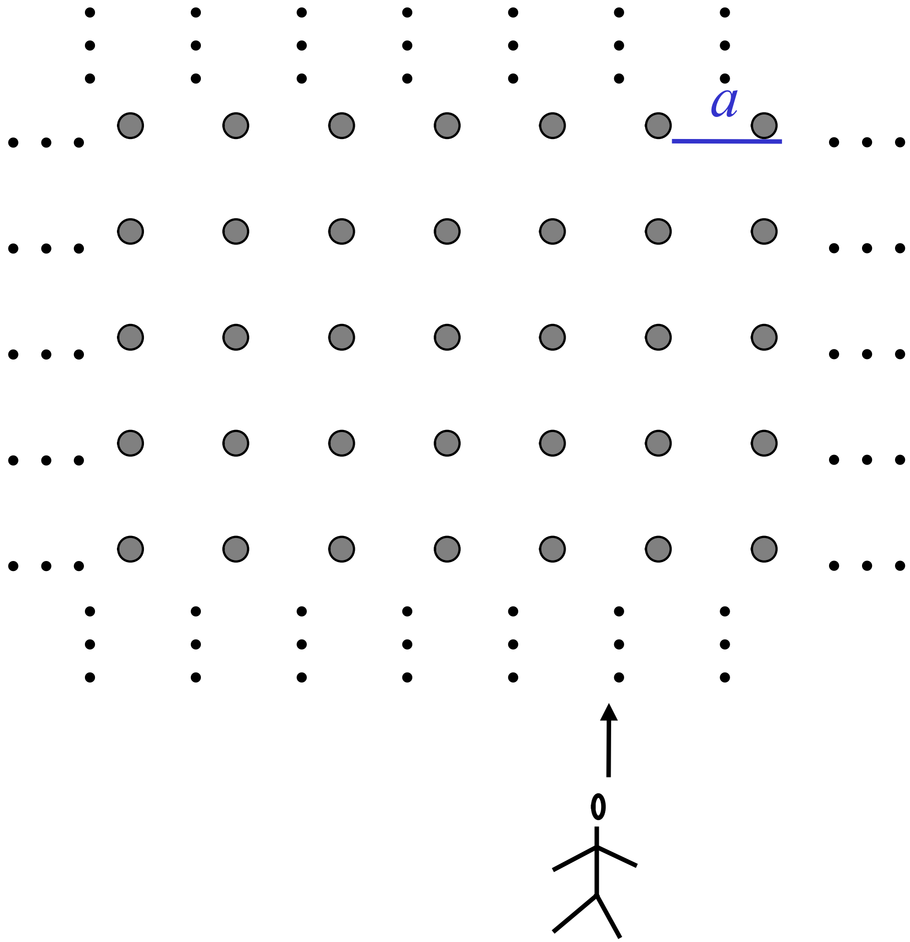
Multiple Scattering

here: scattering off three specks of silicon



can be solved on a computer, but not terribly interesting…





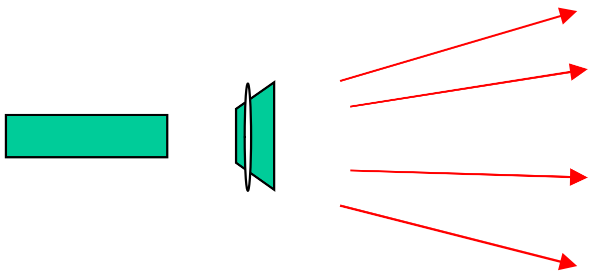
planewave

$$\vec{E}, \vec{H} \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

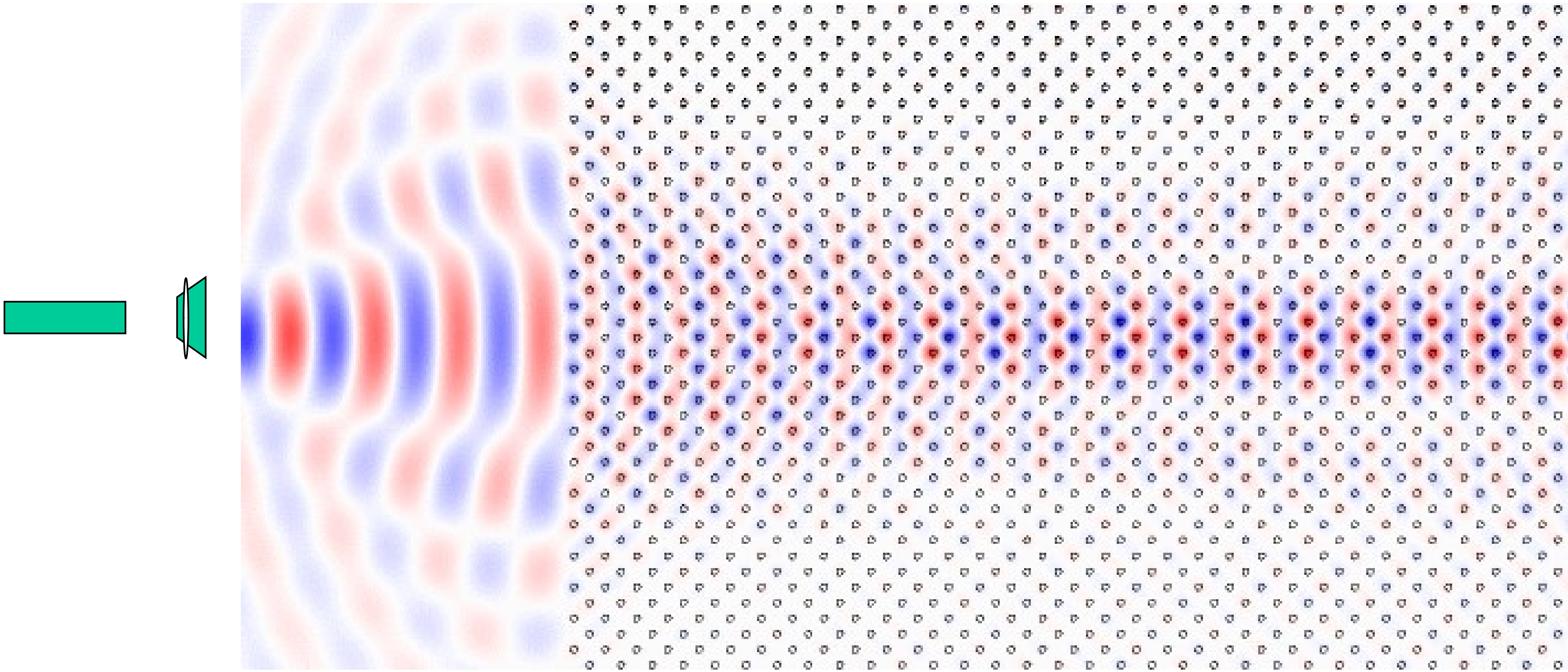
for most λ, beam(s) propagate through crystal without scattering (scattering cancels coherently)

...but for some λ (~ 2a), no light can propagate: a photonic band gap increasing the number of scatterers



... light will just scatter like crazy
and go all over the place …

Not so messy....



the light seems to form several *coherent beams*
that propagate *without scattering*
… and almost *without diffraction* (*supercollimation*)

…the magic of symmetry…

[Emmy Noether, 1915]

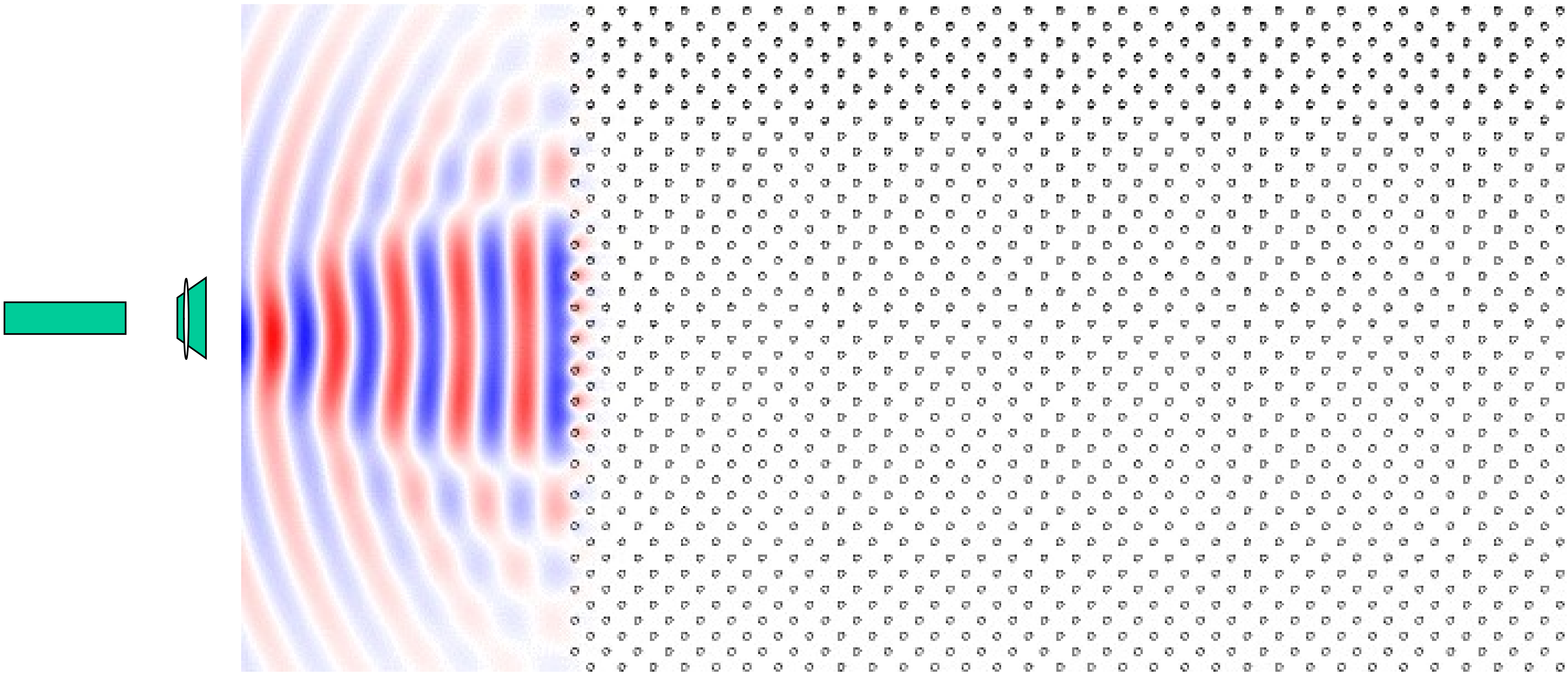
Noether's theorem:
symmetry = conservation laws

In this case, periodicity
= conserved "momentum"
= wave solutions without scattering
[Bloch waves]

Felix Bloch
(1928)

A slight change? Shrink λ by 20%

an "optical insulator" (photonic bandgap)

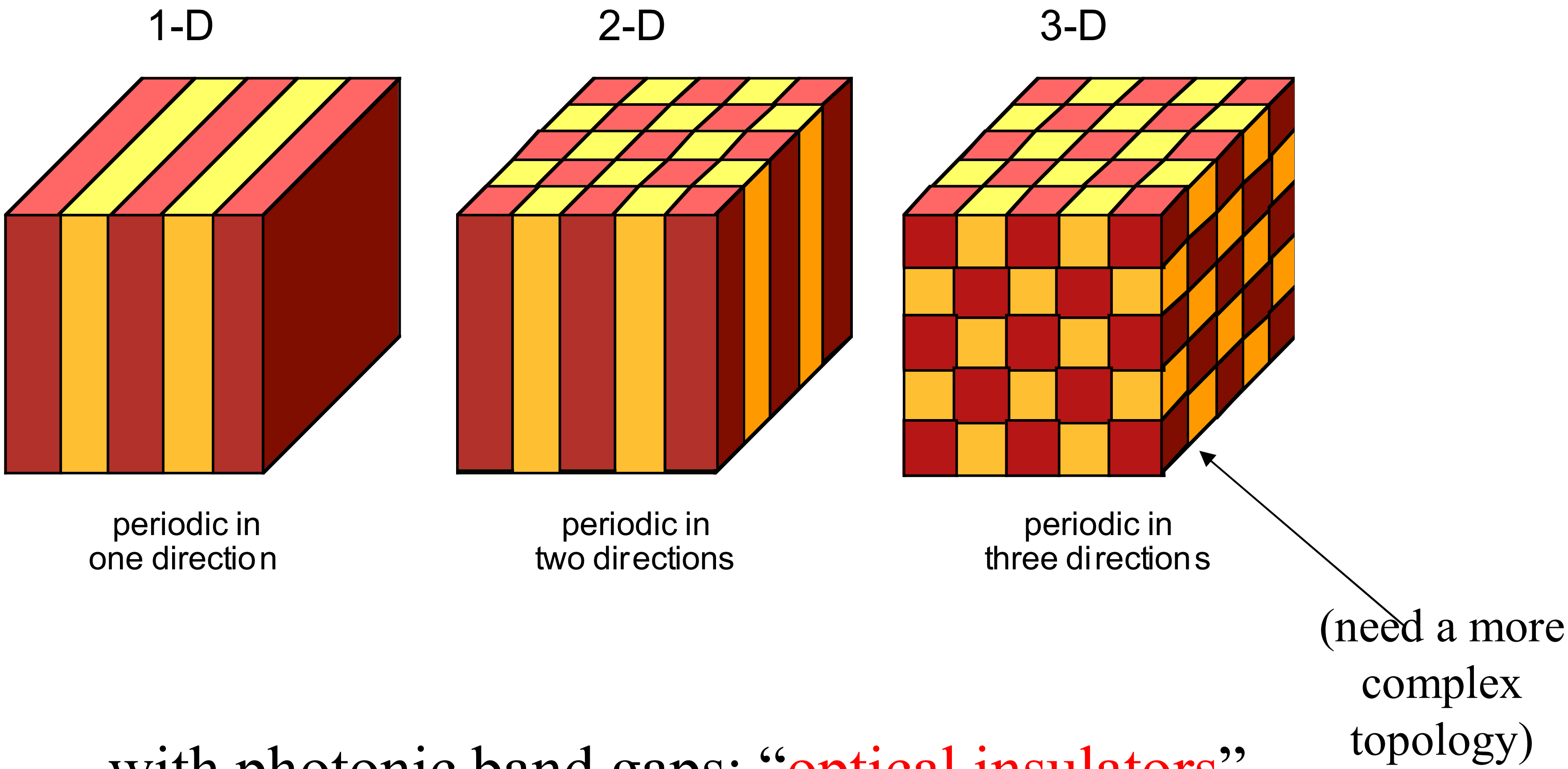


light cannot penetrate the structure at this wavelength!

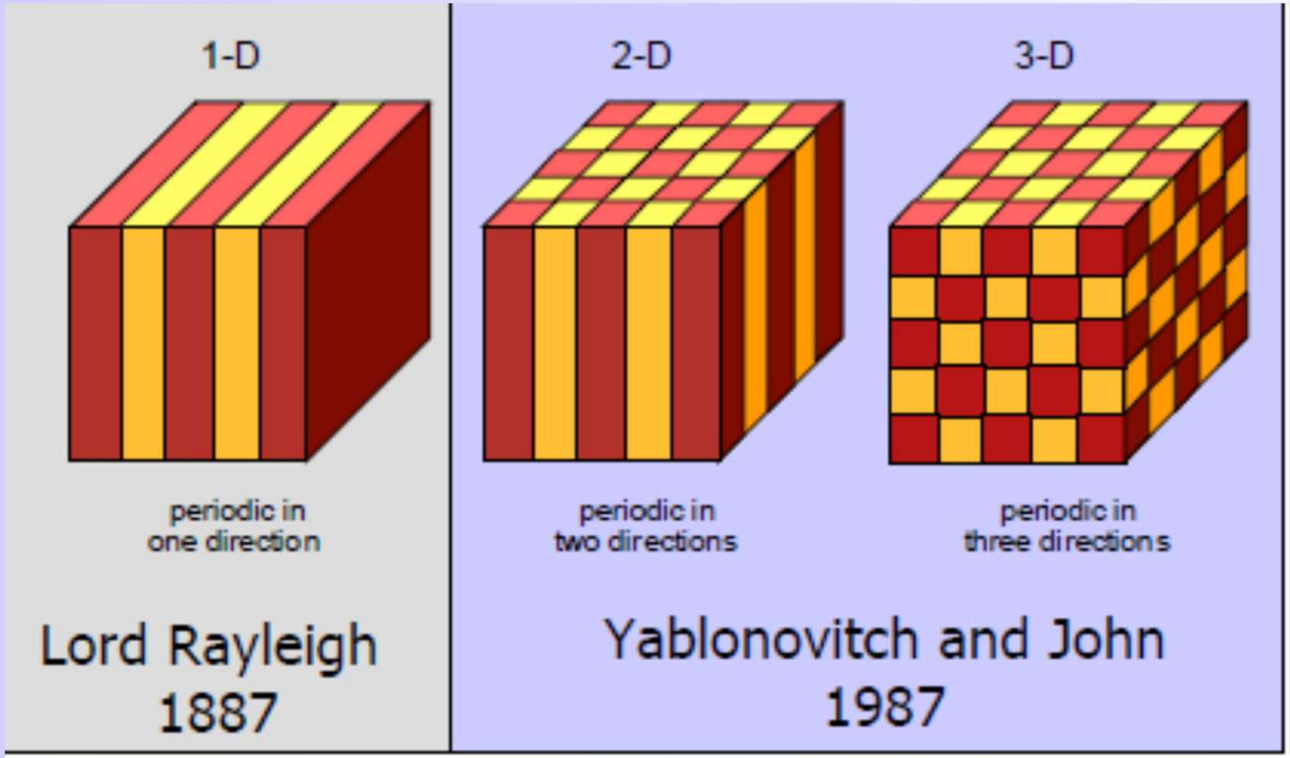
all of the scattering destructively interferes

Photonic Crystals

periodic electromagnetic media

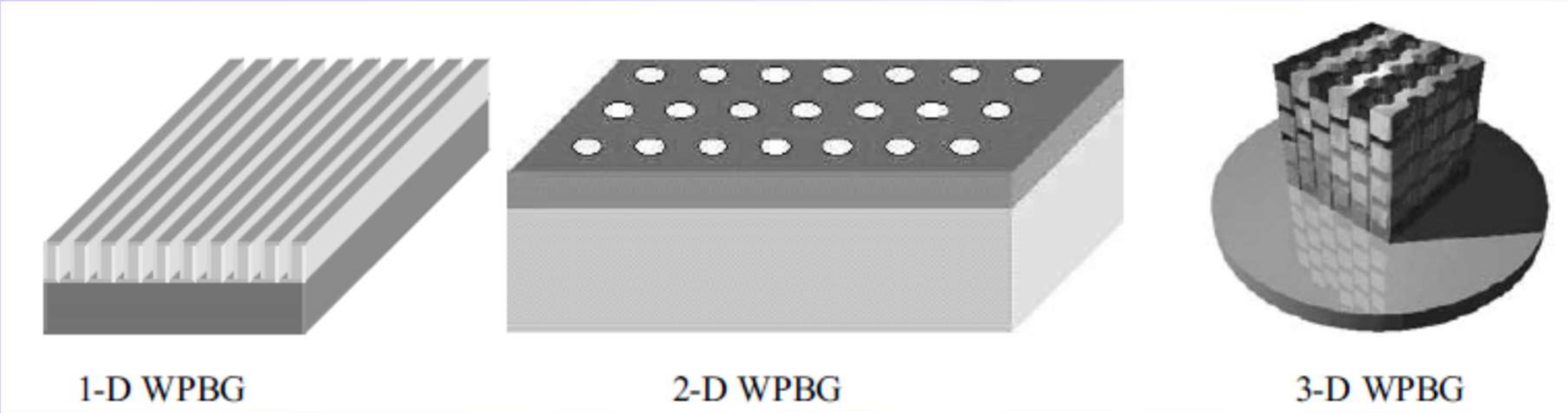


with photonic band gaps: "optical insulators"

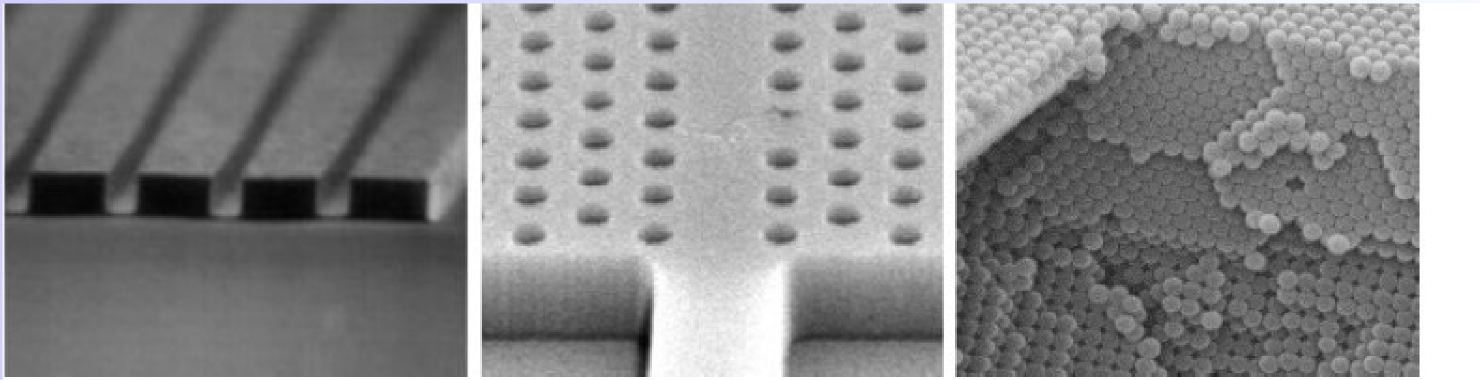


Lord Rayleigh, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," Philosophical Magazine **24**, 145–159 (1887)
E. Yablonovitch, "Inhibited spontaneous emission in solid-state physics and electronics", Phys. Rev. Lett.**58**, 2059 (1987)
S. John, "Strong localization of photons in certain disordered dielectric superlattices", Phys. Rev. Lett.**58**, 2486 (1987)

PC Structures



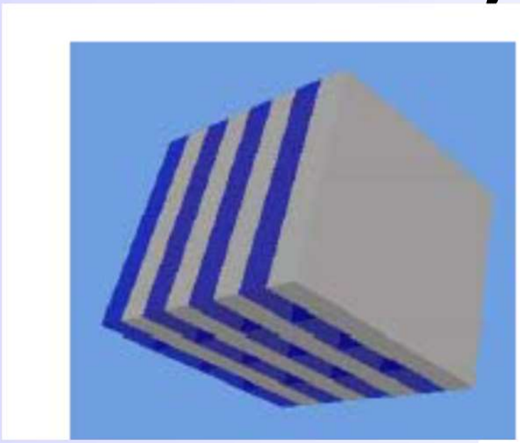
- Waveguiding 1-D, 2-D, 3-D

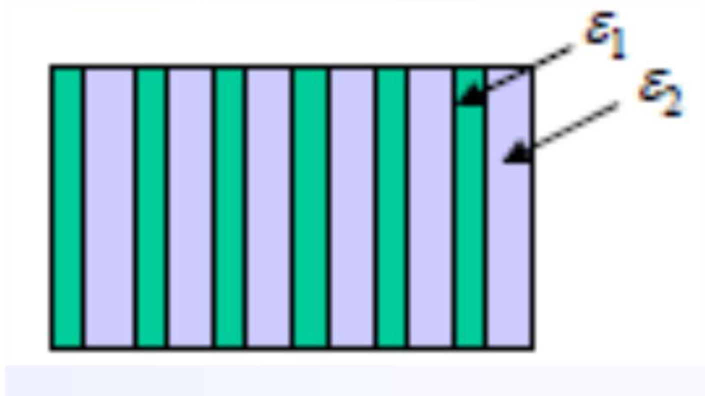


Photonic crystals 1-D, 2-D,3-D

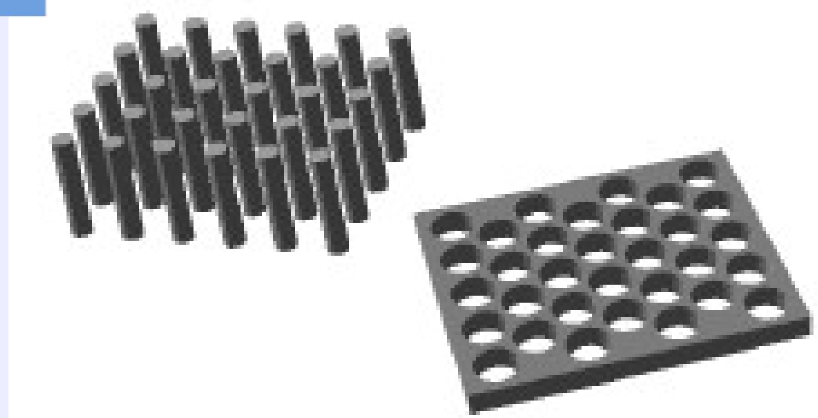
- Bulk

1-D: Bragg gratings

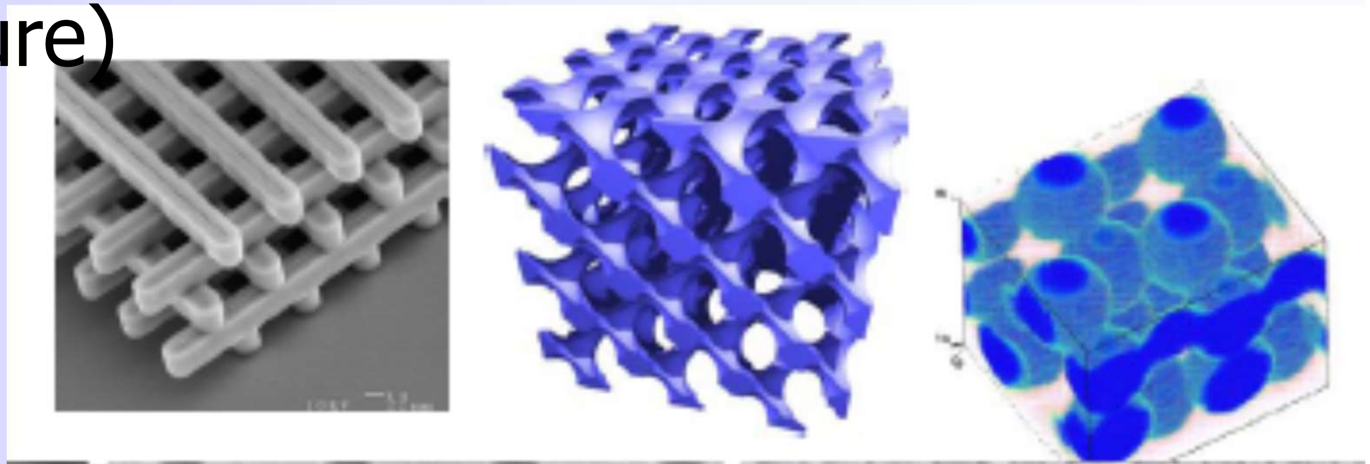




2-D: PhC Slab Silicon on insulator



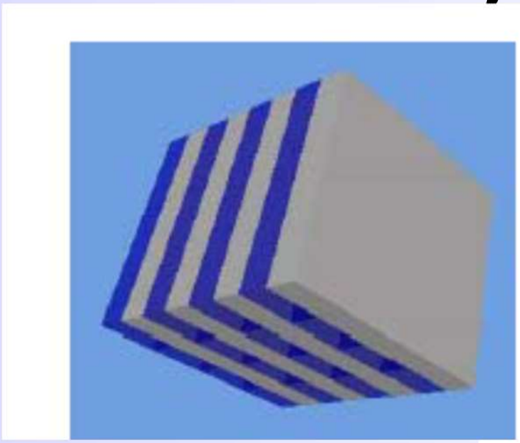
3-D: Yablonovite, woodpile structure, opal and (inverse opal structure)

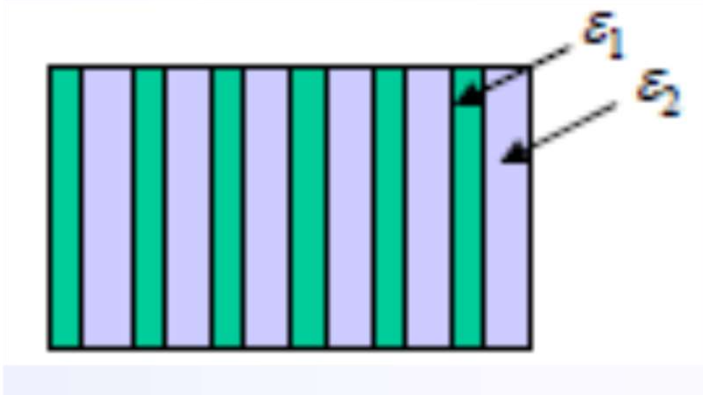


Photonic crystals 1-D, 2-D,3-D

- Bulk

1-D: Bragg gratings





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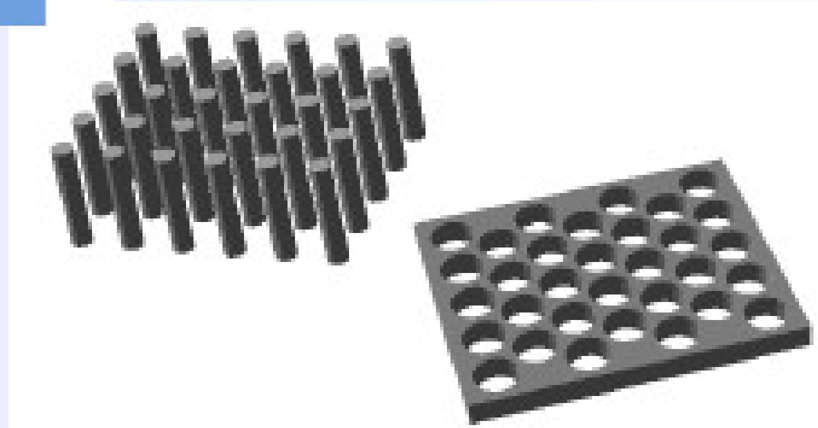

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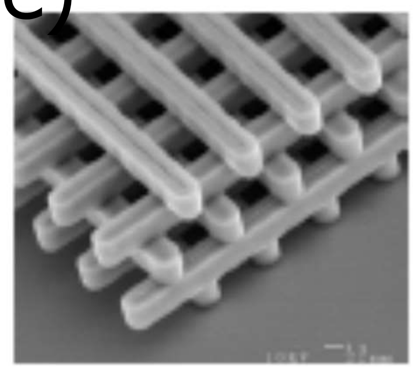
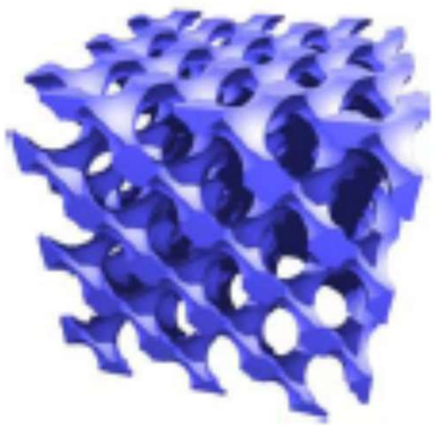
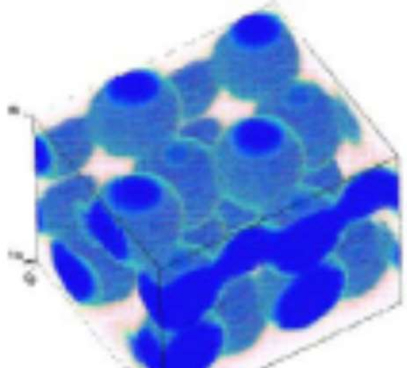




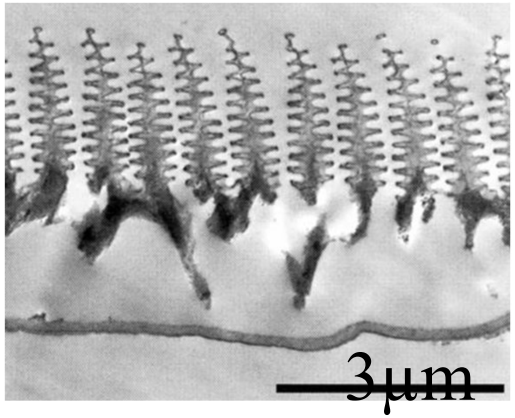


Photonic Crystals in Nature

Morpho rhetenor butterfly

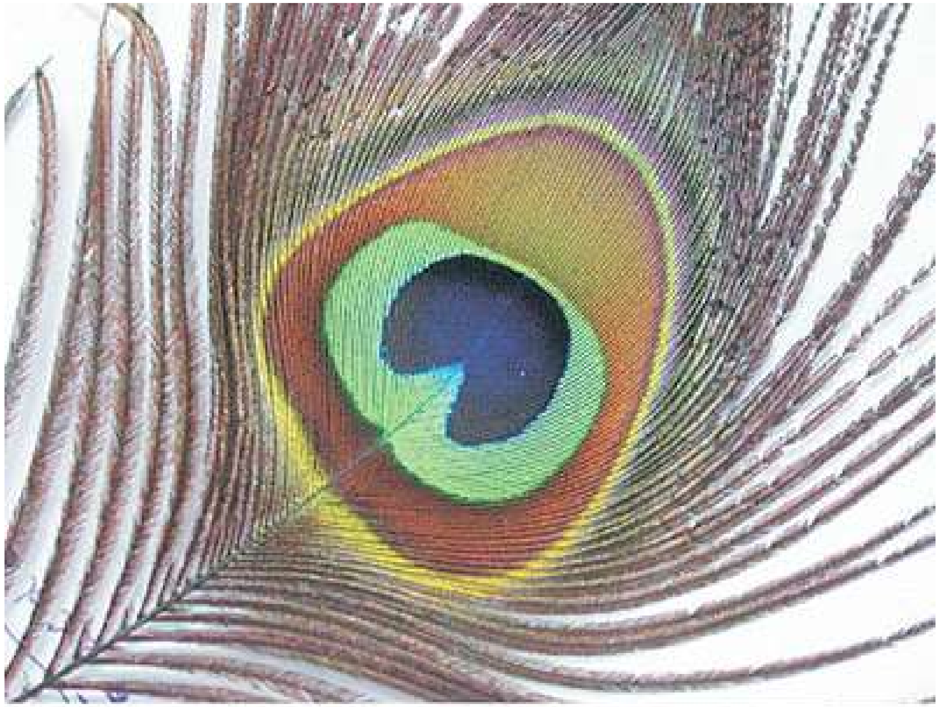
wing scale:

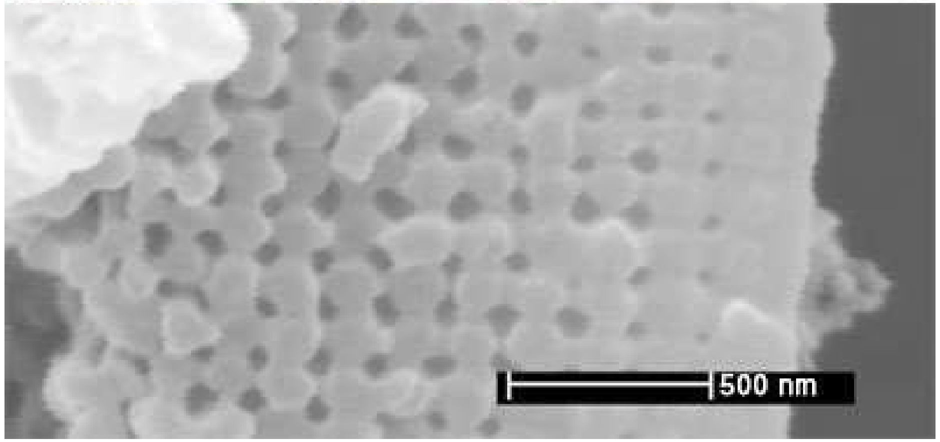
[P. Vukosic *et al.*, *Proc. Roy. Soc: Bio. Sci.* **266**, 1403 (1999)]



[also: B. Gralak *et al.*, *Opt. Express* **9**, 567 (2001)]

Peacock feather

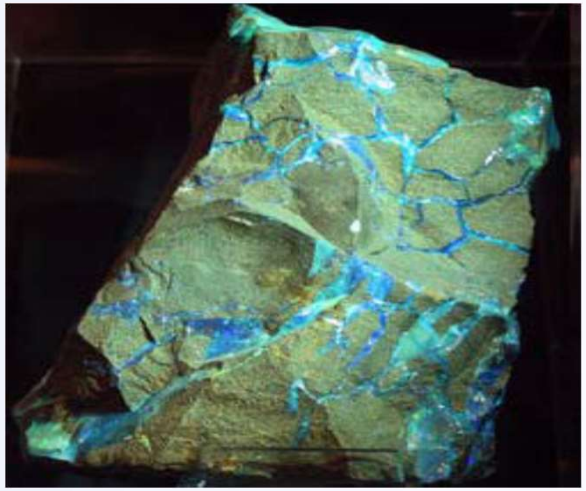


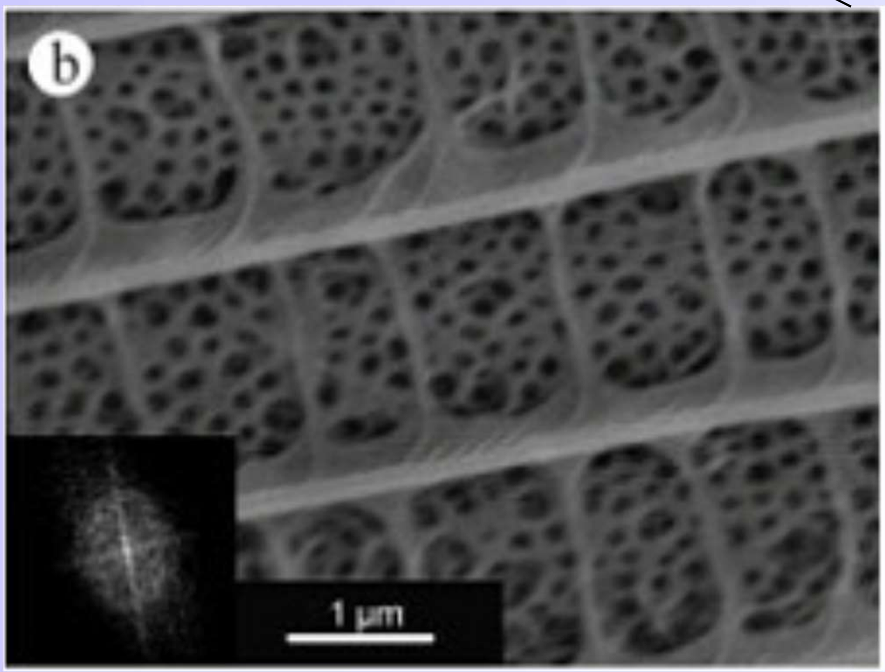


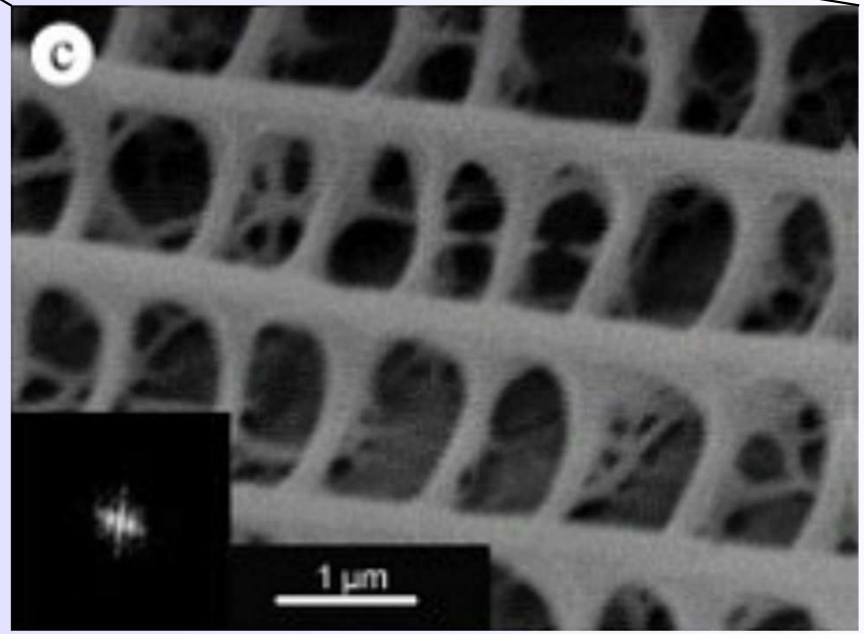
[J. Zi *et al*, *Proc. Nat. Acad. Sci. USA*, **100**, 12576 (2003)]

[figs: Blau, *Physics Today* **57**, 18 (2004)]

Photonic Crystals in Nature

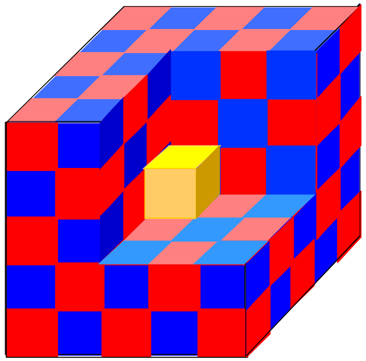




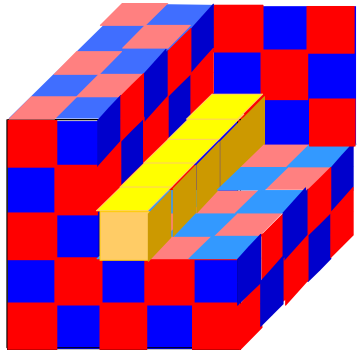


Photonic Crystals

periodic electromagnetic media



can trap light in cavities

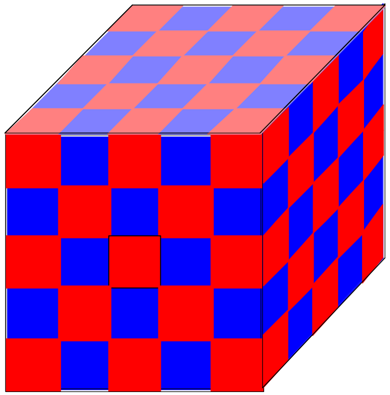


and waveguides ("wires")

with photonic band gaps:
"optical insulators"
for holding and controlling light

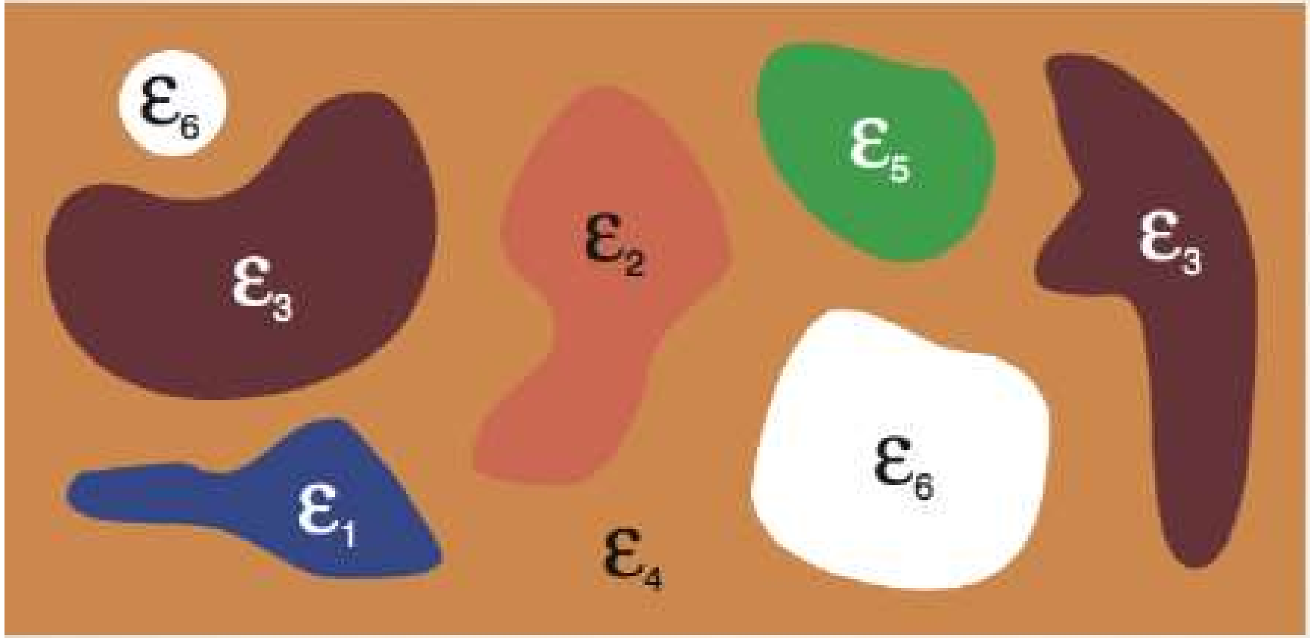
Photonic Crystals

periodic electromagnetic media



But how can we understand such complex systems?
Add up the infinite sum of scattering?

Electromagnetism in mixed dielectric media



$\varepsilon_0 \approx 8.854 \times 10^{-12}$ Farad/m

$\mu_0 = 4\pi \times 10^{-7}$ Henry/m

$$n = \sqrt{\varepsilon\mu}$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0 \qquad \nabla \times \mathbf{E}(\mathbf{r}, t) + \mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} = 0$$

$$\nabla \cdot [\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}, t)] = 0 \qquad \nabla \times \mathbf{H}(\mathbf{r}, t) - \varepsilon_0 \varepsilon(\mathbf{r}) \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} = 0.$$

$$\mathbf{B}(\mathbf{r}) = \mu_0 \mu(\mathbf{r}) \mathbf{H}(\mathbf{r})$$

$$\mathbf{D}(\mathbf{r}) = \varepsilon_0 \varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r})$$

Math Model

Maxwell Equations

in linear regimes, in a macroscopic isotropic loseless material a linear isotropic medium, in absence of free current and charge densities.

$$\begin{cases} \nabla \cdot \mathbf{H}(\mathbf{r},t) = 0 \\ \nabla \cdot \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t) = 0 \\ \nabla \times \mathbf{E}(\mathbf{r},t) = -\mu_0 \dfrac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} \\ \nabla \times \mathbf{H}(\mathbf{r},t) = \varepsilon_0 \varepsilon(\mathbf{r}) \dfrac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} \end{cases}$$

Harmonic modes

$$\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})e^{i\omega t} \qquad \mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})e^{i\omega t}$$

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -i\omega\mu_0 \mathbf{H}(\mathbf{r},t) \qquad \nabla \times \mathbf{H}(\mathbf{r},t) = i\omega\varepsilon_0\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t)$$

Master equation

$$\nabla \times \left(\frac{\nabla \times \mathbf{H}(\mathbf{r})}{\varepsilon_r(\mathbf{r})} \right) = \left(\frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r}) \quad \Longrightarrow \quad \Theta \mathbf{H}(\mathbf{r}) = \left(\frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r})$$

Linear Hermitian operator $\Theta = \nabla \times \left(\frac{\nabla \times}{\varepsilon_r(\mathbf{r})} \right)$

eigenvectors $\mathbf{H}(\mathbf{r})$ orthogonal complete set

Eigenvalues real numbers $\left(\frac{\omega}{c} \right)^2$

Harmonic Modes

- Because the linearity of the Maxwell equations we can separate the time dependence from the spatial dependence

$$\mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})e^{-i\omega t}$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})e^{-i\omega t}$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}) = 0 \qquad \nabla \cdot [\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r})] = 0$$

H and E are **transverse** waves

$$\nabla \times \left(\frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = \left(\frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r})$$

Master Equation

- ε(**r**) —> **H** from **master equation** and then **E** from Maxwell equations

Electromagnetism as Eigenvalue Problem

- The master equation can be seen as an eigenvalue problem.

$$\hat{\Theta}\mathbf{H}(\mathbf{r}) = \left(\frac{\omega}{c}\right)^2 \mathbf{H}(\mathbf{r})$$

$$\hat{\Theta}\mathbf{H}(\mathbf{r}) \triangleq \nabla \times \left(\frac{1}{\varepsilon(\mathbf{r})}\nabla \times \mathbf{H}(\mathbf{r})\right)$$

- The eigen vectors $\mathbf{H}(\mathbf{r})$ are the spatial patterns of the harmonic mode, and the eigen values are proportional to the squared frequencies.

Properties of Θ

- Θ satsifies the following properties:

  ✓ Linear

  ✓ Hermitian

$$(\mathbf{F},\mathbf{G}) \triangleq \int d^3\mathbf{r}\,\mathbf{F}^*(\mathbf{r})\cdot\mathbf{G}(\mathbf{r}) \quad \Rightarrow \quad (\mathbf{F},\hat{\Theta}\mathbf{G}) = (\hat{\Theta}\mathbf{F},\mathbf{G})$$

  ✓ Positive semi-definite. Hence the eigenvalues ω^2 are real and nonnegative

Hermitian Eigenproblems

$$\underbrace{\nabla \times \frac{1}{\varepsilon} \nabla \times}_{\text{eigen-operator}} \underbrace{\vec{H}}_{\text{eigen-state}} = \underbrace{\left(\frac{\omega}{c}\right)^2}_{\text{eigen-value}} \vec{H}$$

+ constraint

$$\nabla \cdot \vec{H} = 0$$

Hermitian for real (lossless) ε

→ well-known properties from linear algebra:

ω are real (lossless)
eigen-states are orthogonal
eigen-states are complete (give all solutions)*

* Technically, completeness requires slightly more than just Hermitian-ness.

Electromagnetic Energy and the Variational Principle

- **Variational Theorem:** the smallest eigenvalue ω_0^2/c^2 corresponds to the field pattern that minimizes the functional:

$$U_f(\mathbf{H}) \triangleq \frac{(\mathbf{H}, \hat{\Theta}\mathbf{H})}{(\mathbf{H}, \mathbf{H})}$$

Rayleigh quotient

- ω_0^2/c^2 is the minimum of U over all conceivable field patterns H (subject to the transversality constrain)

Electromagnetic Energy and the Variational Principle

- Using the Maxwell equations the Rayleigh functional can be written in this way:

$$U_f(\mathbf{H}) = \frac{\int d^3\mathbf{r}\, |\nabla \times \mathbf{E}(\mathbf{r})|^2}{\int d^3\mathbf{r}\, \varepsilon(\mathbf{r})|\mathbf{E}(\mathbf{r})|^2}$$

E tends to concentrate its electric field energy in regions of high dielectric constant

- The energy functional must be distinguished from the physical energy

$$U_{\mathbf{E}} \triangleq \frac{\varepsilon_0}{4} \int d^3\mathbf{r}\, \varepsilon(\mathbf{r})|\mathbf{E}(\mathbf{r})|^2$$

$$U_{\mathbf{H}} \triangleq \frac{\mu_0}{4} \int d^3\mathbf{r}\, |\mathbf{H}(\mathbf{r})|^2$$

$$\mathbf{S} \triangleq \frac{1}{2} \mathrm{Re}\left[\mathbf{E}^* \times \mathbf{H}\right]$$

Properties and symmetries

- Scaling properties: the master equation is scale invariant. ε(r), H(r), ω —> ε(r'/s), H(r'/s), ω/s
- Continous translational symmetry ε(r-d)=ε(r):

$$\hat{T}_{d\hat{\mathbf{z}}} e^{ikz} = e^{ik(z-d)} = (e^{-ikd}) e^{ikz}$$

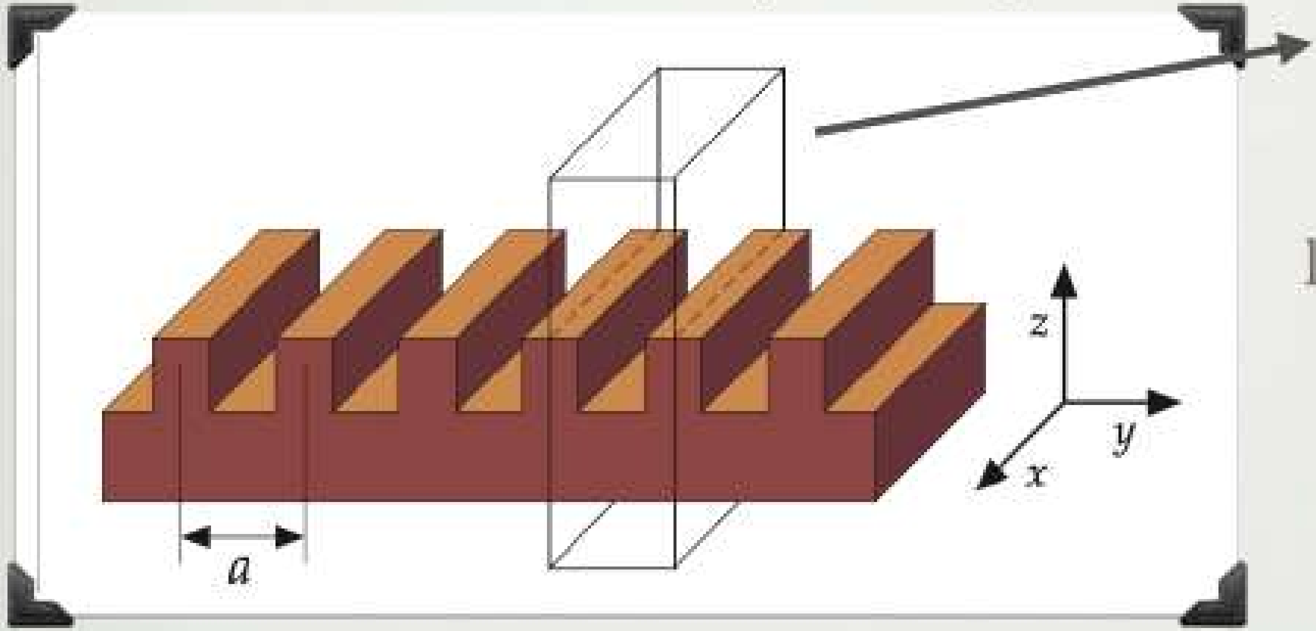
exp(ikz) is the eigenfunction and exp(-ikd) is the eigen value. k is conserved along the symmetry direction. In the case of homegeneus medium (ε(r)=cost.):

$$\mathbf{H}_{\mathbf{k}}(\mathbf{r}) = \mathbf{H}_0 e^{i\mathbf{k}\cdot\mathbf{r}} \quad \longrightarrow \quad \omega = c|\mathbf{k}|/\sqrt{\varepsilon}.$$

Dispersion Relation

Properties and symmetries

- Discrete translational symmetry



Primitive lattice vector: $\mathbf{a}=a\hat{\mathbf{y}}$

$\varepsilon(\mathbf{r})=\varepsilon(\mathbf{r}+\mathbf{R})$
$\mathbf{R}=l*\mathbf{a}$

$$\hat{T}_{\mathbf{R}}e^{ik_y y} = e^{ik_y(y-\ell a)} = (e^{-ik_y \ell a})e^{ik_y y}$$

$\mathbf{b}=b\hat{\mathbf{y}}$, $b=2\pi/a$ Primitive reciprocal lattice vector

$k_y \longrightarrow k_y+(2\pi/a)*m$ set of degenerate modes

Bloch's theorem

- The periodicity in the y direction leads to a particular form of the field H:

$$\mathbf{H}(\ldots, y, \ldots) \propto e^{ik_y y} \cdot \mathbf{u}_{k_y}(y, \ldots)$$

where u is a periodic function of y. u(y)=u(y+l*a). Also ω must be periodic:

$-\pi/a < k_y \leq \pi/a$

$\omega_n(k_y) = \omega_n(k_y + m*b)$

Brillouin zone

Properties and symmetries

- Rotational symmetry:

$$\hat{\Theta}(\hat{O}_{\mathcal{R}}\mathbf{H}_{\mathbf{k}n}) = \hat{O}_{\mathcal{R}}(\hat{\Theta}\mathbf{H}_{\mathbf{k}n}) = \left(\frac{\omega_n(\mathbf{k})}{c}\right)^2 (\hat{O}_{\mathcal{R}}\mathbf{H}_{\mathbf{k}n})$$

$$\omega_n(\mathcal{R}\mathbf{k}) = \omega_n(\mathbf{k})$$

the smallest region within the Brillouin zone for which $\omega_n(k)$ are not related by symmetry is called the **irridducible Brillouin zone**.

Properties and symmetries

- Mirror symmetry:

$$\hat{O}_{M_x}\mathbf{H_k}(\mathbf{r}) = \pm\mathbf{H_k}(\mathbf{r}) = M_x\mathbf{H_k}(M_x\mathbf{r})$$

$$M_x\mathbf{r} = \mathbf{r}$$ yz plane

TM mode

(E_x,H_y,H_z)

TE mode

(H_x,E_y,E_z)

Properties and symmetries

- Time reversal inavariance: from the fact that H and H* satisfy the same equation (master equation) we get:

$$\omega_n(\mathbf{k}) = \omega_n(-\mathbf{k})$$

- This is a consequence of the time-reversal symmetry of the Maxwell equation.

Periodic Hermitian Eigenproblems

[G. Floquet, “Sur les équations différentielles linéaries à coefficients périodiques,” *Ann. École Norm. Sup.* **12**, 47–88 (1883).]
[F. Bloch, “Über die quantenmechanik der electronen in kristallgittern,” *Z. Physik* **52**, 555–600 (1928).]

if eigen-operator is periodic, then Bloch-Floquet theorem applies:

can choose:

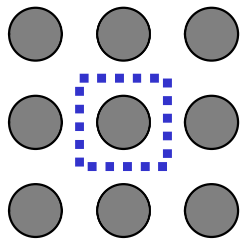
$$\vec{H}(\vec{x},t) = e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}\vec{H}_{\vec{k}}(\vec{x})$$

planewave

periodic “envelope”

Corollary 1: **k** is conserved, *i.e.* no scattering of Bloch wave

Corollary 2: $\vec{H}_{\vec{k}}$ given by finite unit cell,
so ω are discrete $\omega_n(\mathbf{k})$



Periodic Hermitian Eigenproblems

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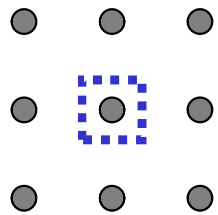
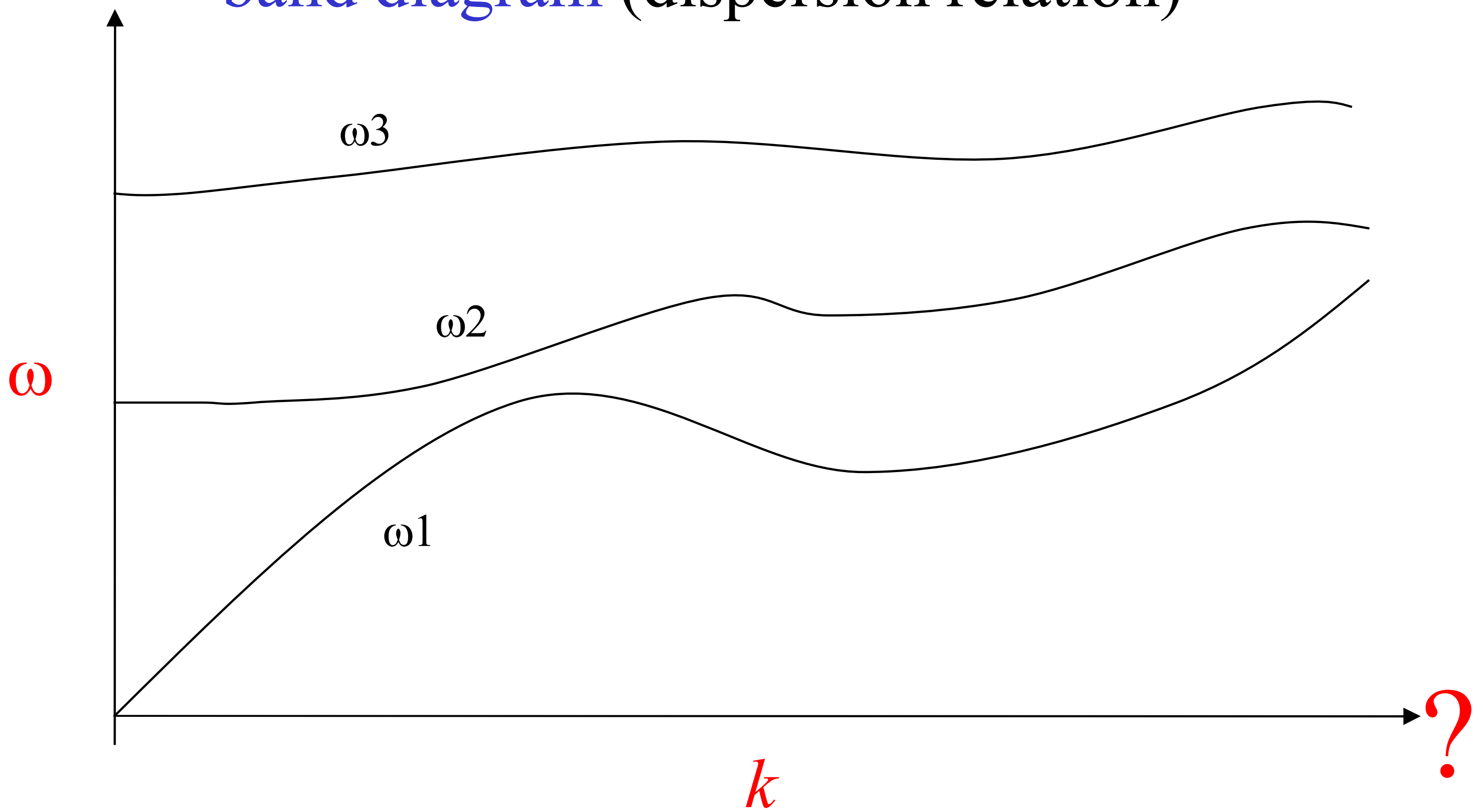


band diagram (dispersion relation)

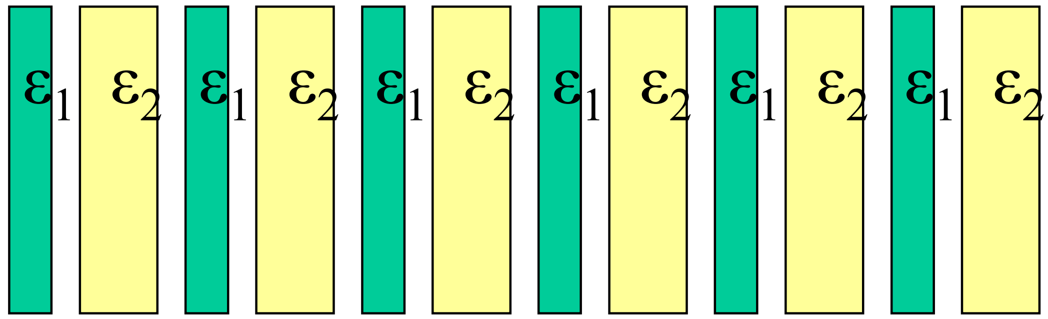


map of
what states
exist &
can interact

range of *k*?

Periodic Hermitian Eigenproblems in 1d

$$H(x) = e^{ikx} H_k(x)$$



a

$$\varepsilon(x) = \varepsilon(x+a)$$

Consider $k+2\pi/a$: $$e^{i(k+\frac{2\pi}{a})x} H_{k+\frac{2\pi}{a}}(x) = e^{ikx}\left[e^{i\frac{2\pi}{a}x} H_{k+\frac{2\pi}{a}}(x) \right]$$

k is periodic:

$k + 2\pi/a$ equivalent to k

"quasi-phase-matching"

periodic!

satisfies same equation as H_k

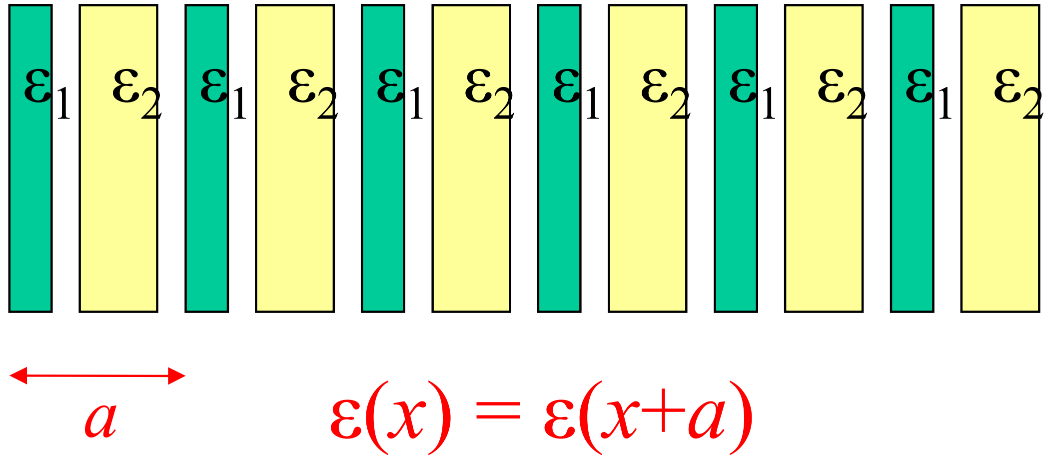
$= H_k$

Periodic Hermitian Eigenproblems in 1d

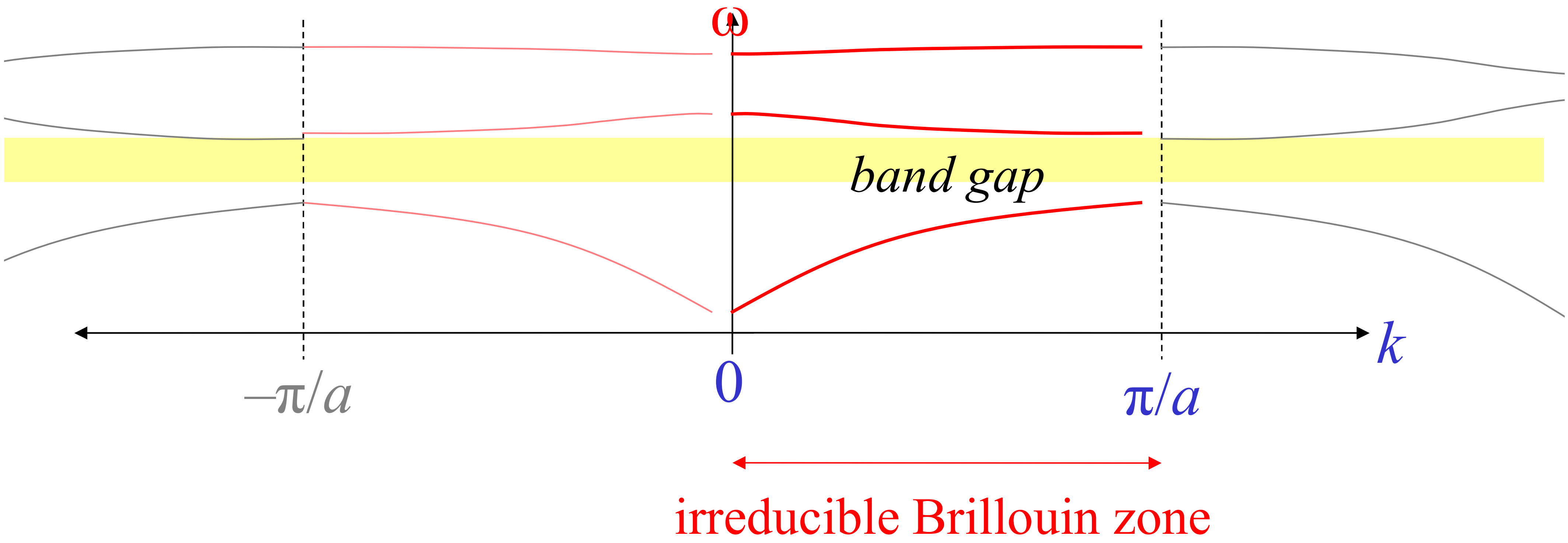
k is periodic:

$k + 2\pi/a$ equivalent to k

"quasi-phase-matching"



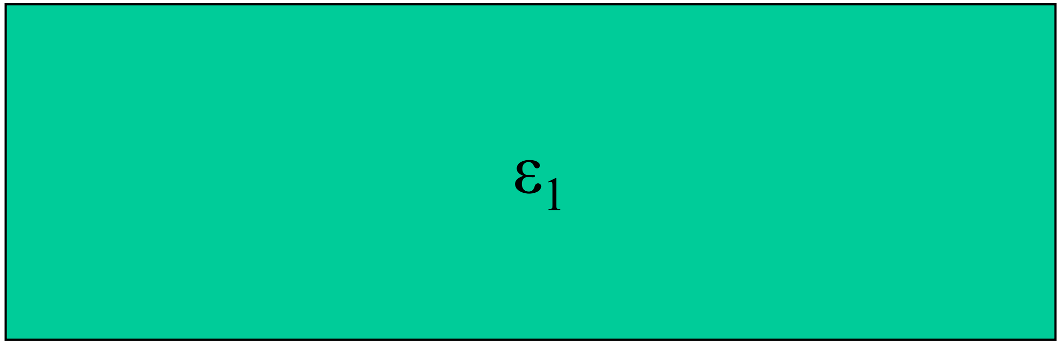
$\varepsilon(x) = \varepsilon(x+a)$

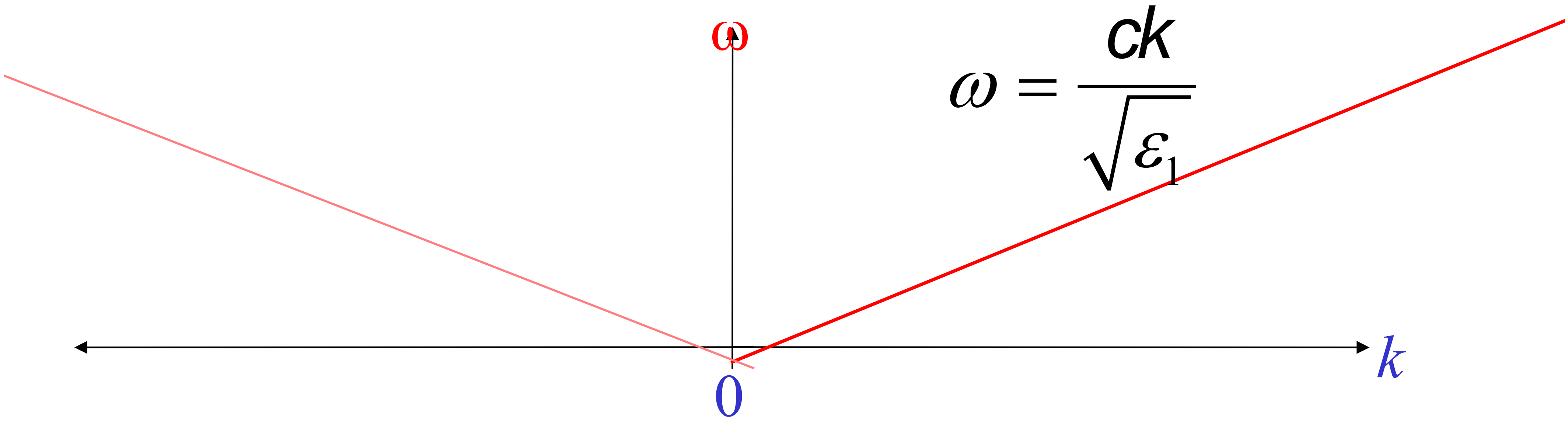


Any 1d Periodic System has a Gap

[Lord Rayleigh, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," *Philosophical Magazine* **24**, 145–159 (1887).]

Start with
a uniform (1d) medium:

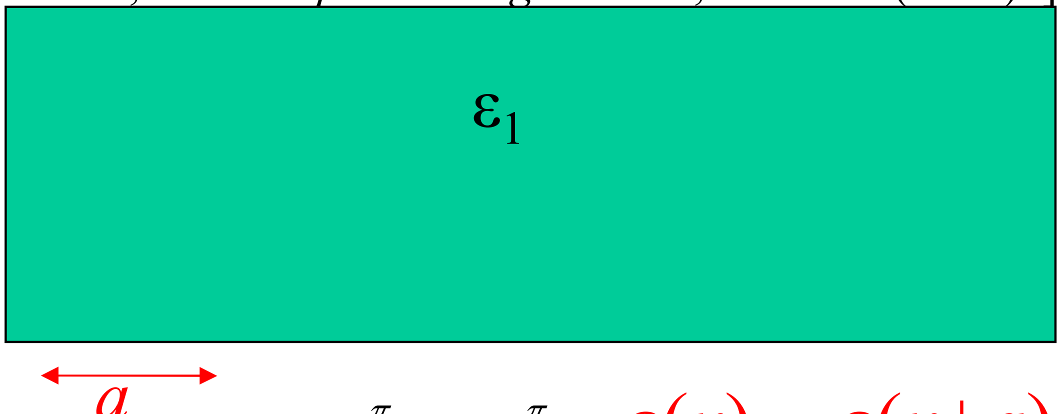




Any 1d Periodic System has a Gap

[Lord Rayleigh, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," *Philosophical Magazine* **24**, 145–159 (1887).]

Treat it as "artificially" periodic

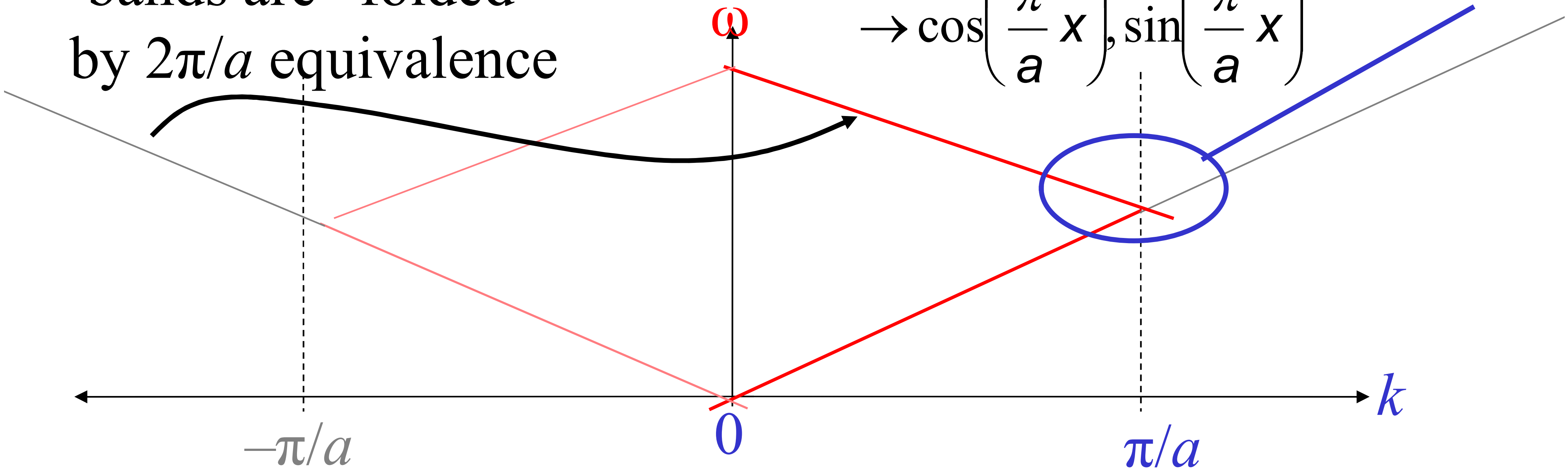


$$e^{+i\frac{\pi}{a}x}, e^{-i\frac{\pi}{a}x}$$

$$\varepsilon(x) = \varepsilon(x+a)$$

$$\rightarrow \cos\left(\frac{\pi}{a}x\right), \sin\left(\frac{\pi}{a}x\right)$$

bands are "folded" by $2\pi/a$ equivalence



Any 1d Periodic System has a Gap

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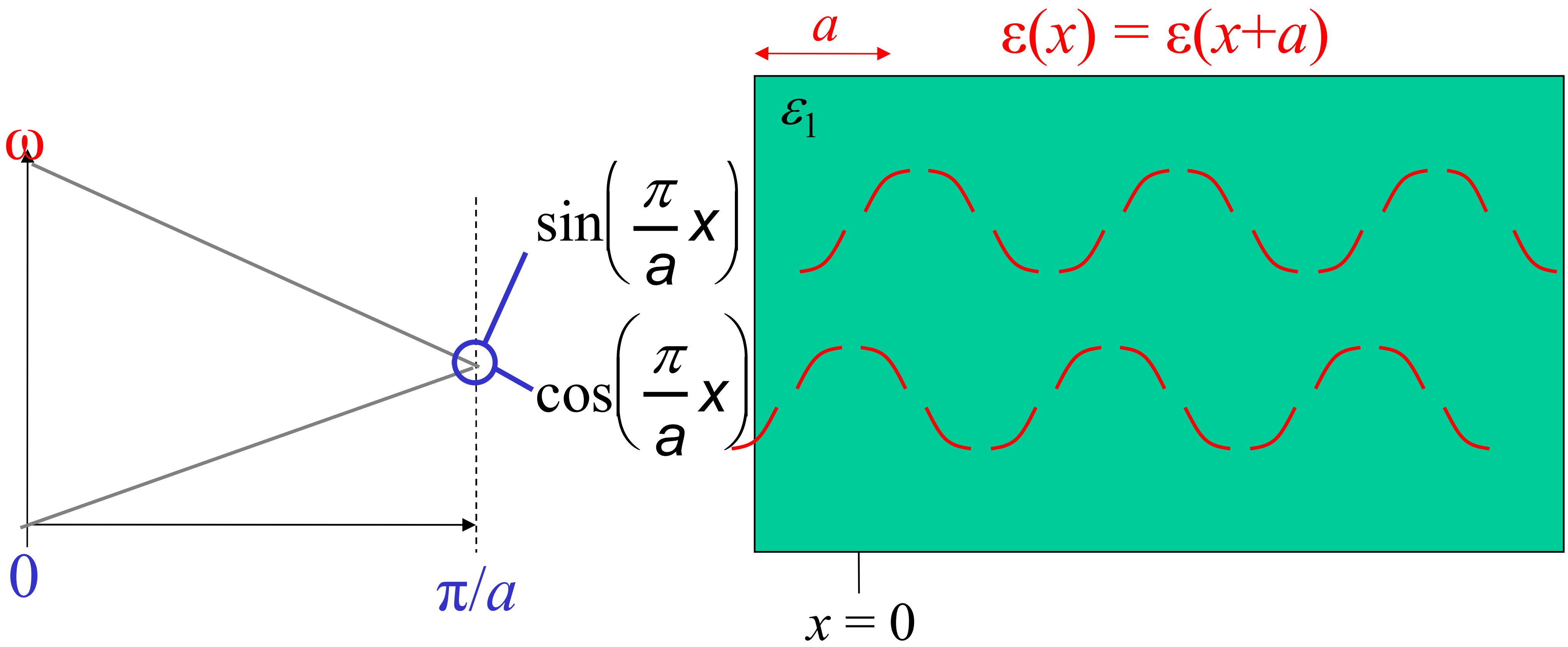
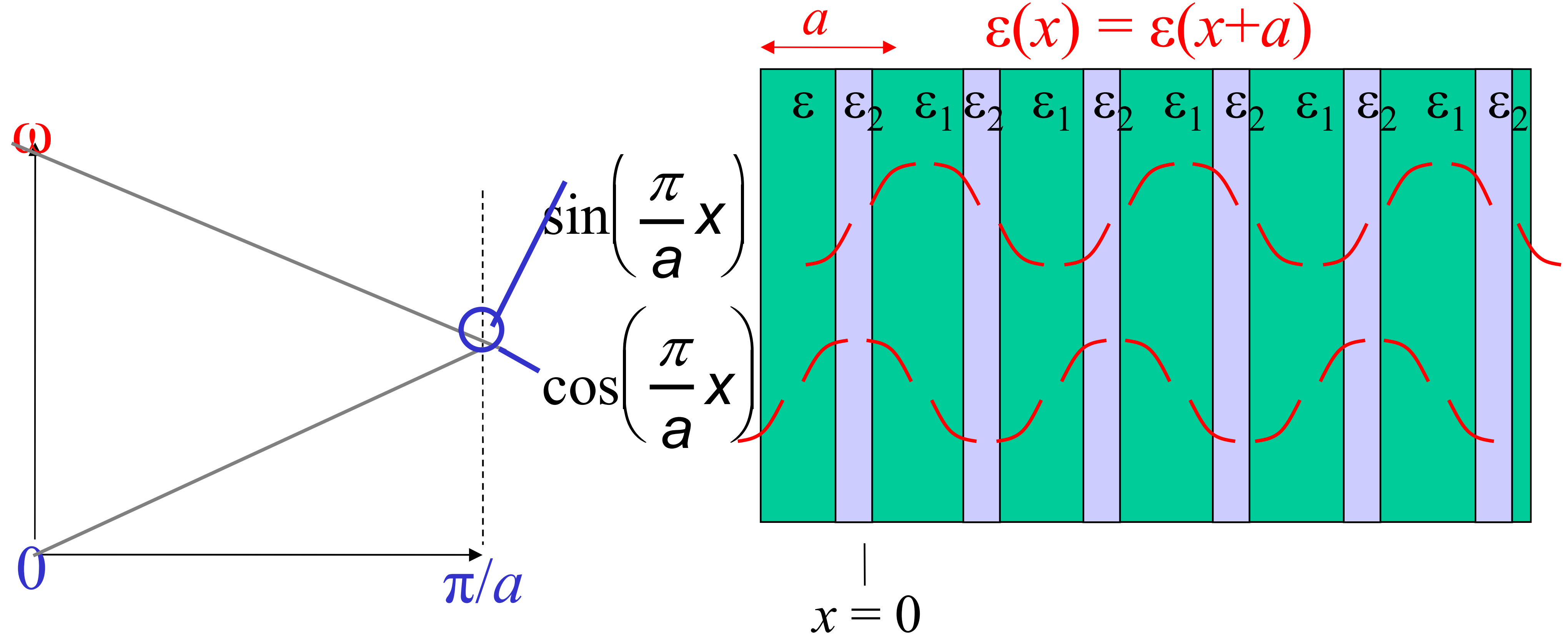


Any 1d Periodic System has a Gap

[Lord Rayleigh, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," *Philosophical Magazine* **24**, 145–159 (1887).]

Add a small "real" periodicity

$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$



Any 1d Periodic System has a Gap

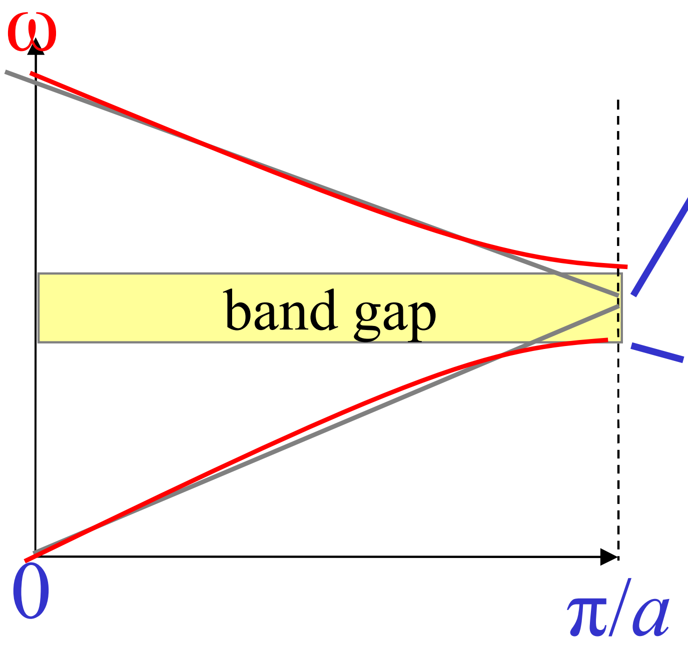
[Lord Rayleigh, "On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure," *Philosophical Magazine* **24**, 145–159 (1887).]

Add a small "real" periodicity

$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$

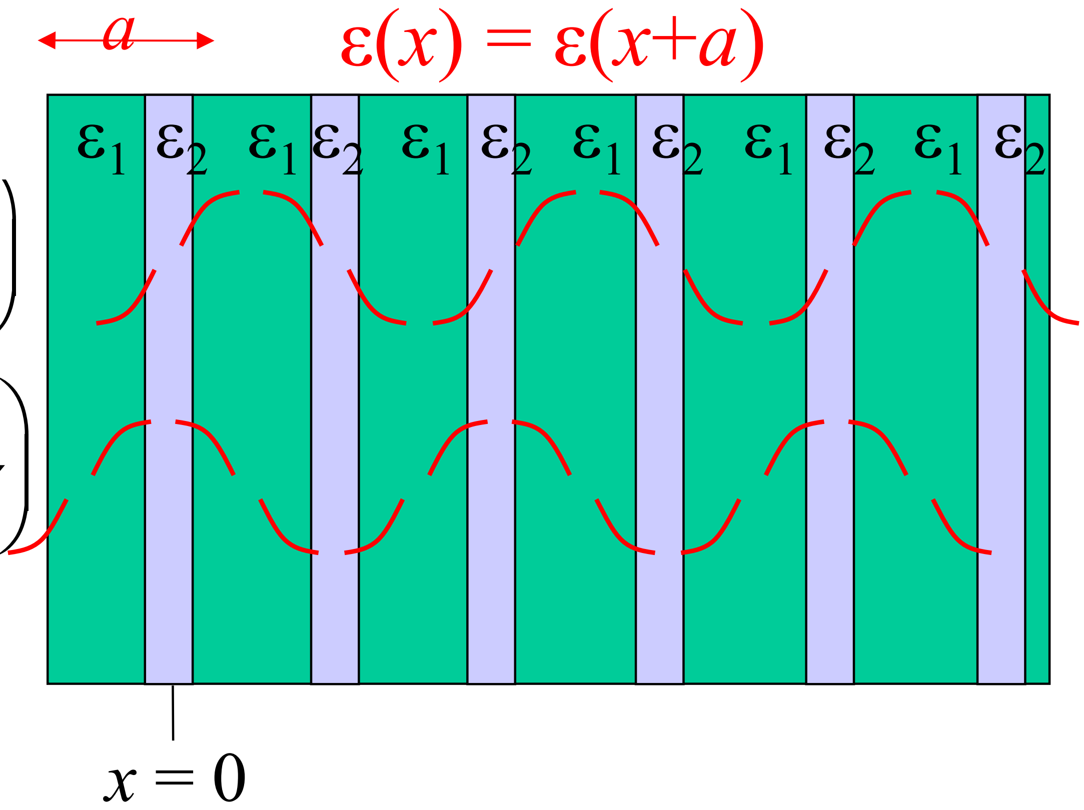
Splitting of degeneracy:

state concentrated in higher index (ε_2) has lower frequency

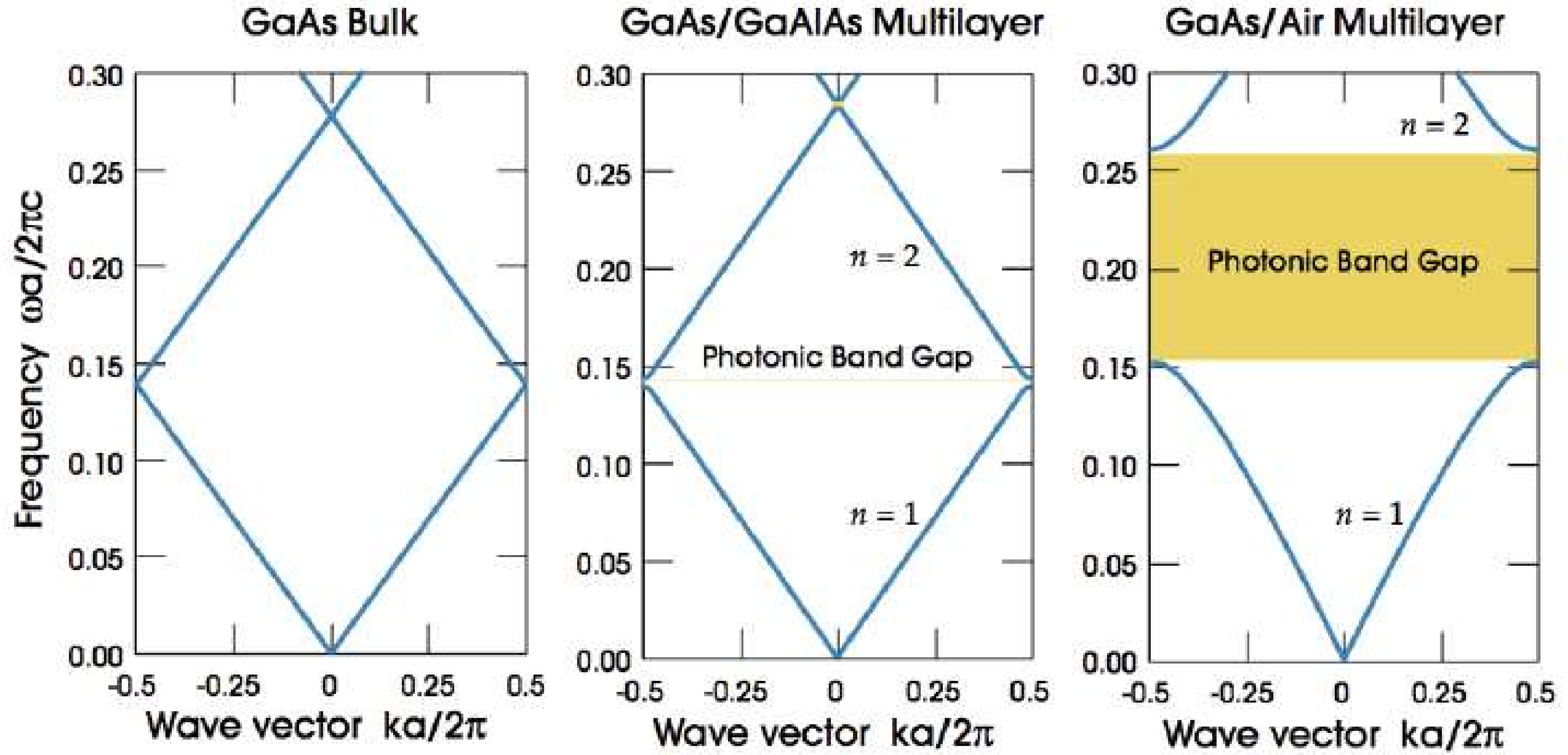


$\sin\left(\frac{\pi}{a}x\right)$

$\cos\left(\frac{\pi}{a}x\right)$

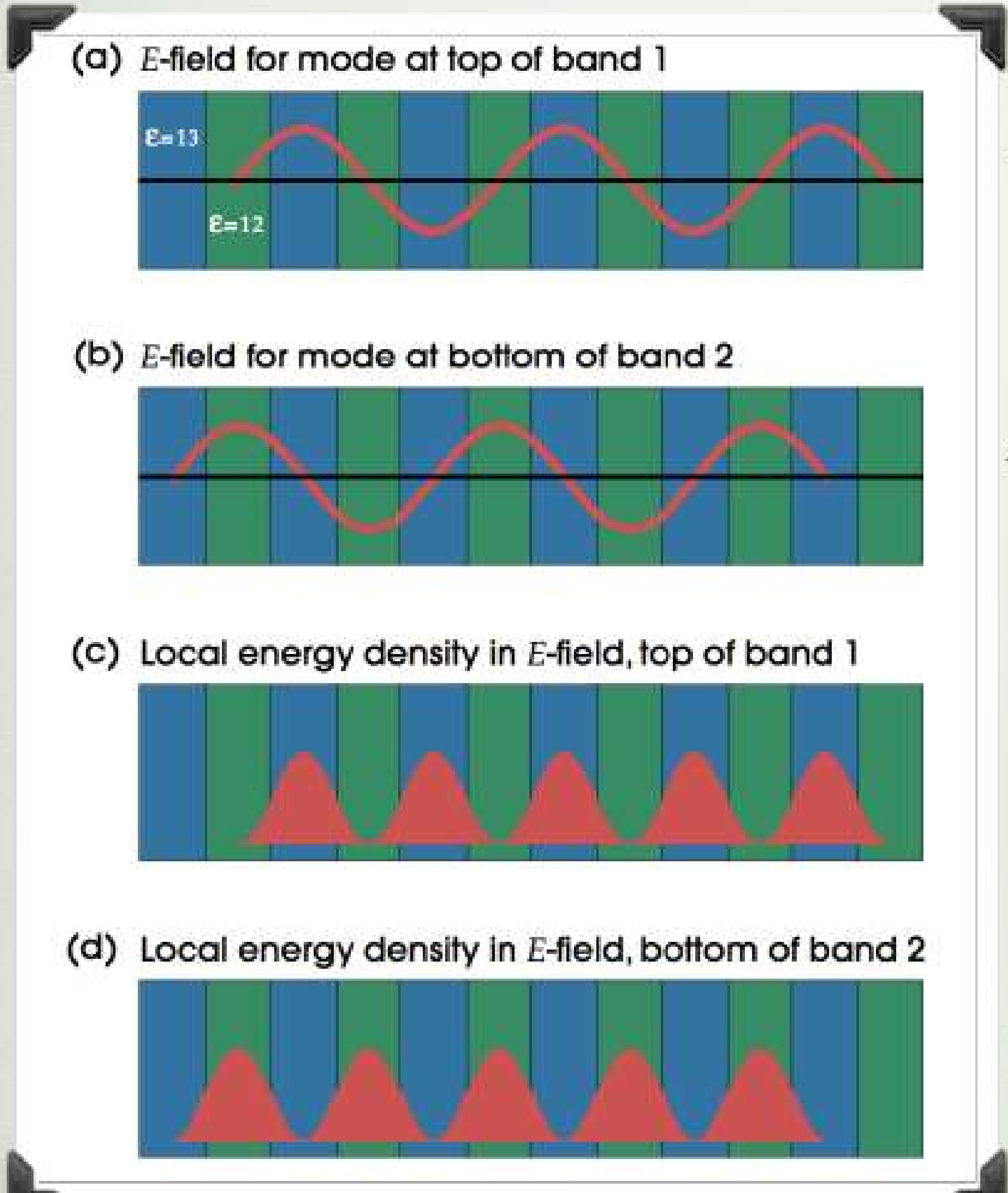


The origin of the photonic gaps



Δε=0 **Δε=1** **Δε=12**

The origin of the photonic gaps



$k=\pi/a \longrightarrow \lambda=2a$

two ways to center a mode of this type

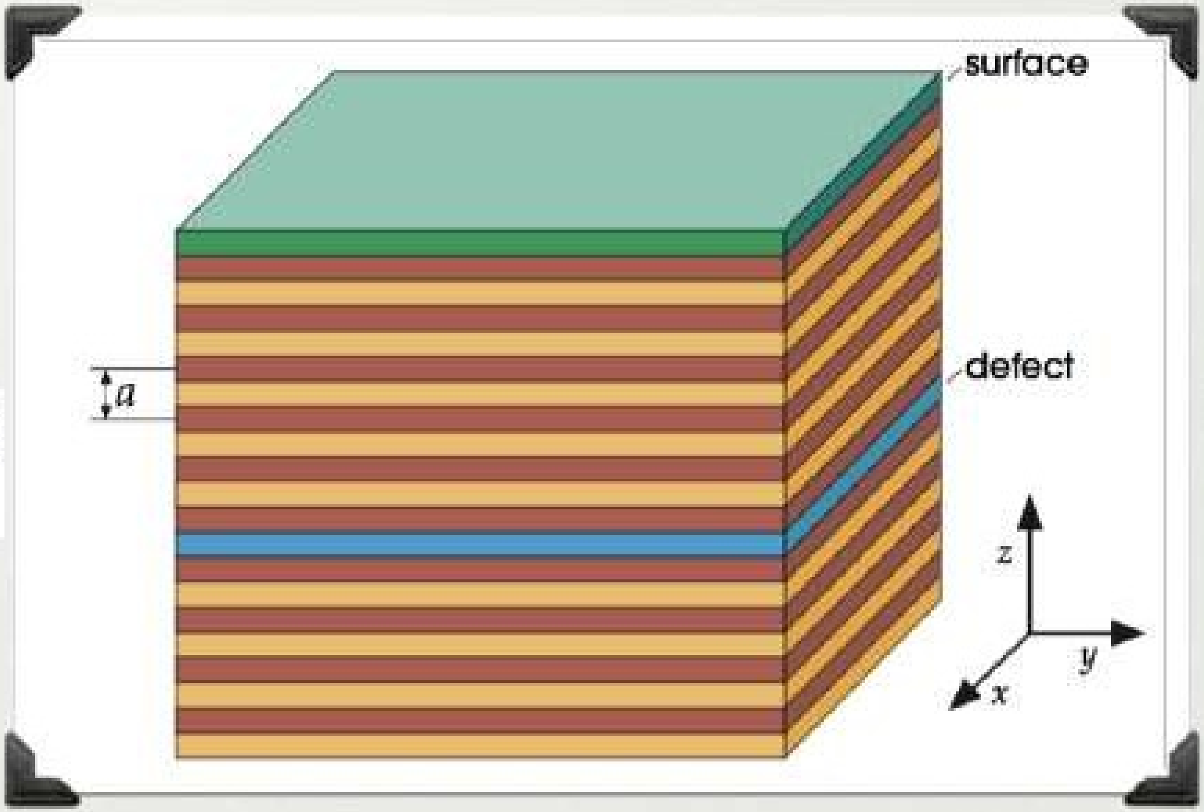
$U_E=\varepsilon|\mathbf{E}|^2/8\pi$

$\Delta\omega/\omega_m$

gap-midgap ratio

Evanescent Modes

- What happens when we send a light wave with a frequency in the photonic gap onto the face of the crystal from outside?



$$\mathbf{H}(\mathbf{r}) = e^{ikz}\mathbf{u}(z)e^{-\kappa z}$$

k is complex and the mode decay exponentially into the crystal on a length scale of 1/k

Some 2d and 3d systems have gaps

- In general, eigen-frequencies satisfy Variational Theorem:

$$\omega_1(\vec{k})^2 = \min_{\substack{\vec{E}_1 \\ \nabla\cdot\varepsilon\vec{E}_1=0}} \frac{\int \left|\left(\nabla + i\vec{k}\right)\times\vec{E}_1\right|^2}{\int \varepsilon\left|\vec{E}_1\right|^2} c^2$$

"kinetic"

inverse "potential"

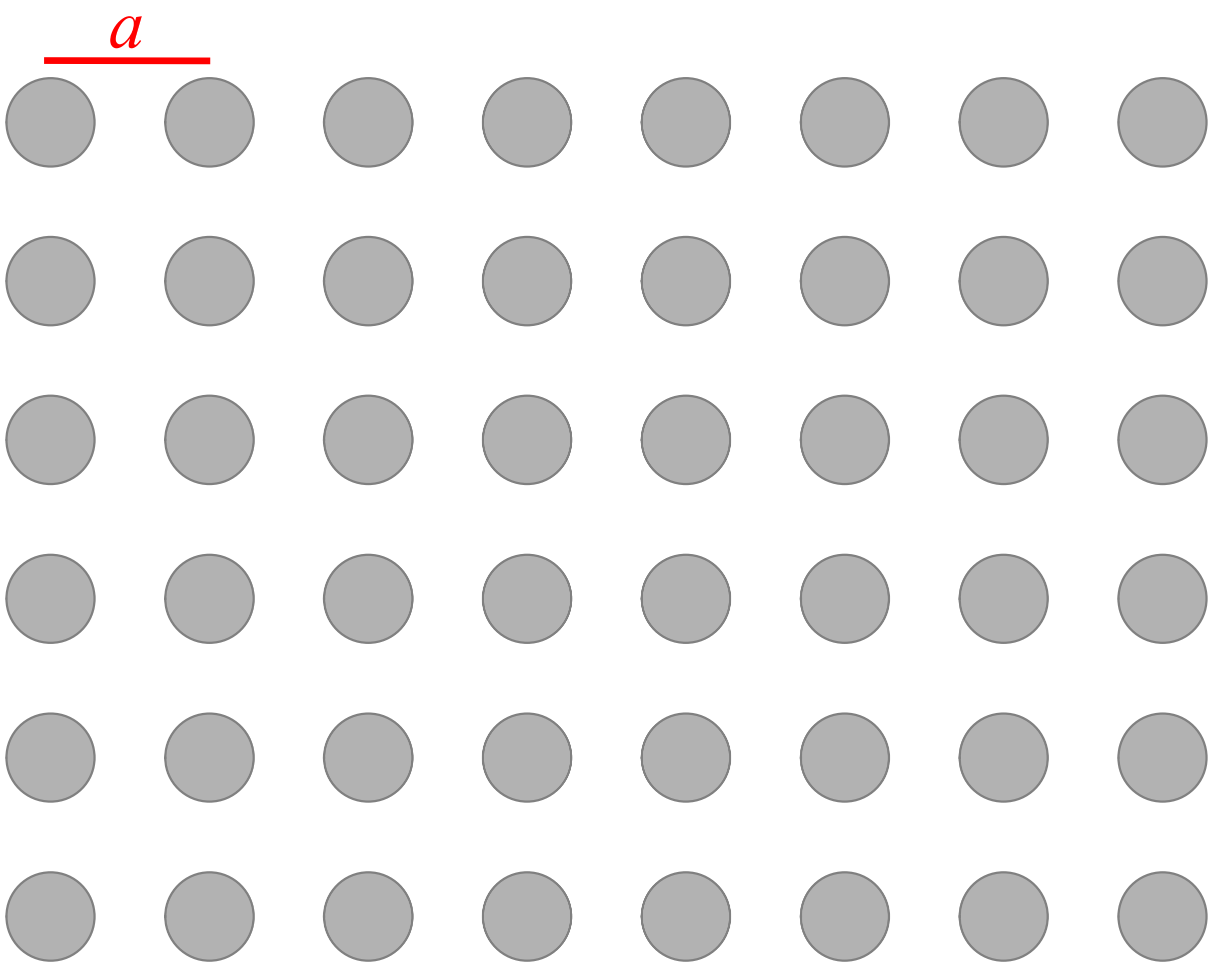
$$\omega_2(\vec{k})^2 = \min_{\substack{\vec{E}_2 \\ \nabla\cdot\varepsilon\vec{E}_2=0 \\ \int \varepsilon E_1^{*}\cdot E_2=0}} \text{"}\cdots\text{"}$$

bands "want" to be in high-ε

…but are forced out by orthogonality

➔ band gap (maybe)

A 2d Model System

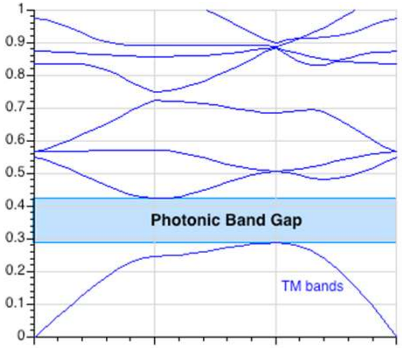


Square lattice of dielectric rods (ε = 12 ~ Si) in air (ε = 1)

Solving the Maxwell Eigenproblem

Finite cell ➔ *discrete* eigenvalues ω_n

Want to solve for $\omega_n(\mathbf{k})$,
& plot vs. "all" **k** for "all" n,



$$(\nabla + i\mathbf{k}) \times \frac{1}{\varepsilon}(\nabla + i\mathbf{k}) \times \mathbf{H}_n = \frac{\omega_n^2}{c^2}\mathbf{H}_n$$

constraint: $(\nabla + i\mathbf{k}) \cdot \mathbf{H} = 0$

where magnetic field = $\mathbf{H}(\mathbf{x})\, e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$

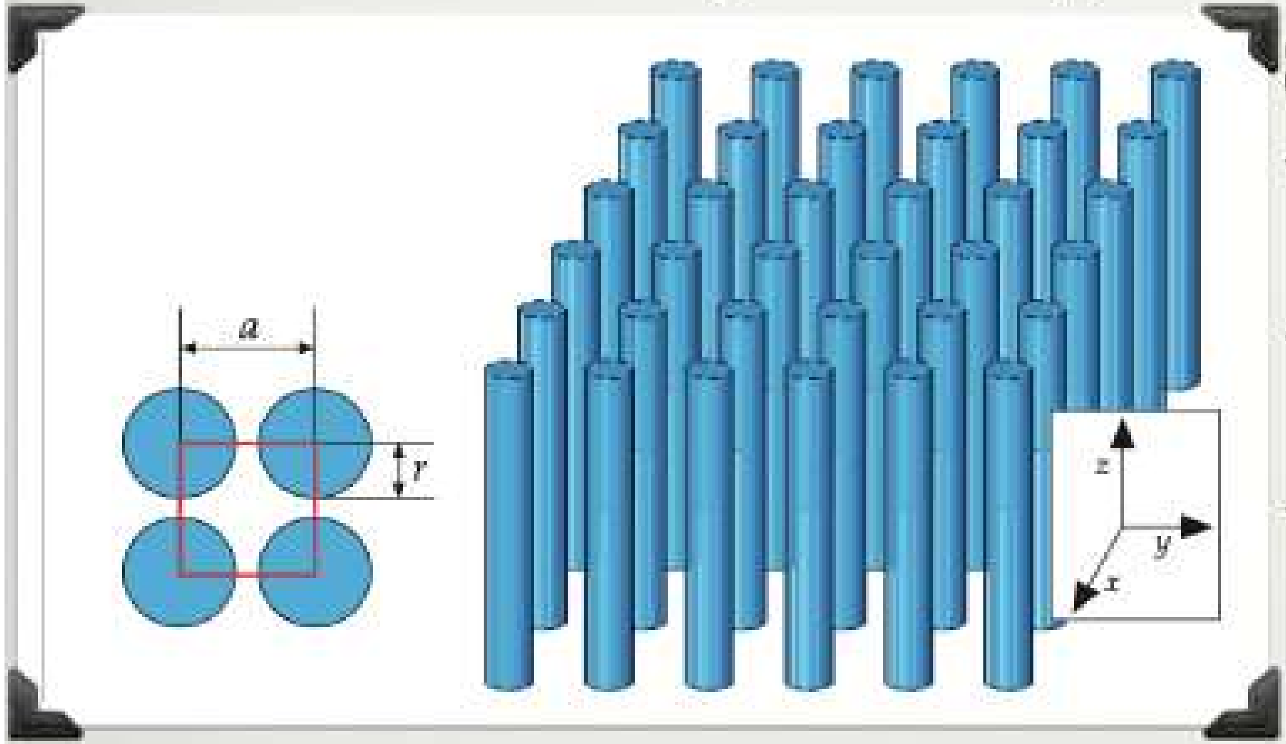
1. Limit range of **k**: irreducible Brillouin zone
2. Limit degrees of freedom: expand **H** in finite basis
3. Efficiently solve eigenproblem: iterative methods

Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

Two-Dimensional Photonic Crystal

- A two dimensional photonic crystal is periodic along two of its axes and homogeneus along the third axis

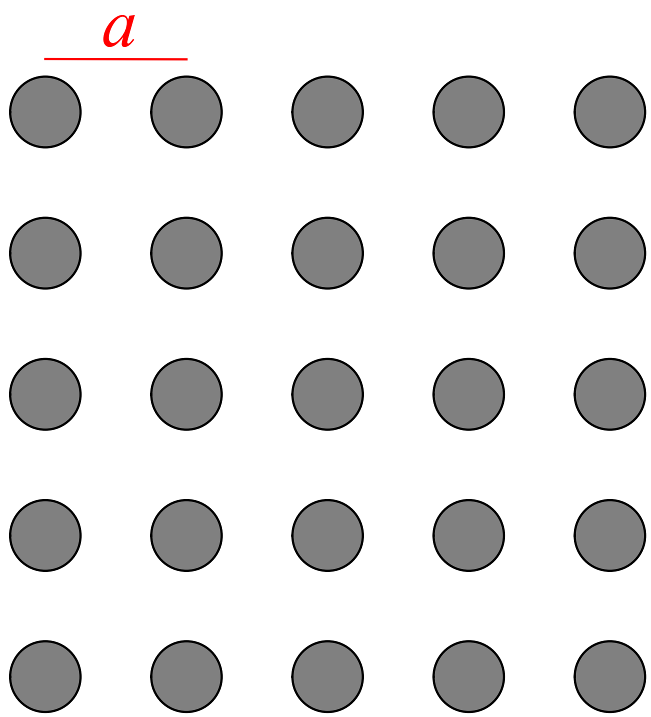


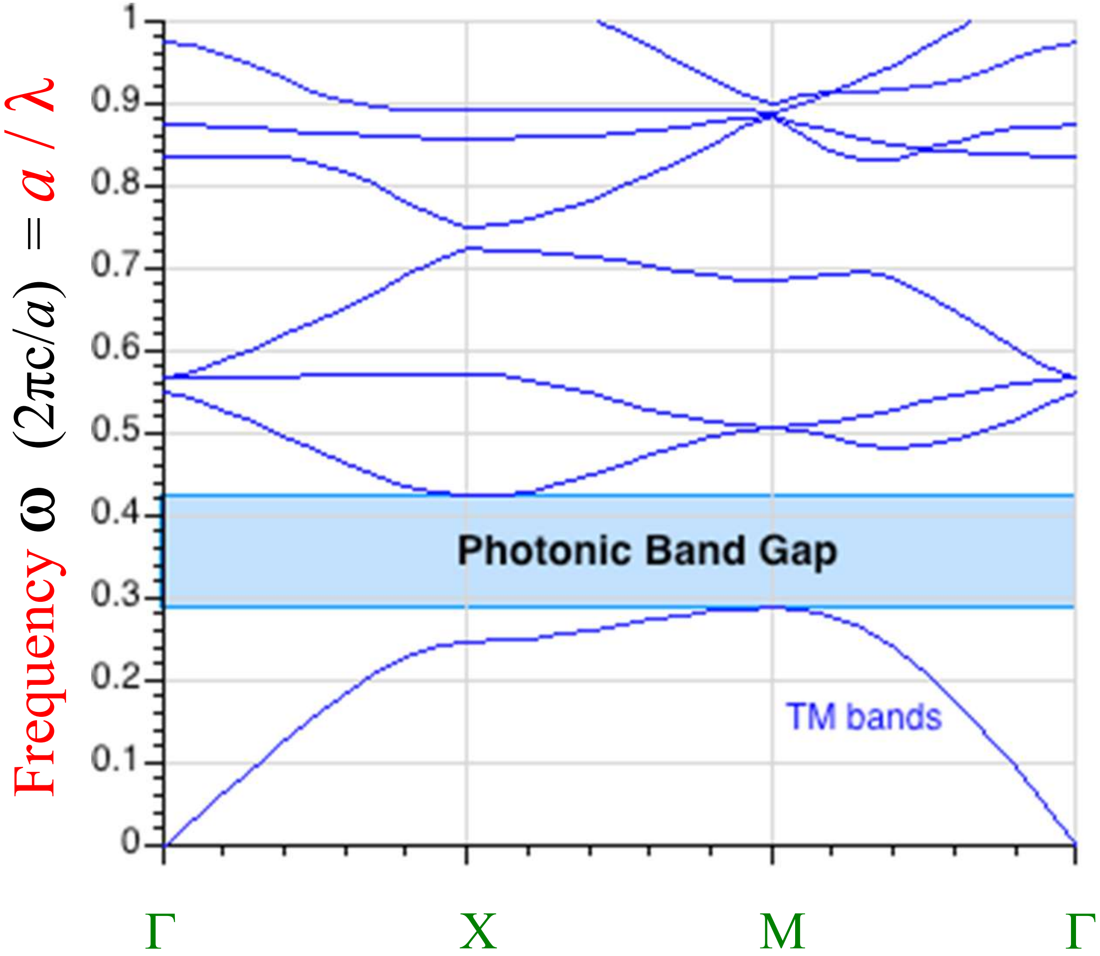
Symmetries:
- ✓ homogeneus translation in the z-direction.
- ✓ discrete translation through the x-y plane
- ✓ mirror symmetry in the x-y plane —> TM and TE modes

$$\mathbf{H}_{(n,k_z,\mathbf{k}_\parallel)}(\mathbf{r}) = e^{i\mathbf{k}_\parallel \cdot \rho} e^{ik_z z} \mathbf{u}_{(n,k_z,\mathbf{k}_\parallel)}(\rho)$$

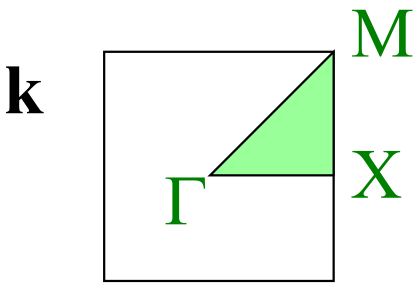
→ **H periodic in the x-y plane**

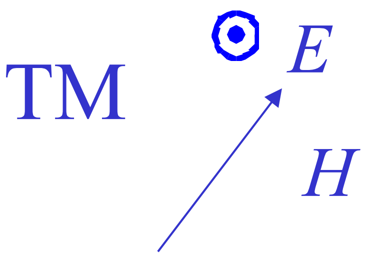
2d periodicity, ε=12:1 silicon/air





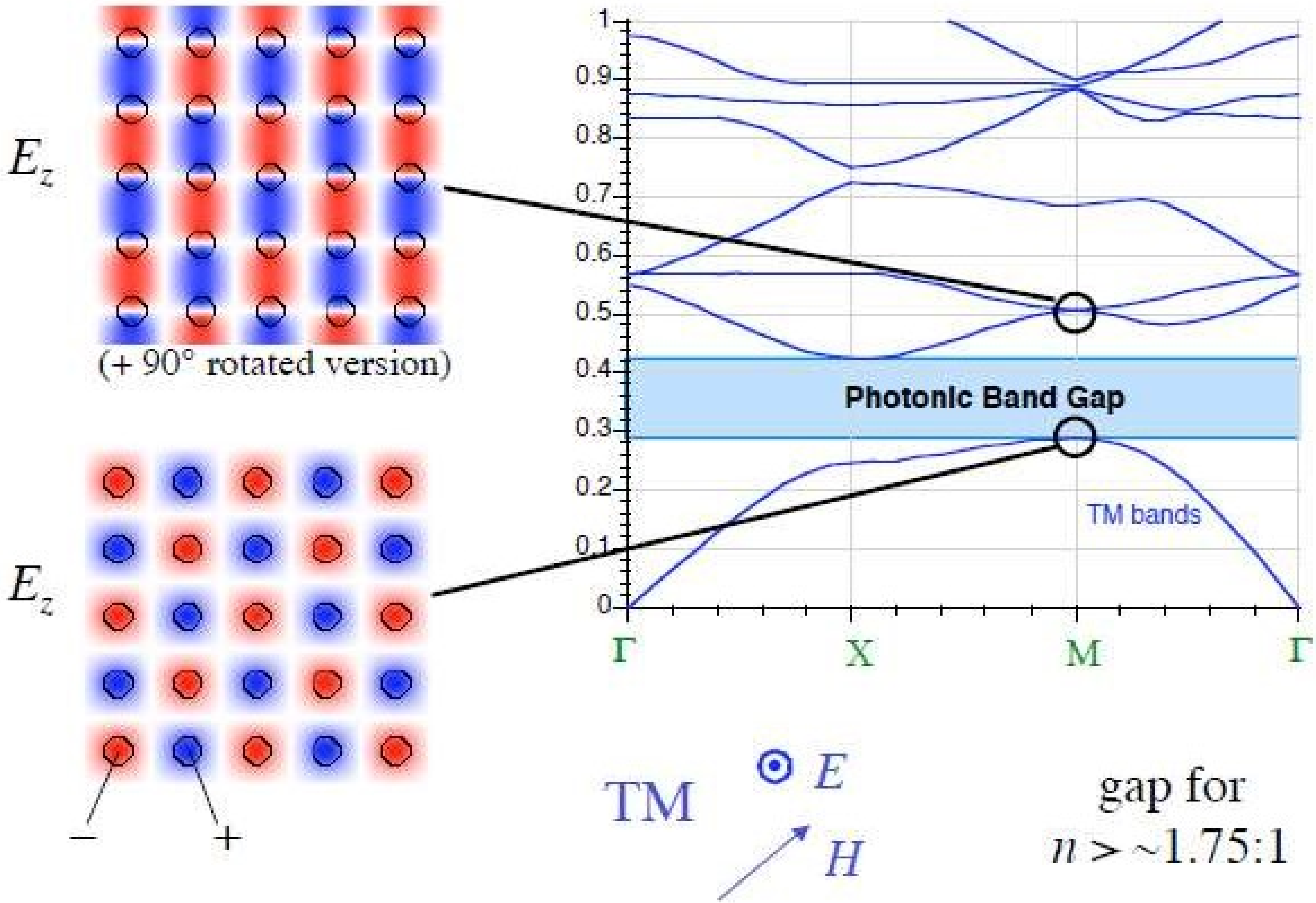
irreducible Brillouin zone



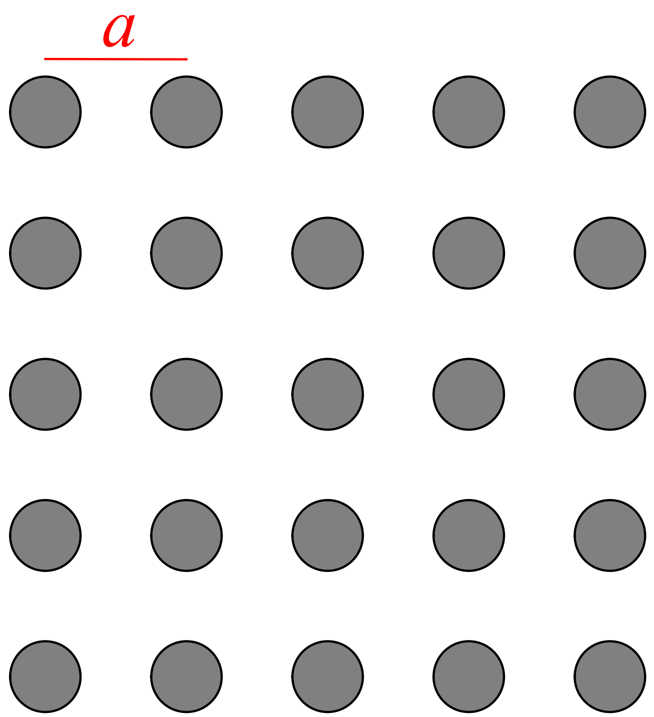


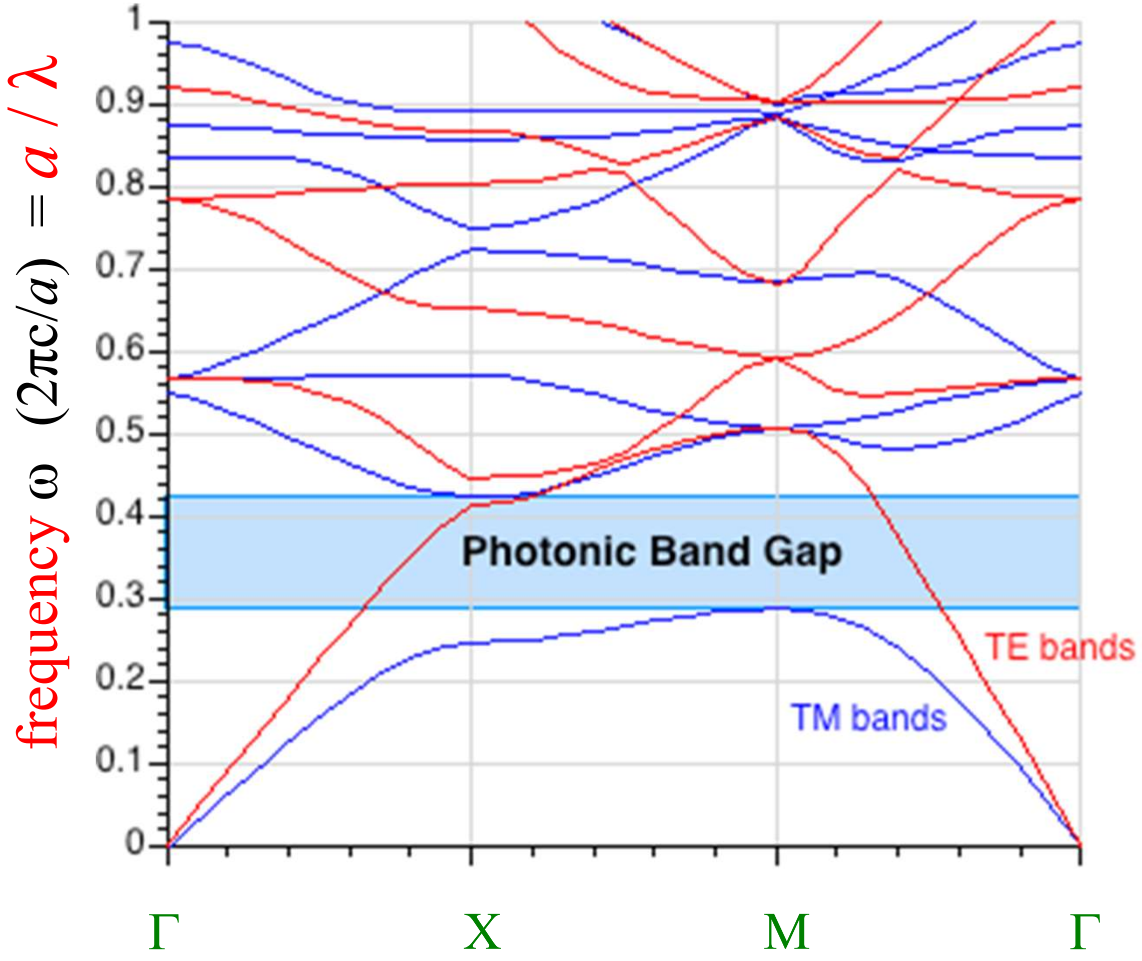
gap for
n > ~1.75:1

2d periodicity, ε=12:1

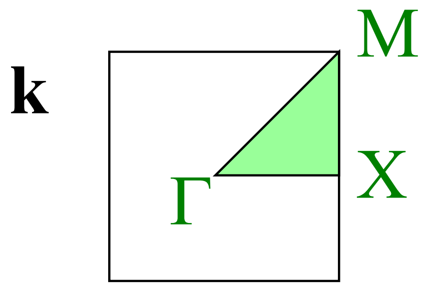


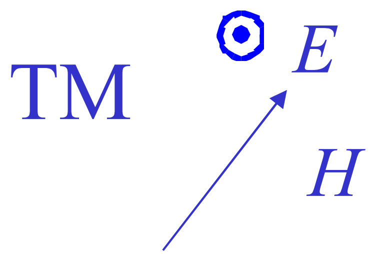
2d periodicity, ε=12:1

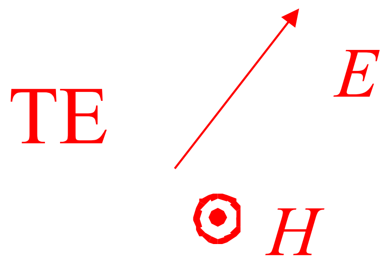




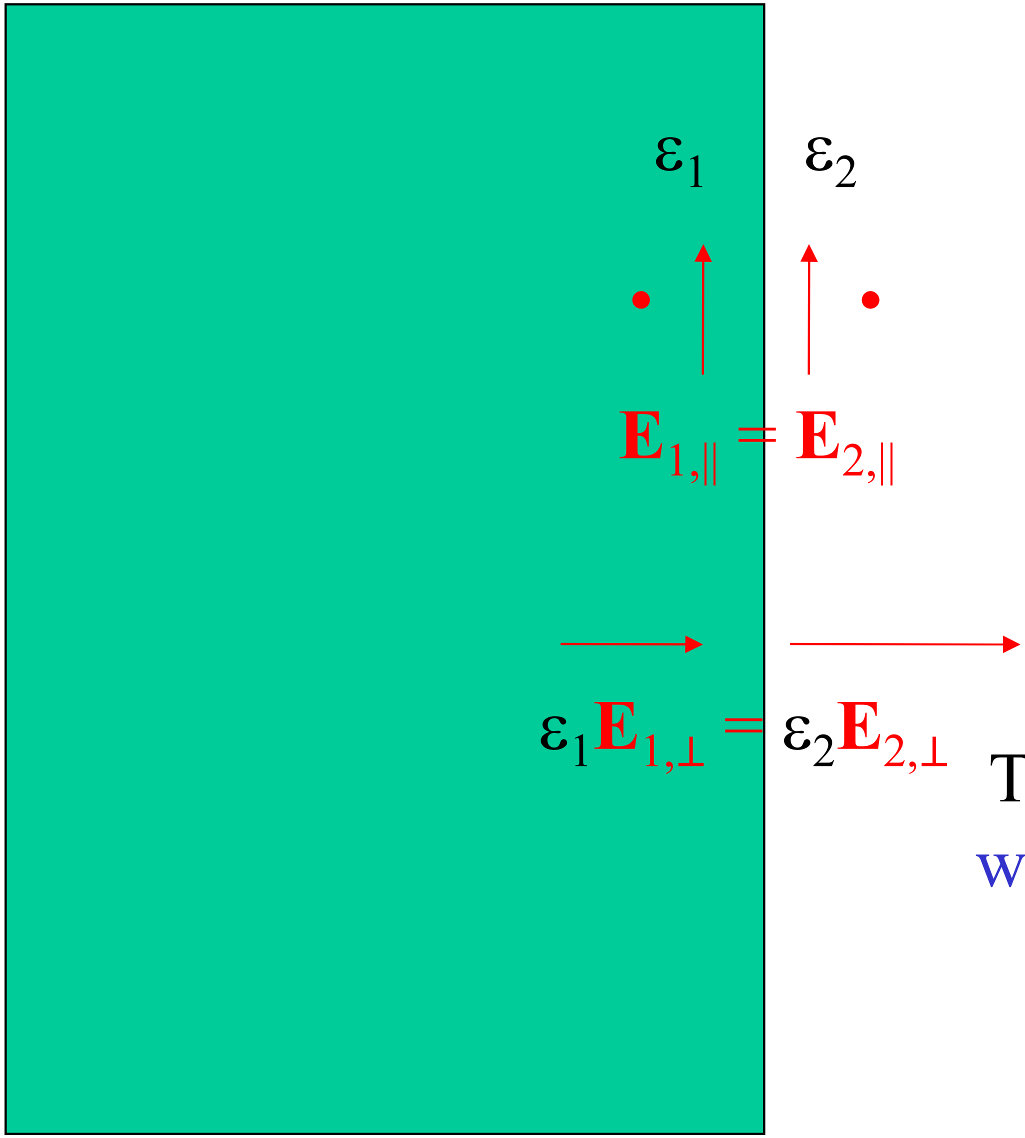
irreducible Brillouin zone







What a difference a boundary condition makes…

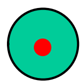


$\mathbf{E}_{||}$ is continuous:
energy density $\varepsilon|\mathbf{E}|^2$
more in larger ε

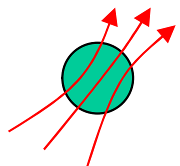
$\varepsilon\mathbf{E}_{\perp}$ is continuous:
energy density $|\varepsilon\mathbf{E}|^2/\varepsilon$
more in smaller ε

To get strong confinement & gaps,
want **E** mostly parallel to interfaces

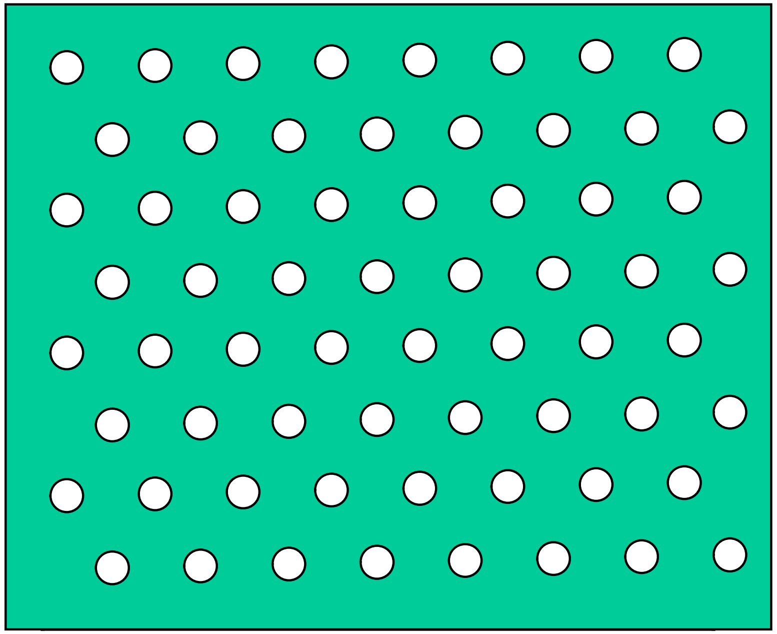
TM: ||

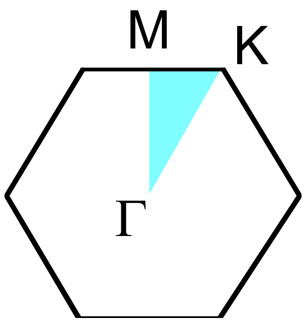


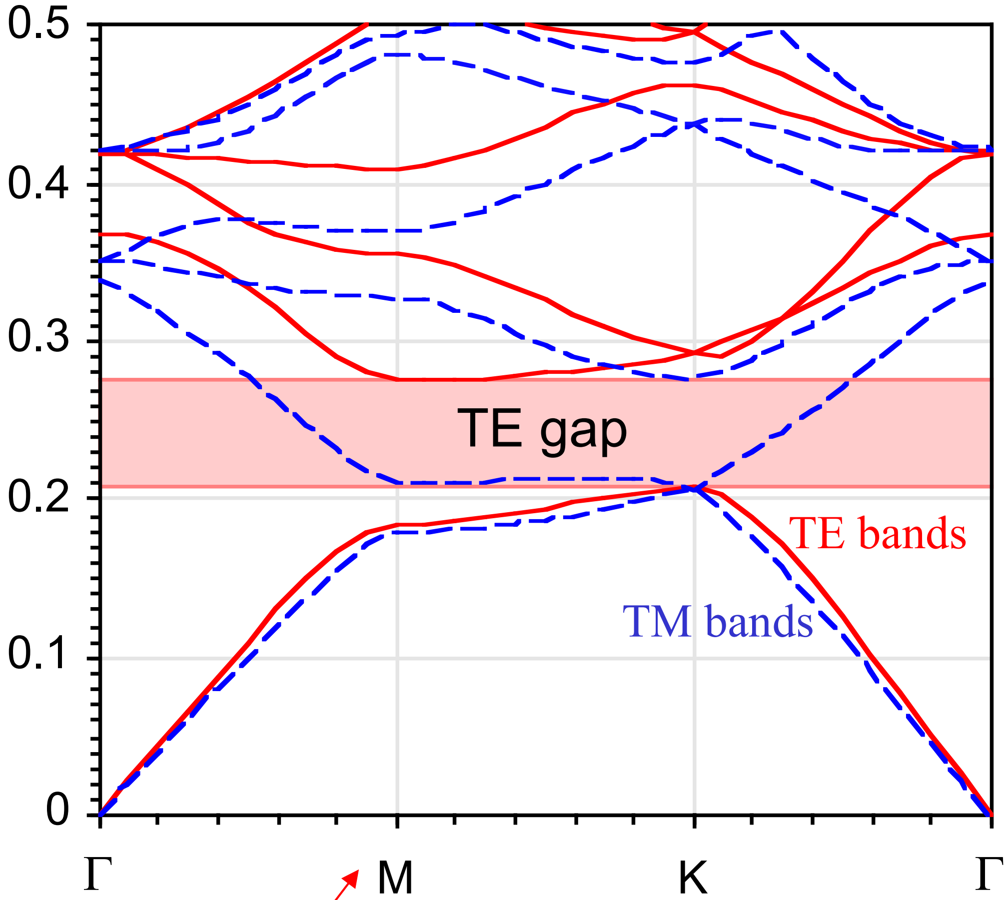
TE: $\perp$

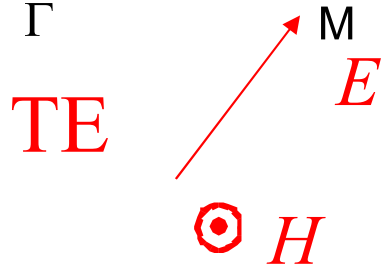


2d photonic crystal: TE gap, ε=12:1



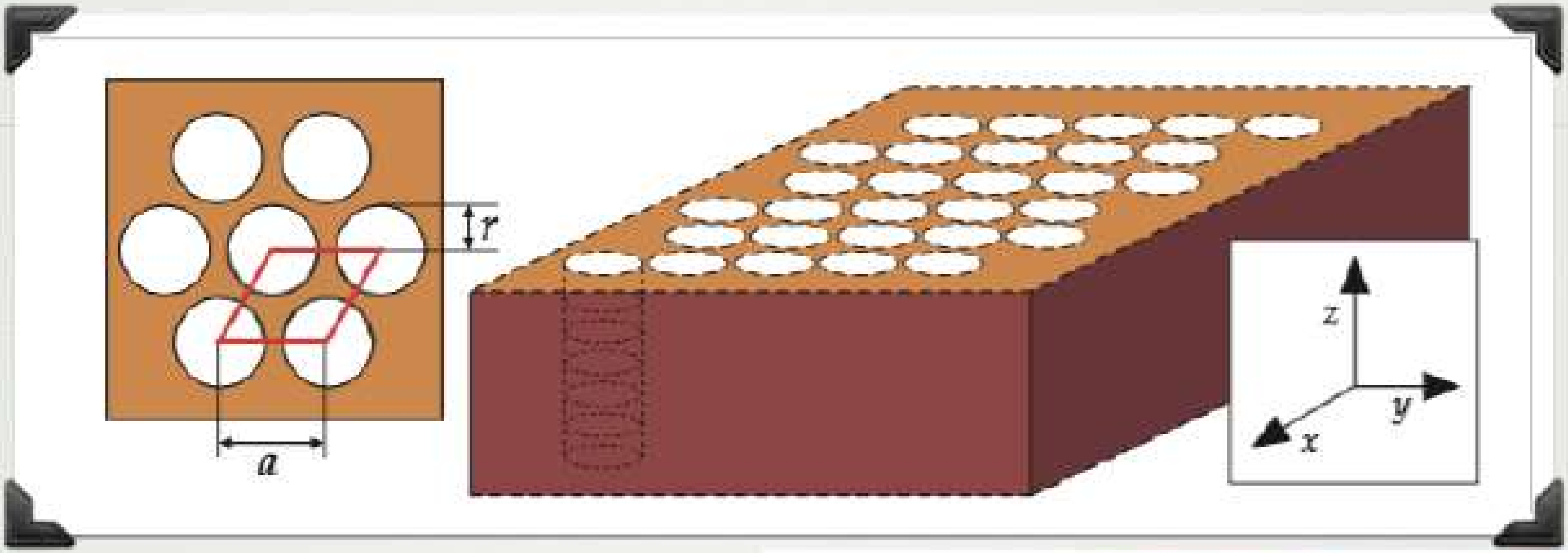


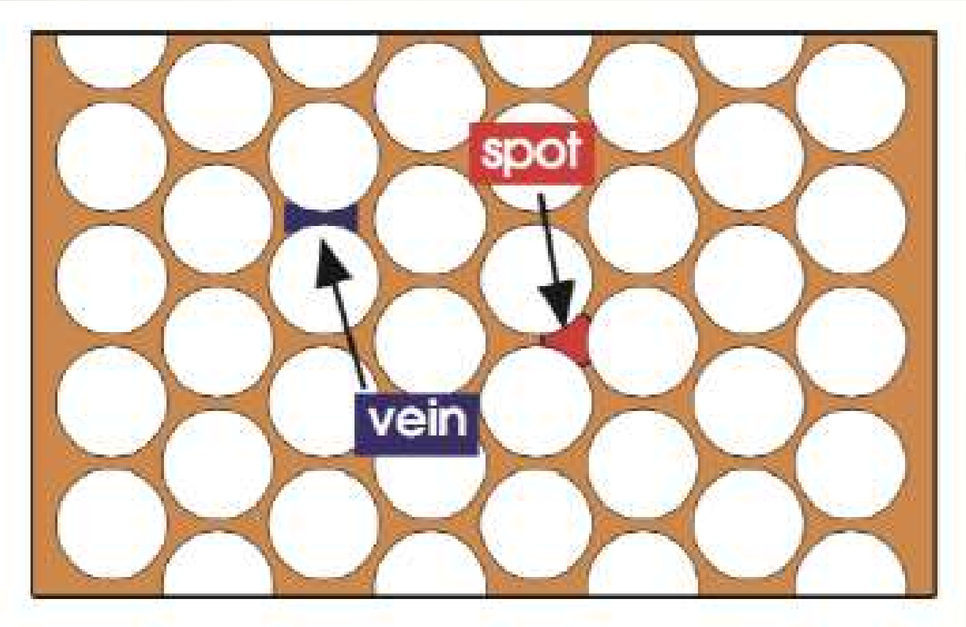


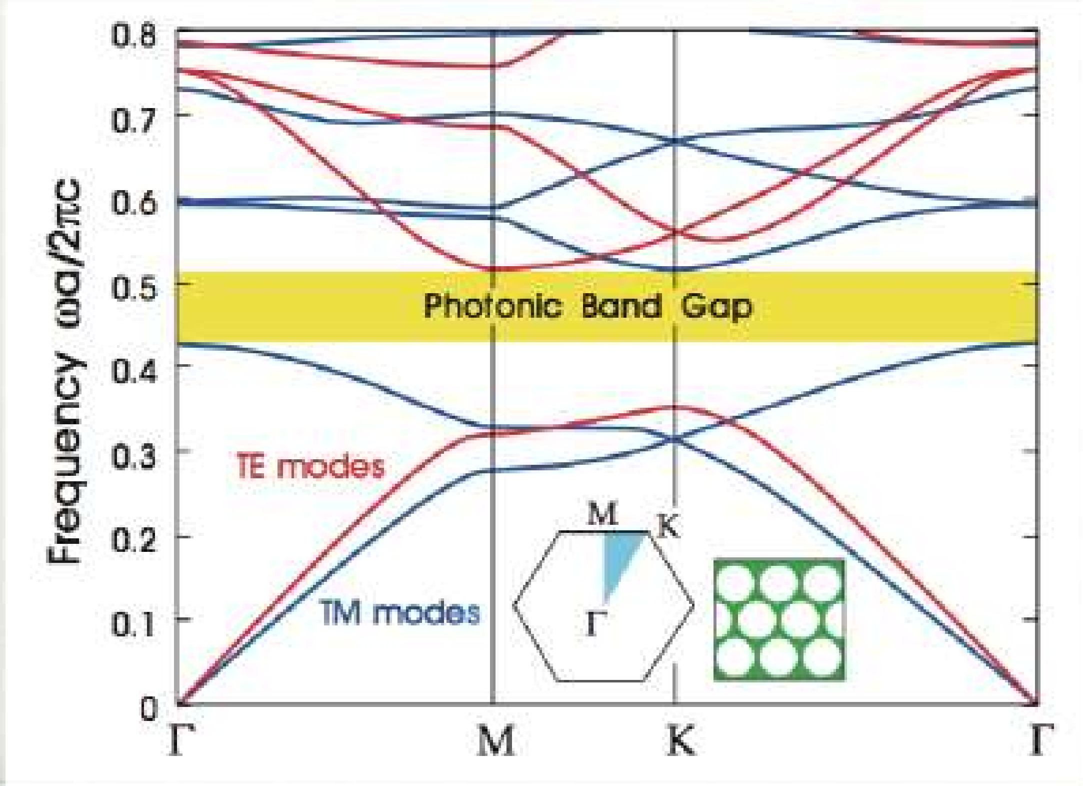


gap for n > ~1.4:1

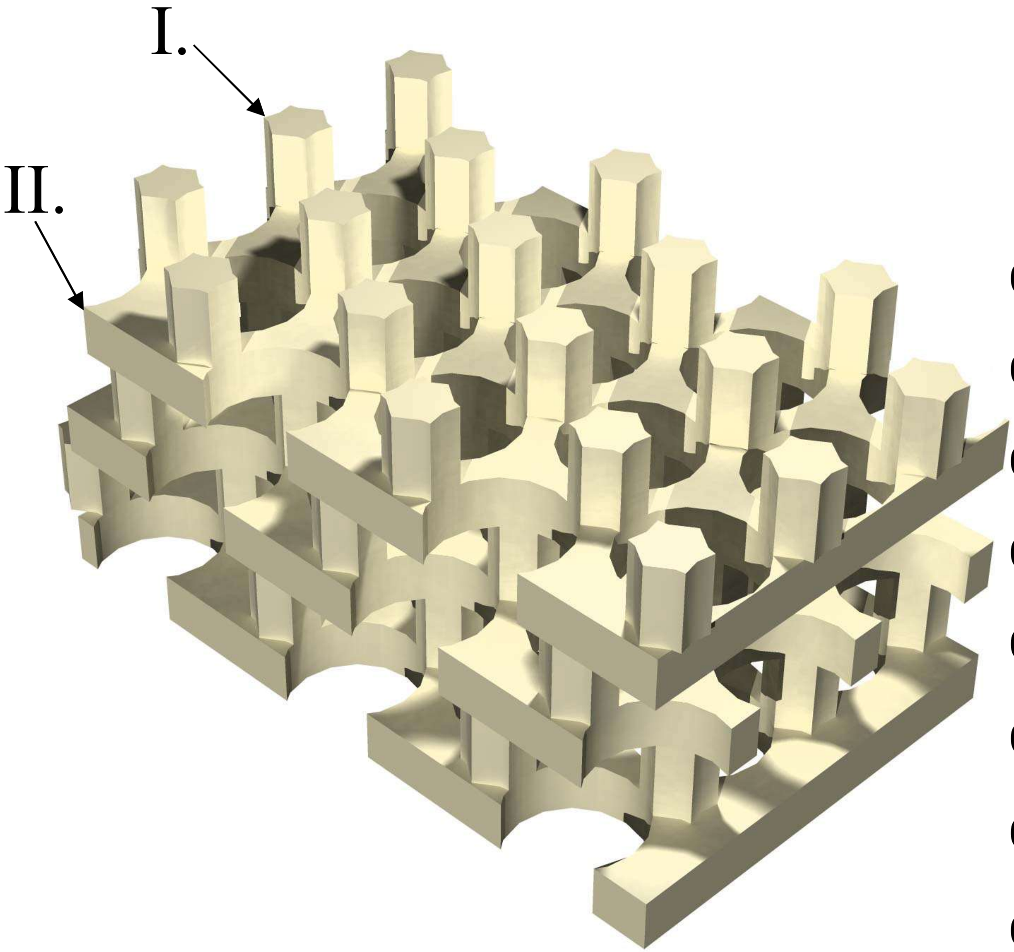
A complete Band Gap for all polarizations

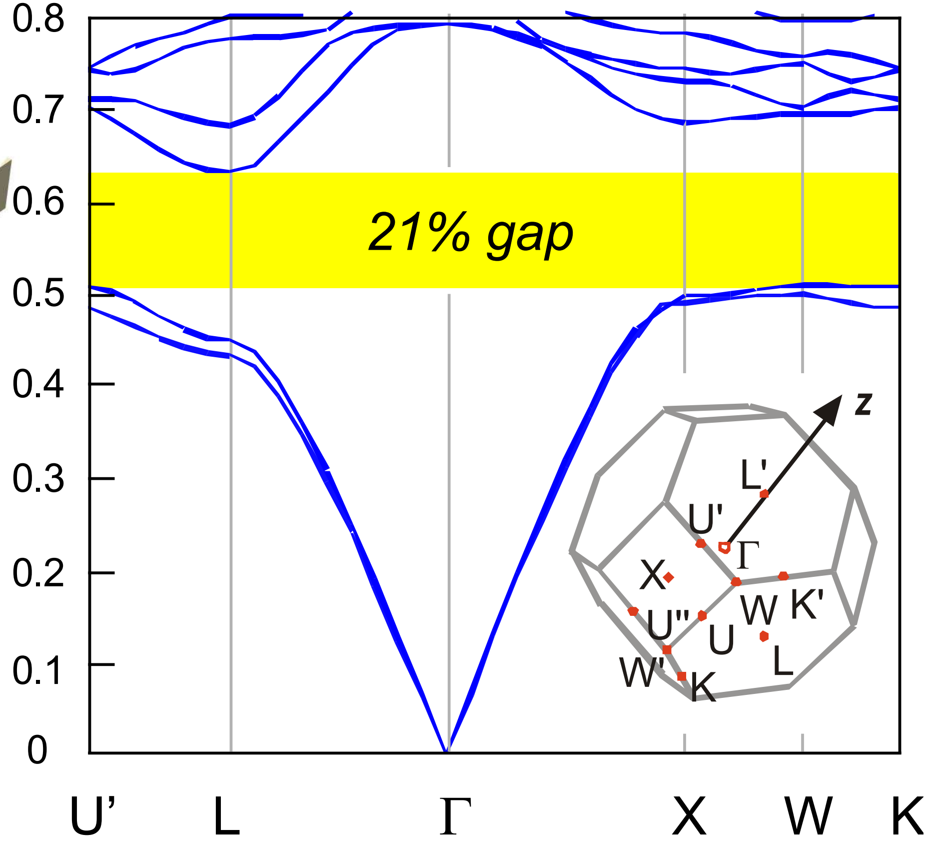




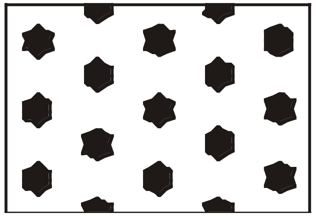


3d photonic crystal: complete gap , ε=12:1

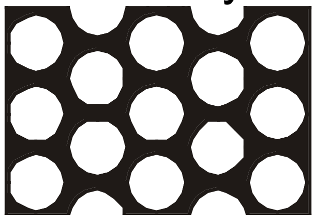




I: rod layer



II: hole layer



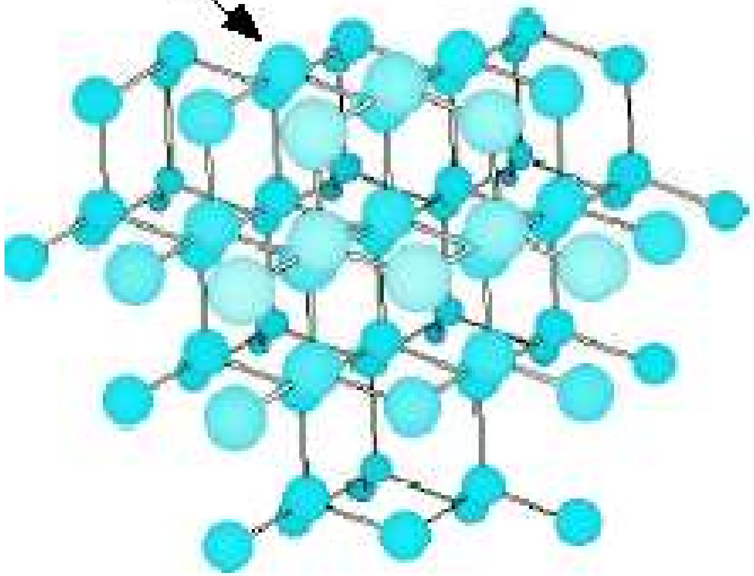
gap for n > ~2:1

[S. G. Johnson *et al.*, *Appl. Phys. Lett.* **77**, 3490 (2000)]

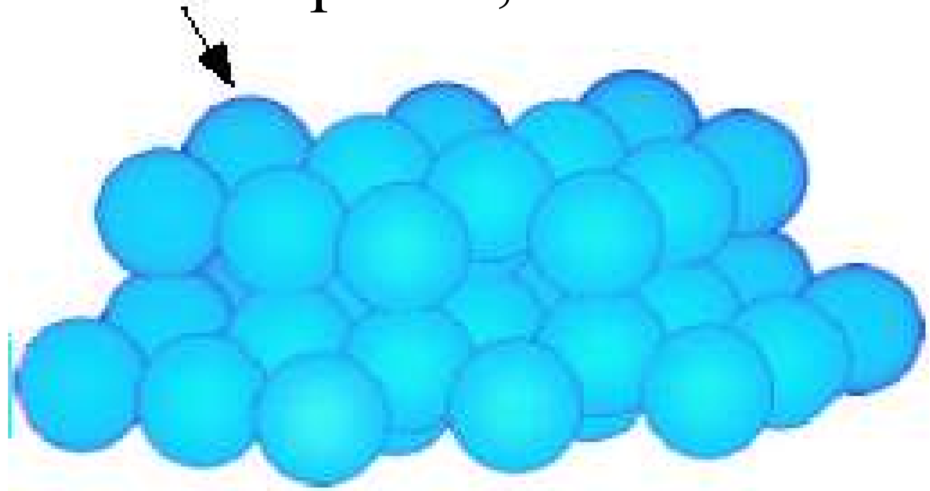
Electronic and Photonic Crystals

Periodic Medium

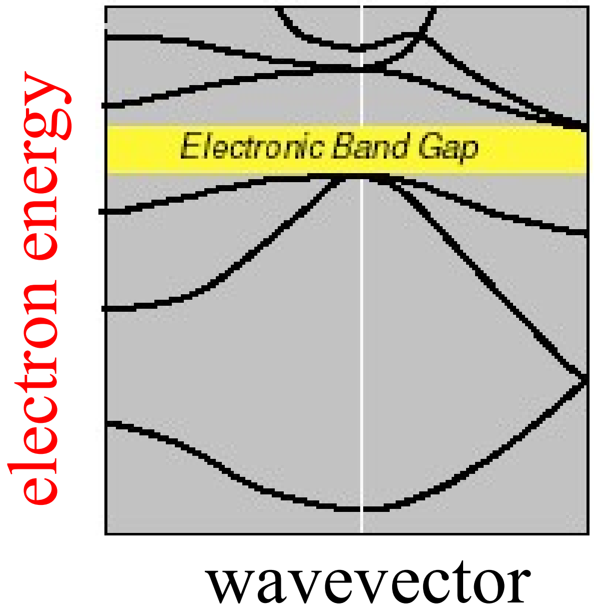
atoms in diamond structure

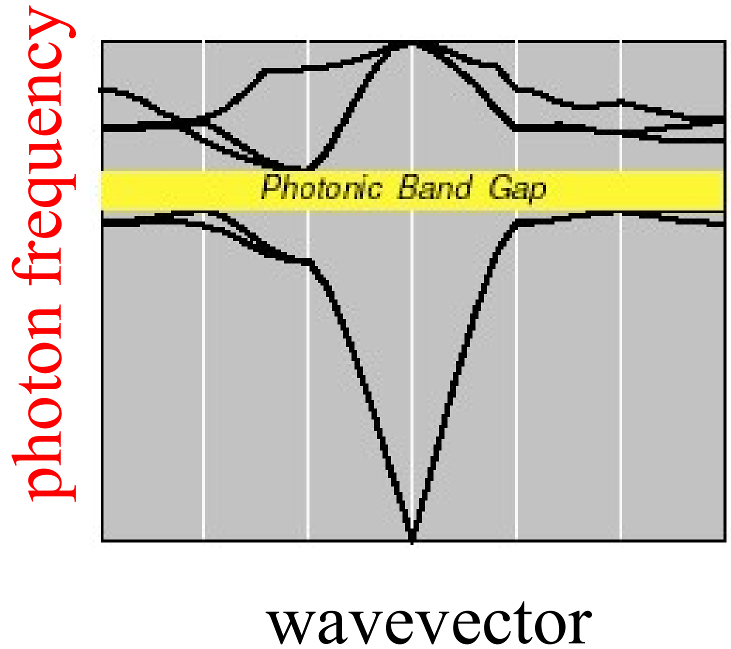


dielectric spheres, diamond lattice



Bloch waves: **Band Diagram**





strongly interacting fermions

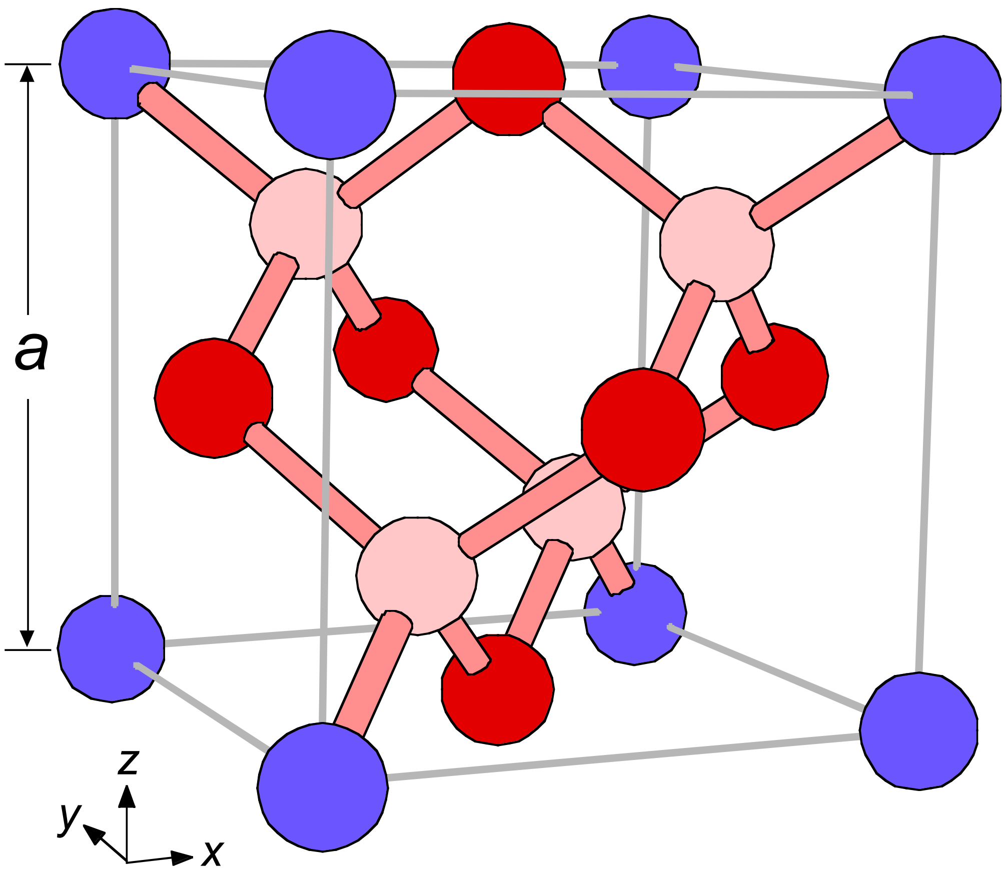
weakly-interacting bosons

The Mother of (almost) All Bandgaps

The diamond lattice:

fcc (face-centered-cubic)
with two “atoms” per unit cell

(primitive)



Recipe for a complete gap:

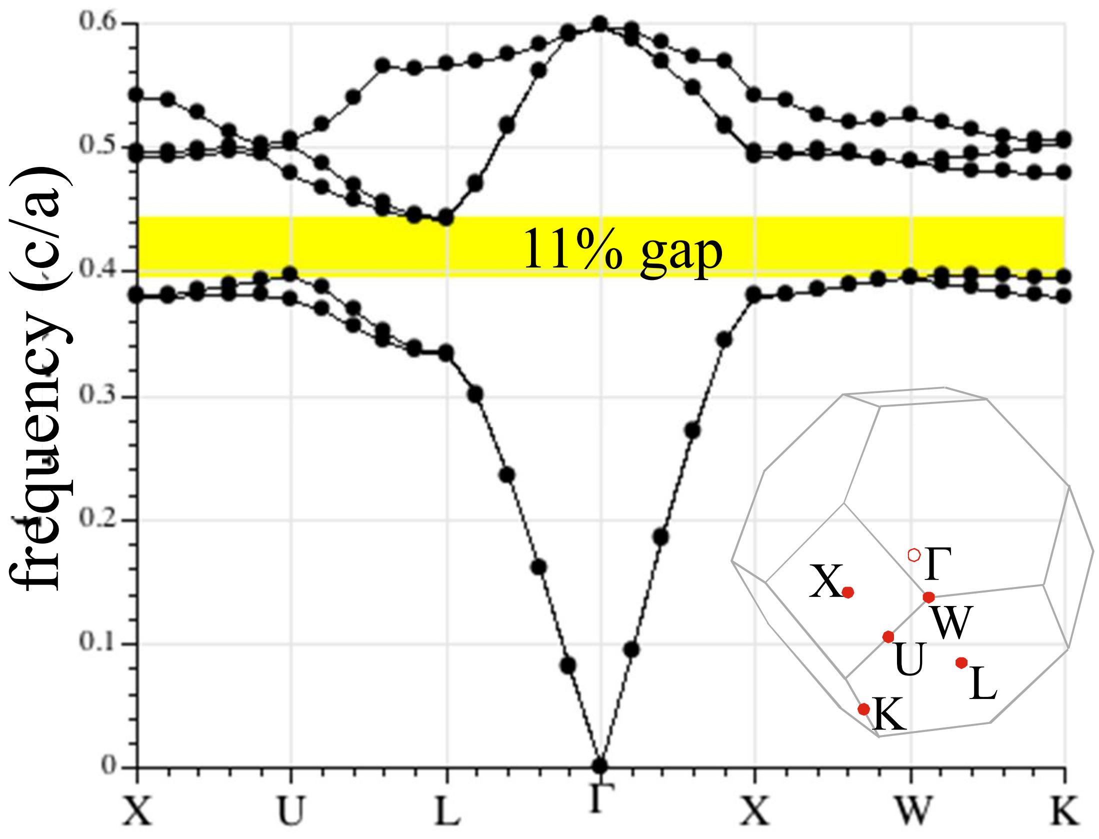
fcc = most-spherical Brillouin zone

+ diamond “bonds” = lowest (two) bands can concentrate in lines

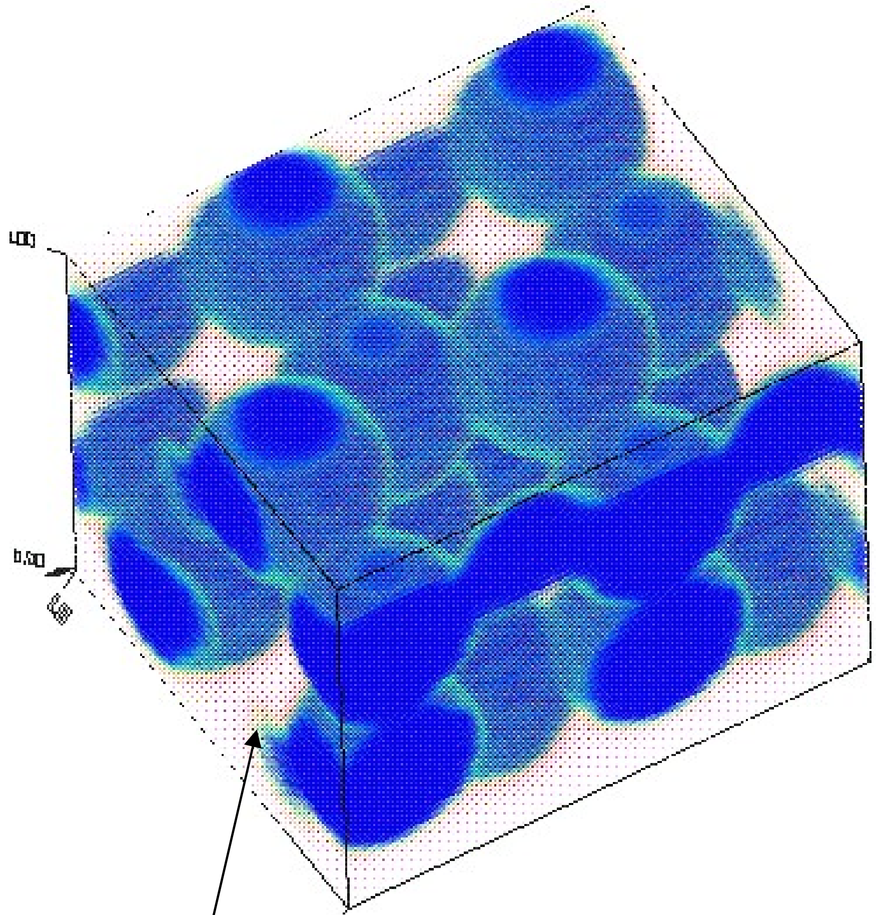
MPB tutorial, http://ab-initio.mit.edu/mpb

The First 3d Bandgap Structure

K. M. Ho, C. T. Chan, and C. M. Soukoulis, *Phys. Rev. Lett.* **65**, 3152 (1990).



for gap at λ = 1.55μm,
sphere diameter ~ 330nm



overlapping Si spheres

http://ab-initio.mit.edu/mpb

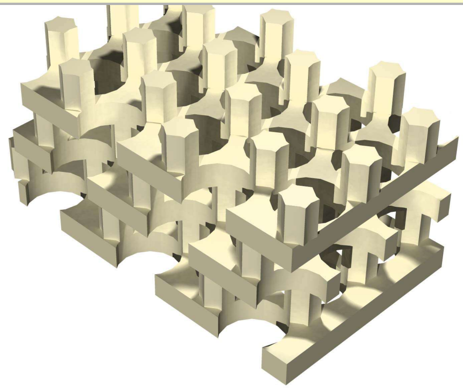
Layer-by-Layer Lithography

- Fabrication of 2d patterns in Si or GaAs is very advanced (think: Pentium IV, 50 million transistors)

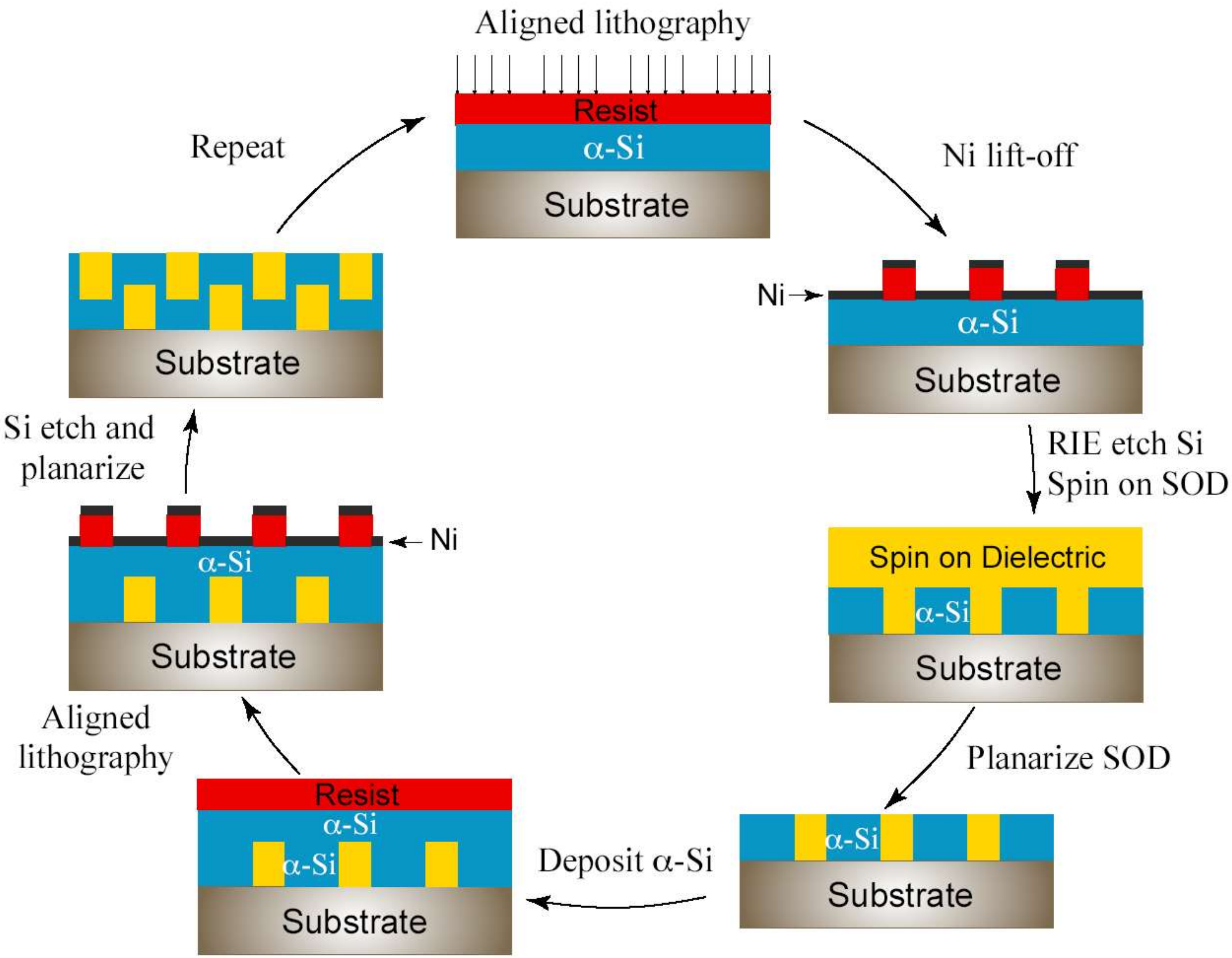
…inter-layer alignment techniques are only slightly more exotic

So, make 3d structure one layer at a time

Need a 3d crystal with constant cross-section layers



A Schematic

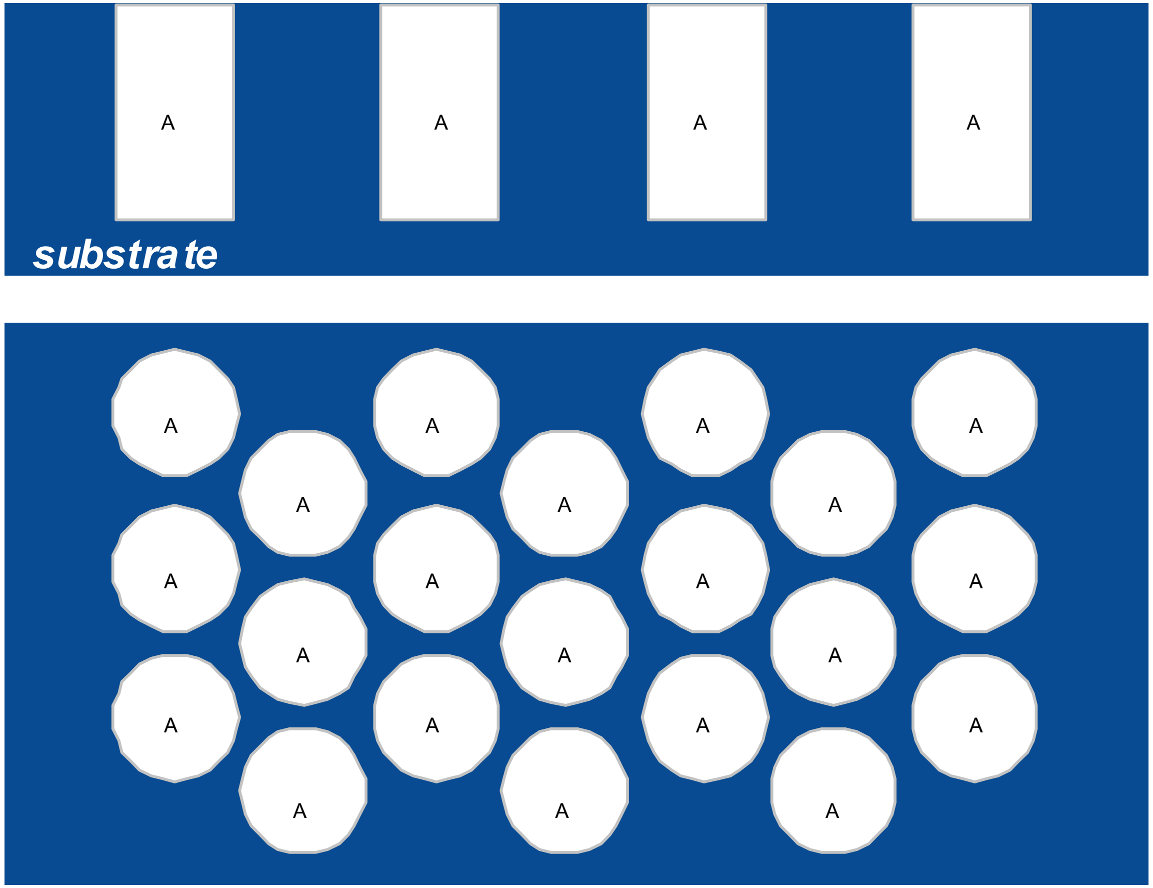


[M. Qi, H. Smith, MIT]

Making Rods & Holes Simultaneously

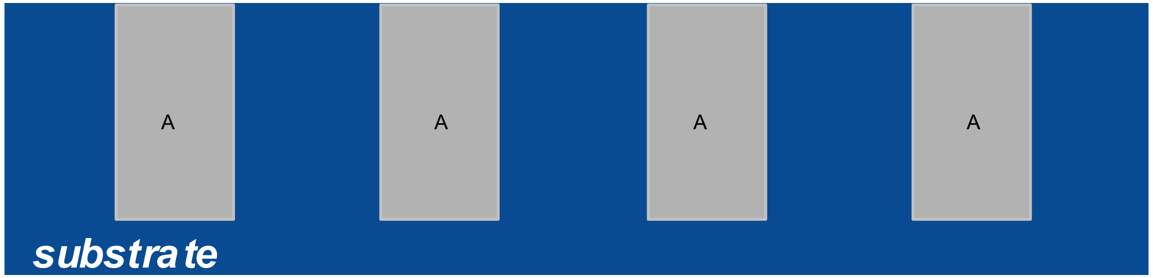
Making Rods & Holes Simultaneously

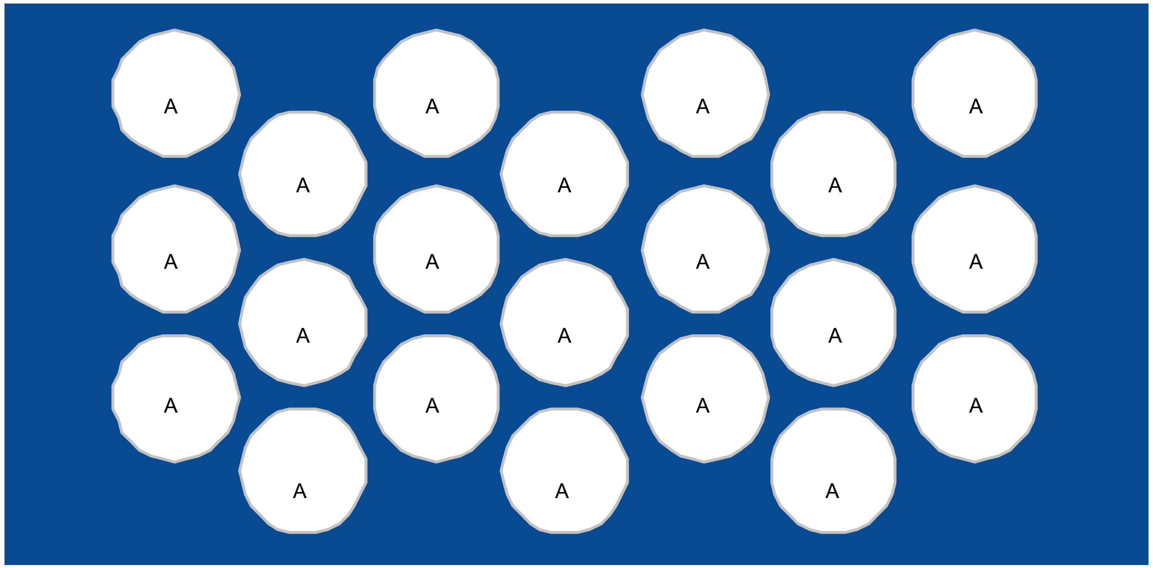
expose/etch
holes



Making Rods & Holes Simultaneously

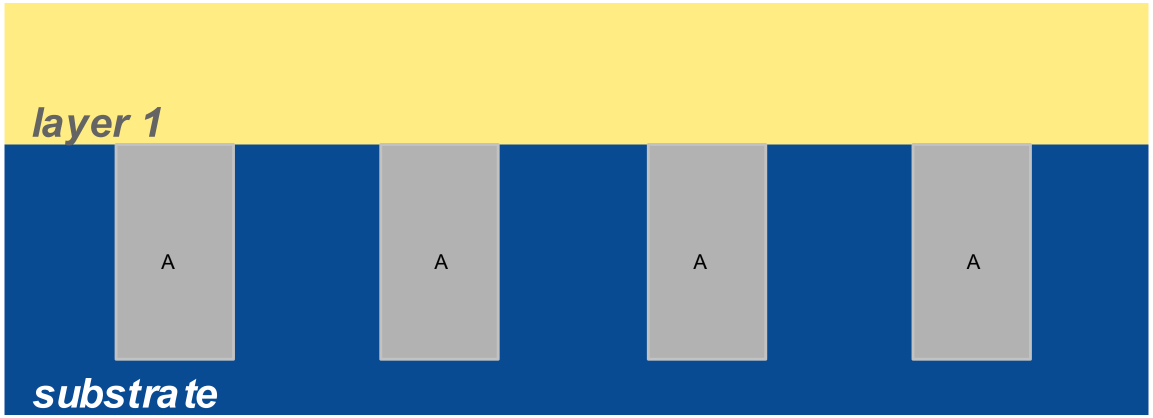
backfill with silica (SiO_2) & polish

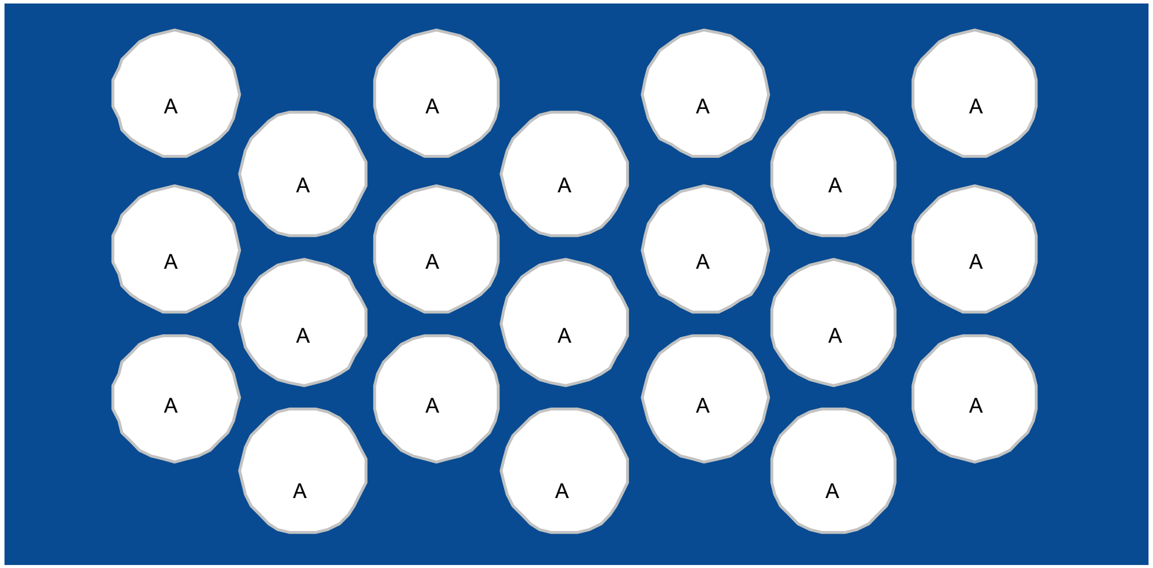




Making Rods & Holes Simultaneously

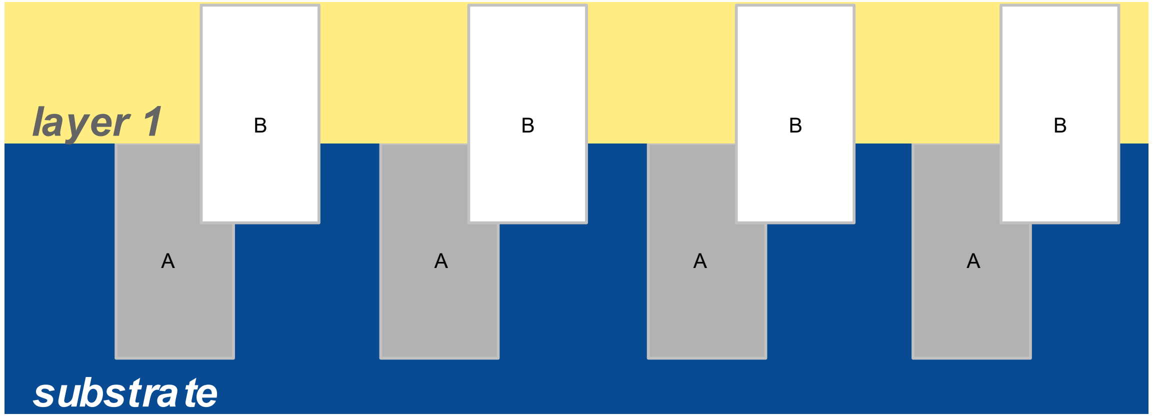
deposit another
Si layer

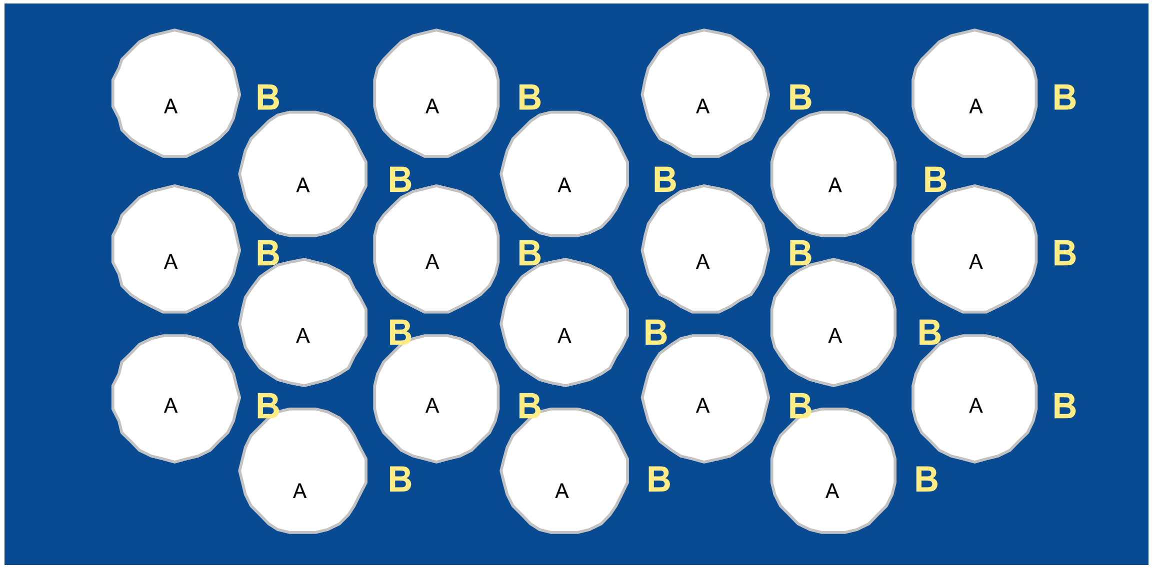




Making Rods & Holes Simultaneously

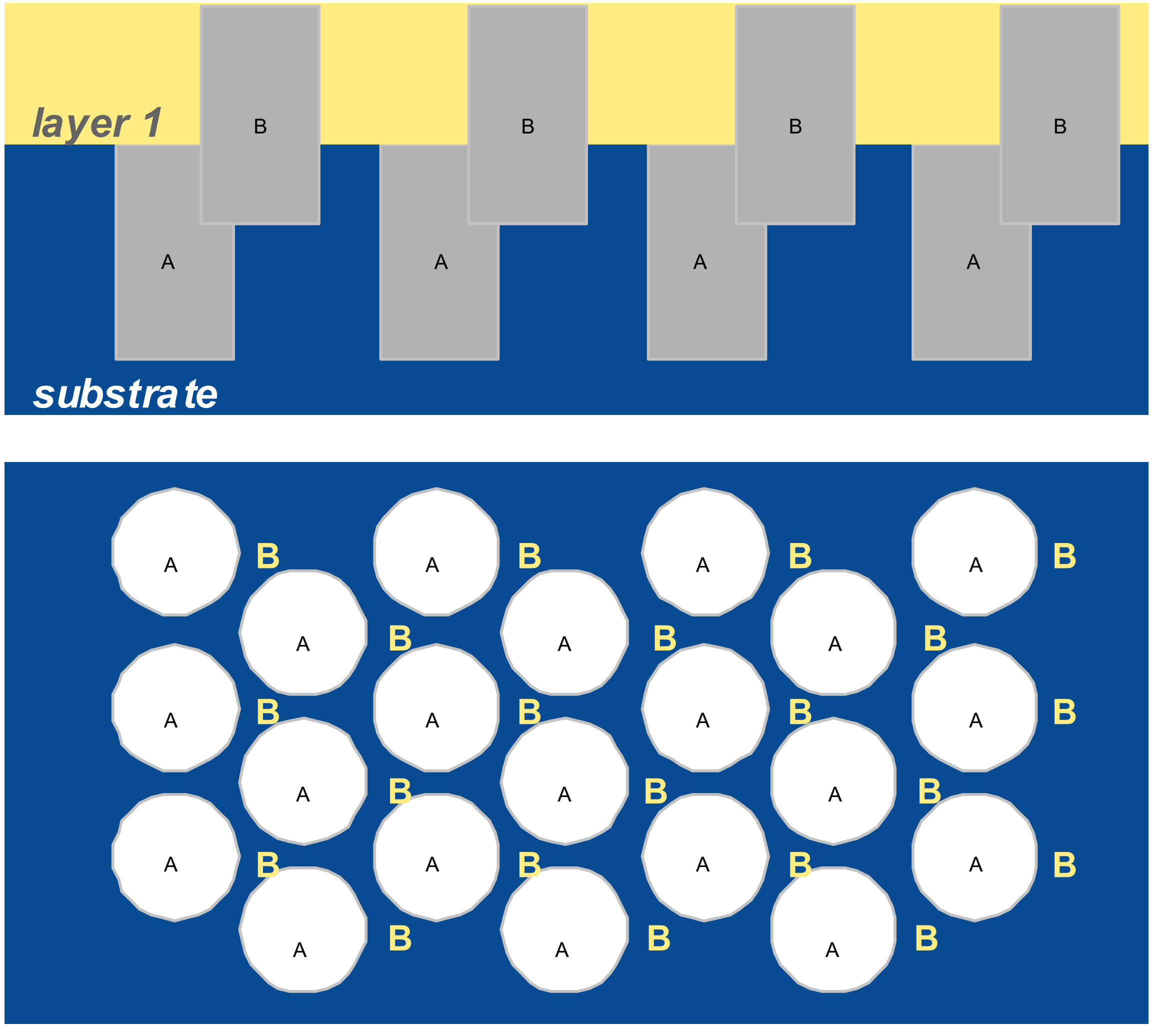
dig more holes
offset
& overlapping





Making Rods & Holes Simultaneously

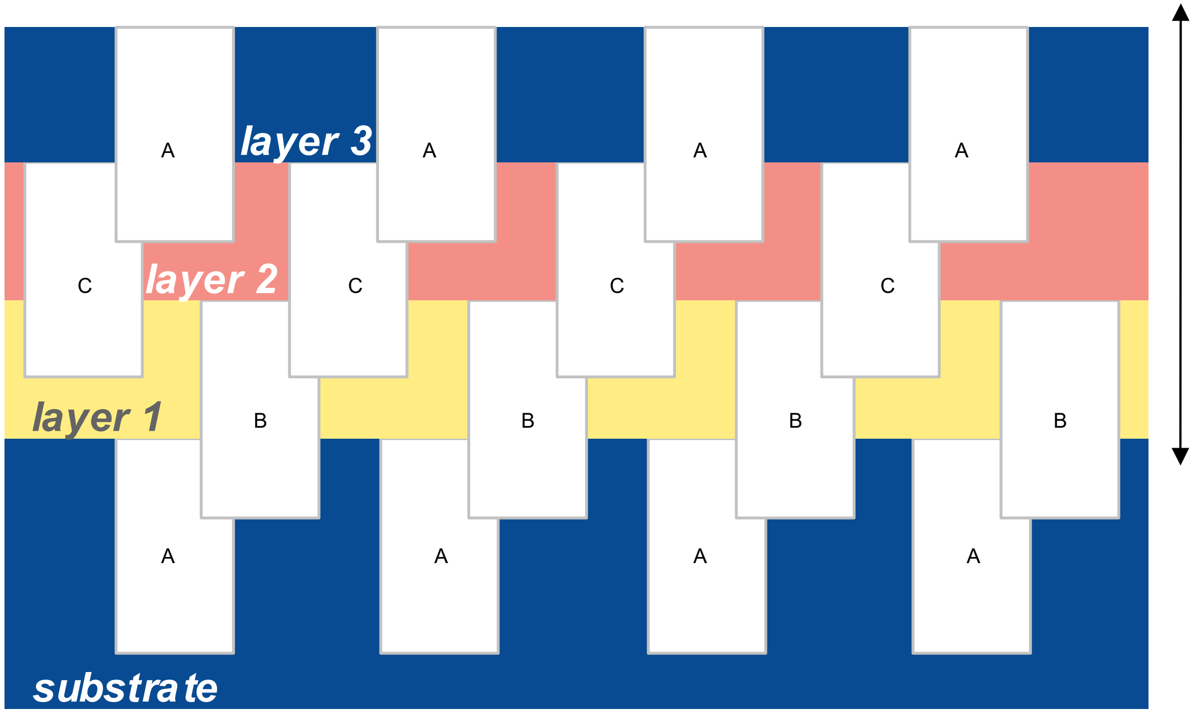
backfill



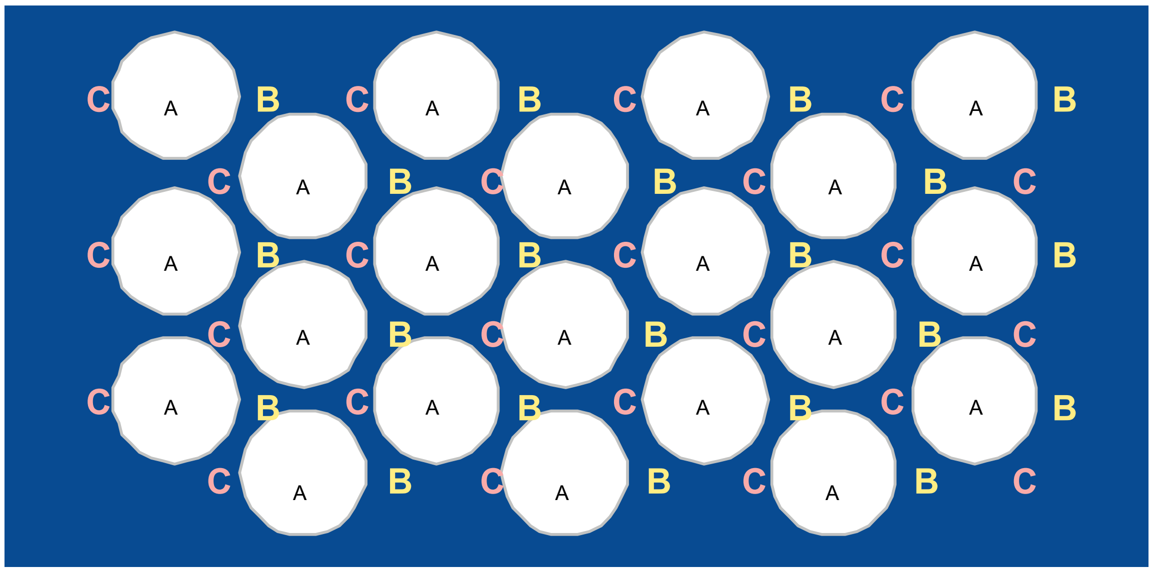
Making Rods & Holes Simultaneously

etcetera

(dissolve silica when done)

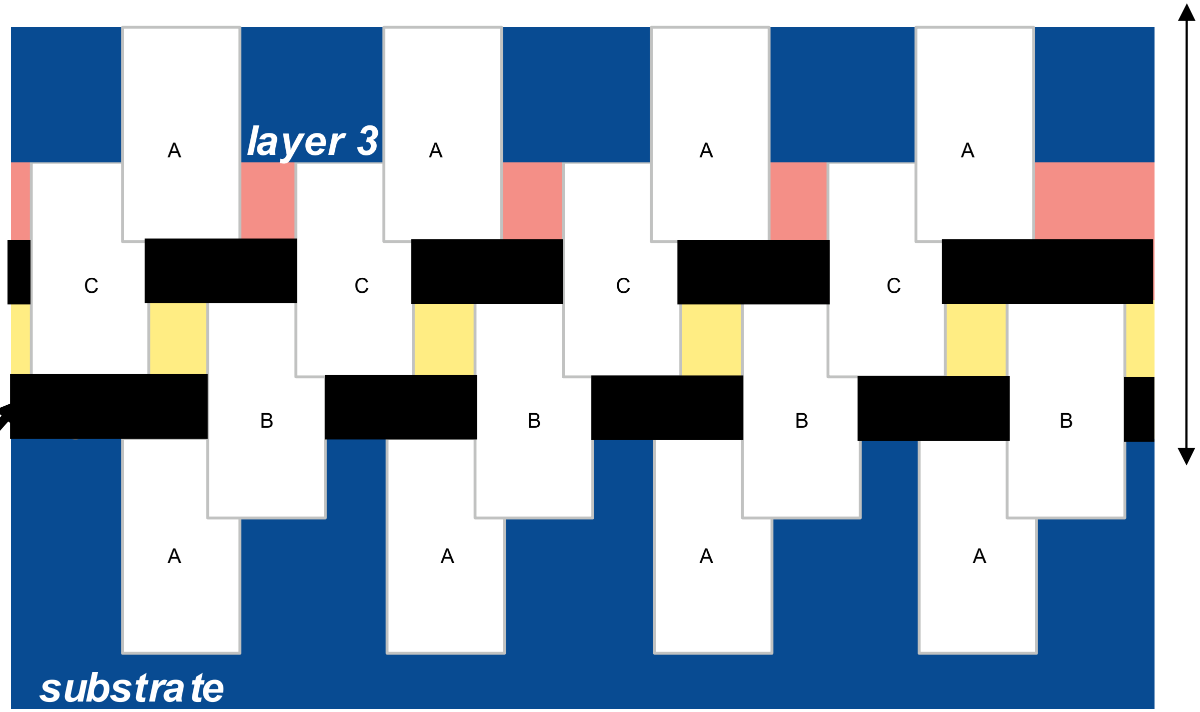


one period



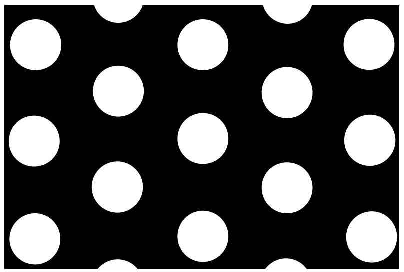
Making Rods & Holes Simultaneously

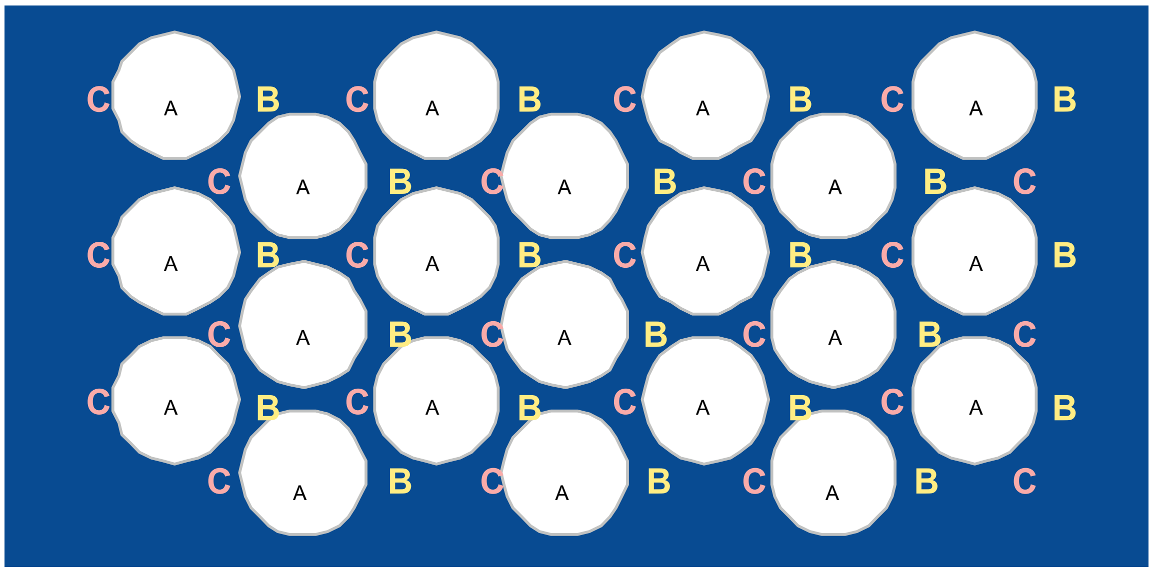
etcetera



one period

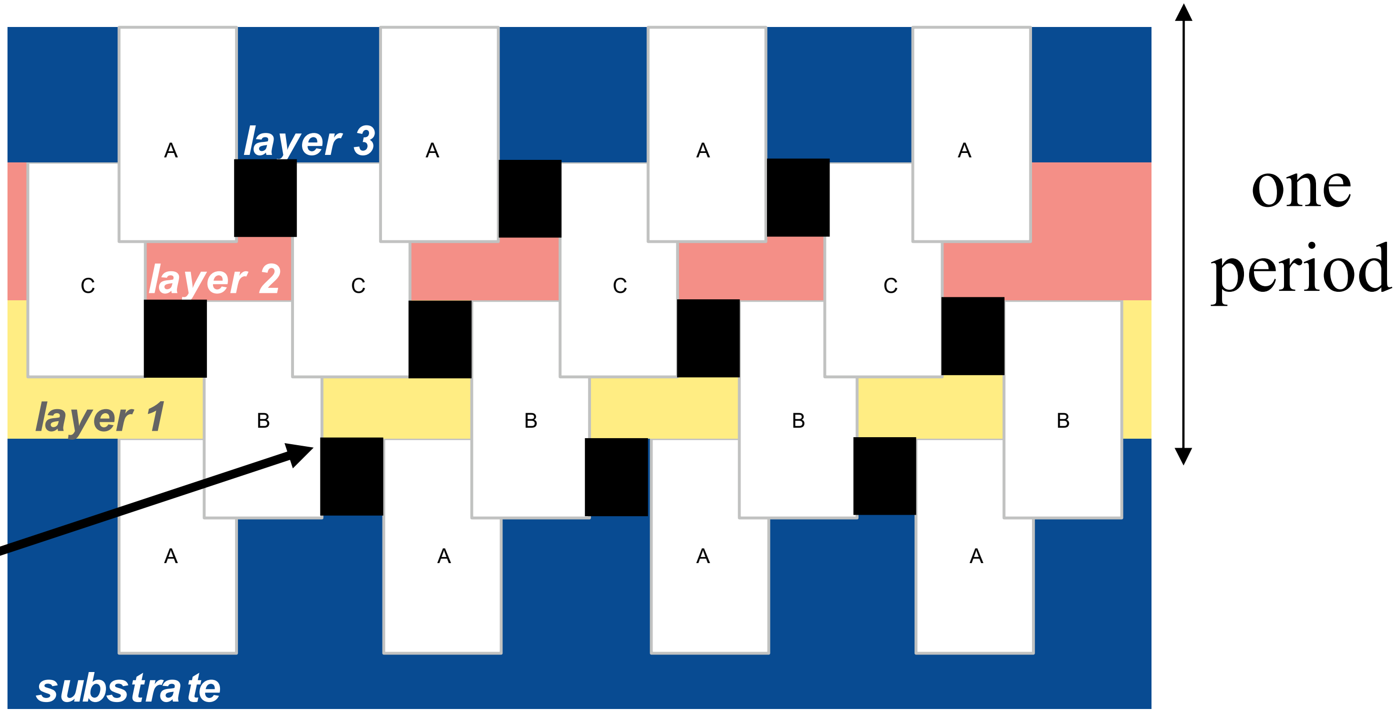
hole layers



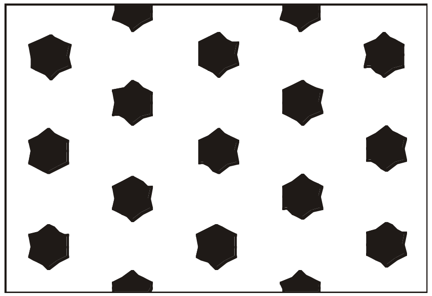


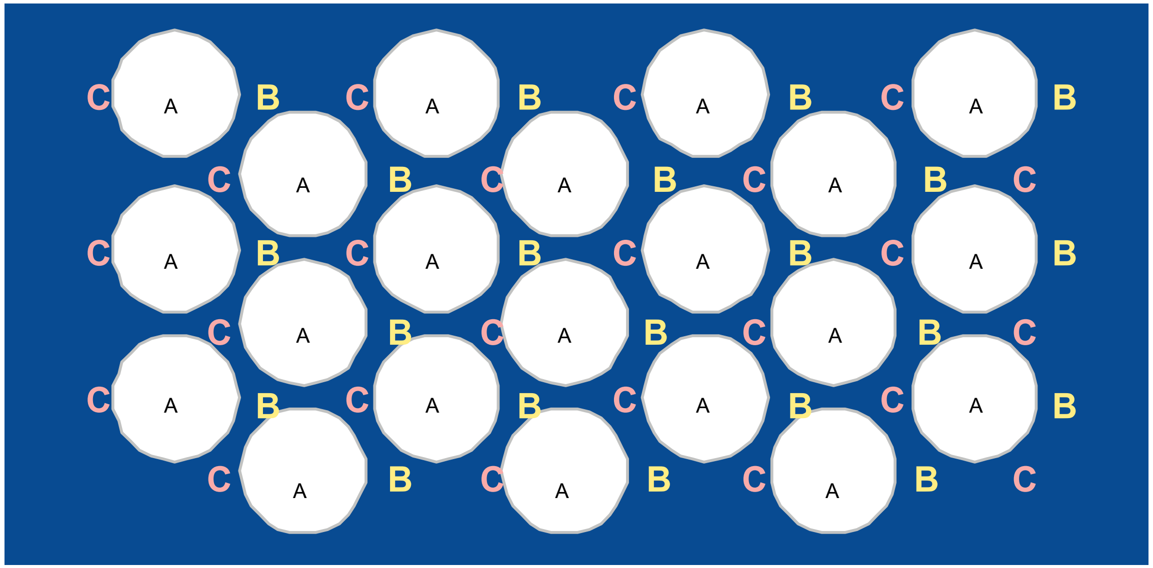
Making Rods & Holes Simultaneously

etcetera

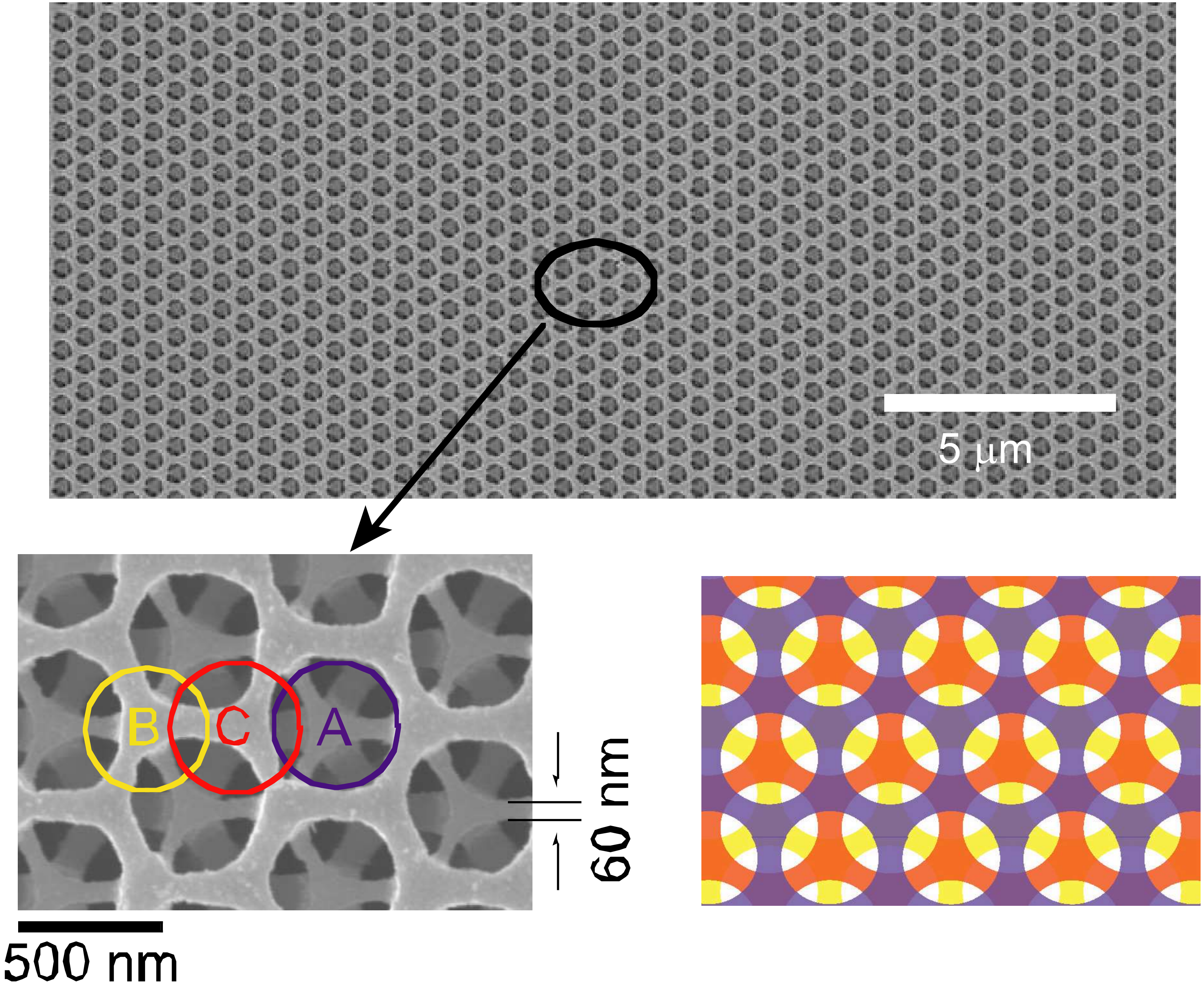


rod layers





7-layer E-Beam Fabrication



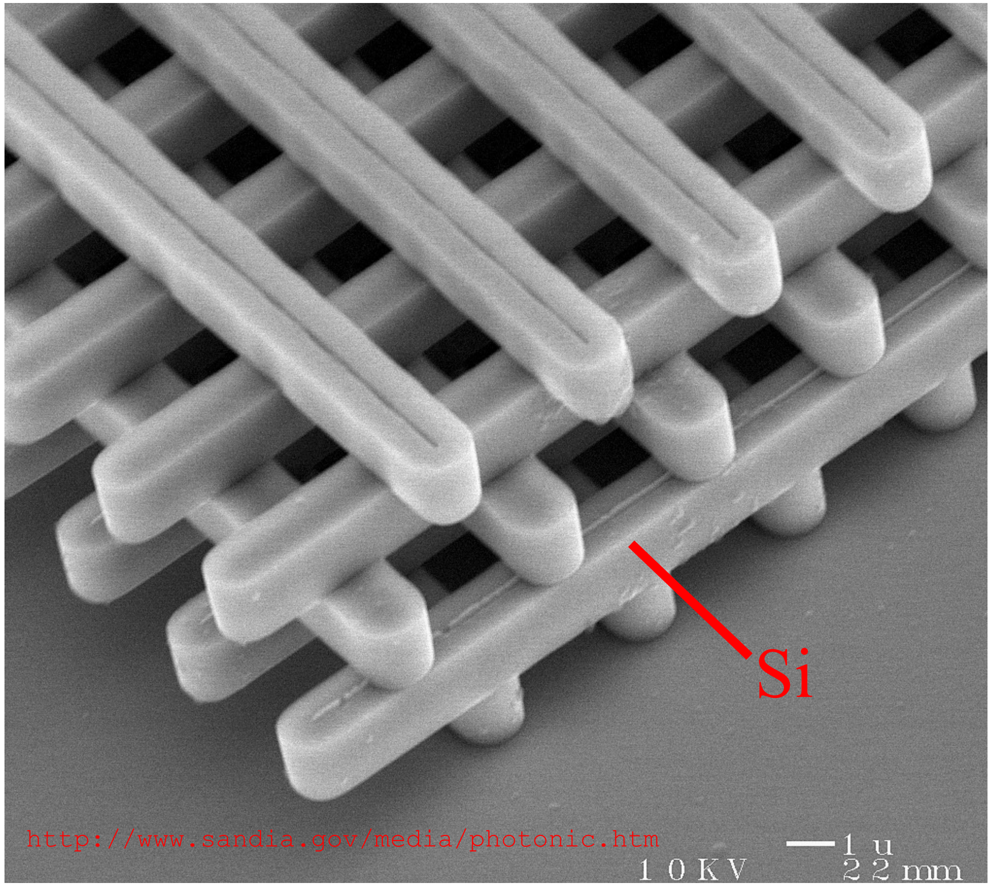
[M. Qi, *et al.*, *Nature* **429**, 538 (2004)]

an earlier design:
(& currently more popular)

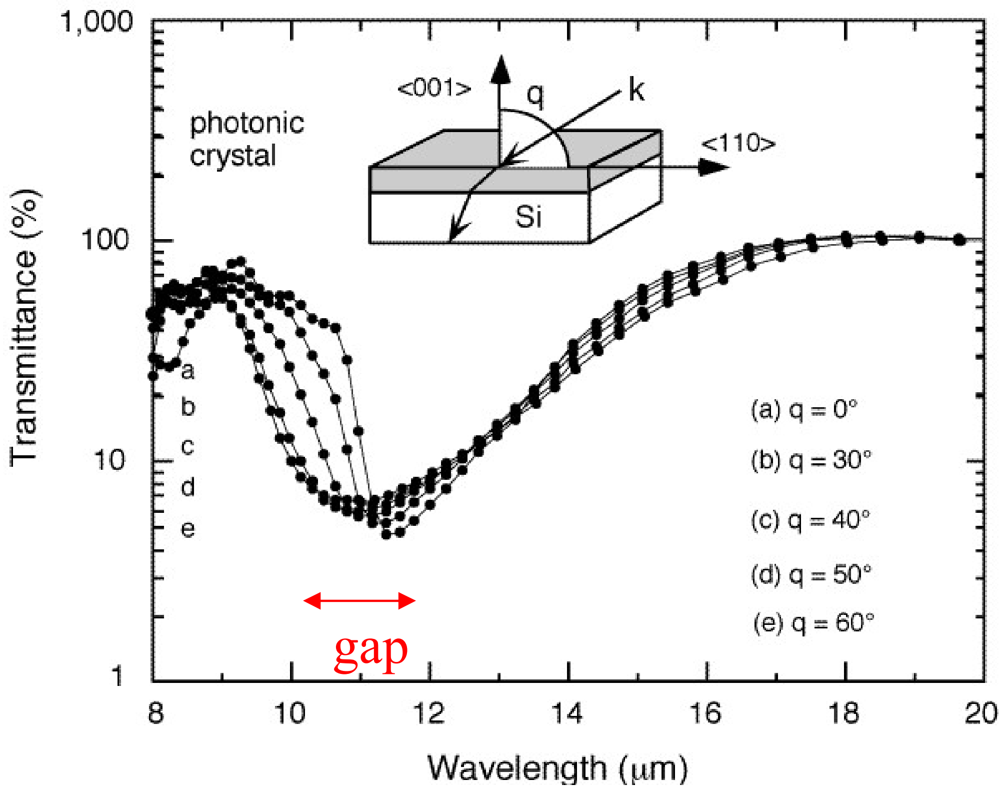
The Woodpile Crystal

[K. Ho *et al.*, *Solid State Comm.* **89**, 413 (1994)] [H. S. Sözüer *et al.*, *J. Mod. Opt.* **41**, 231 (1994)]

(4 "log" layers = 1 period)



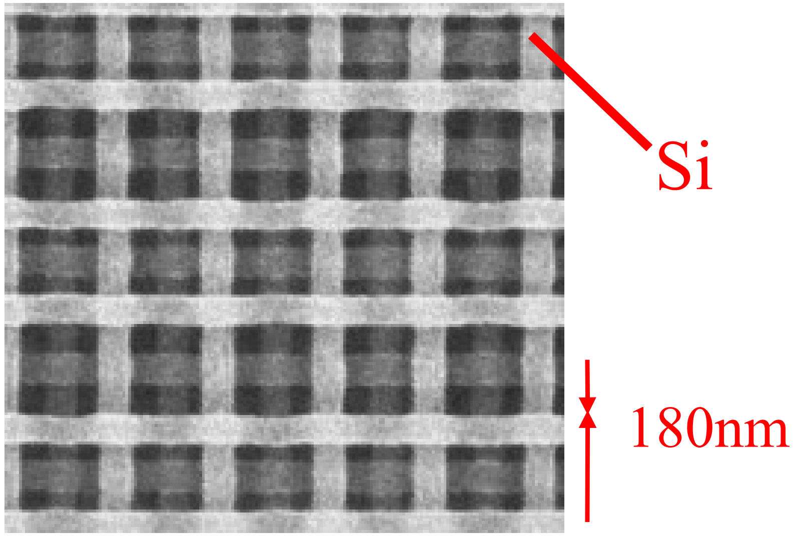
[S. Y. Lin *et al.*, *Nature* **394**, 251 (1998)]

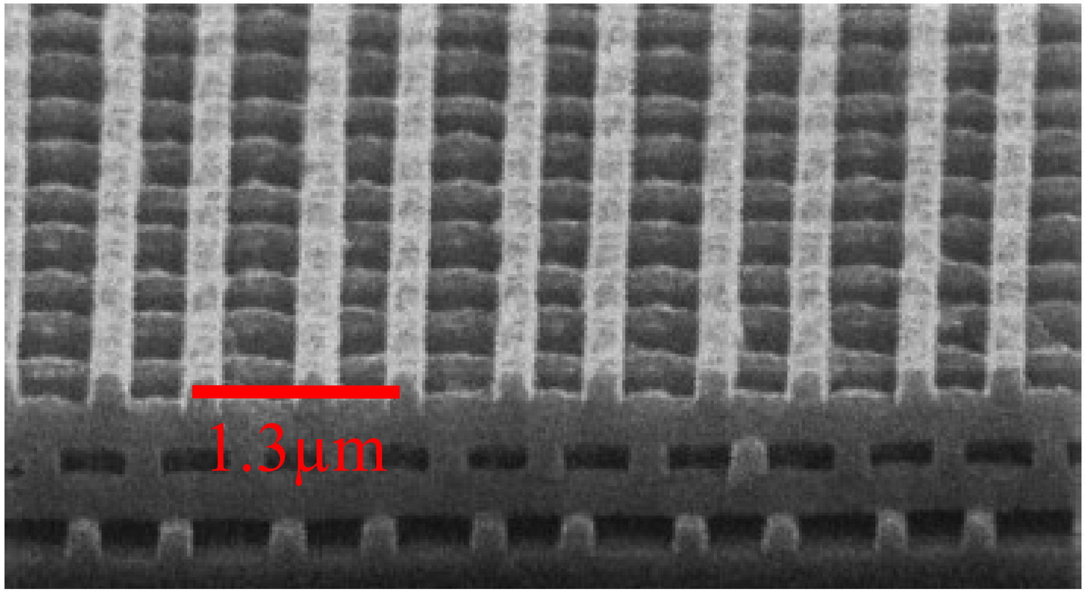


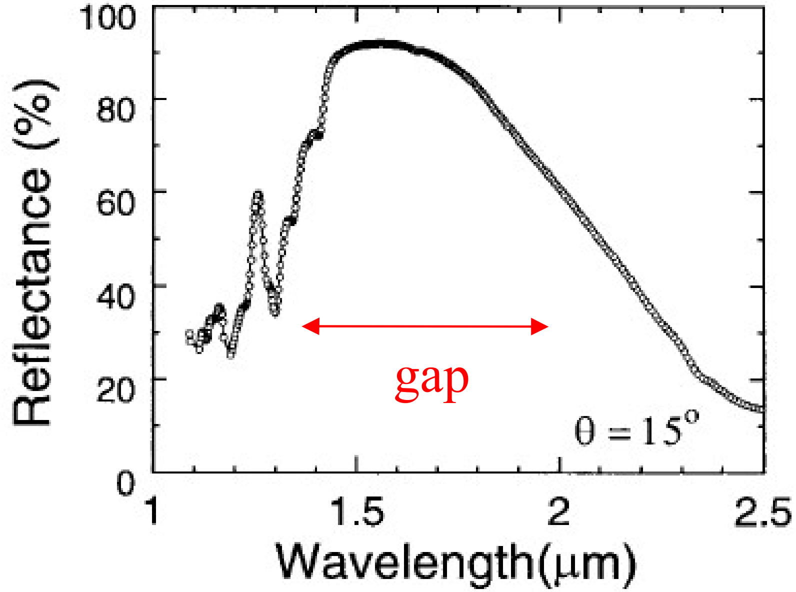
1.25 Periods of Woodpile @ 1.55μm

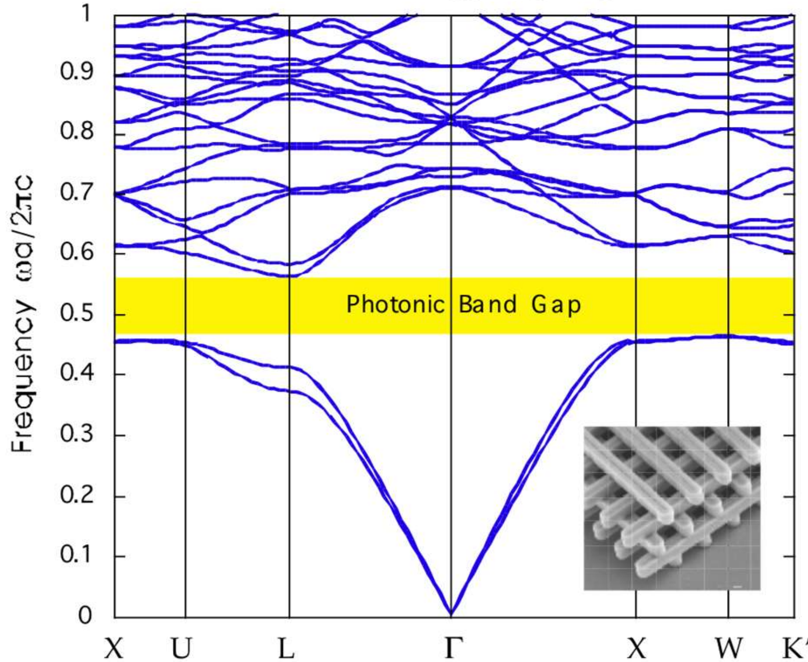
(4 "log" layers = 1 period)

[Lin & Fleming, *JLT* **17**, 1944 (1999)]

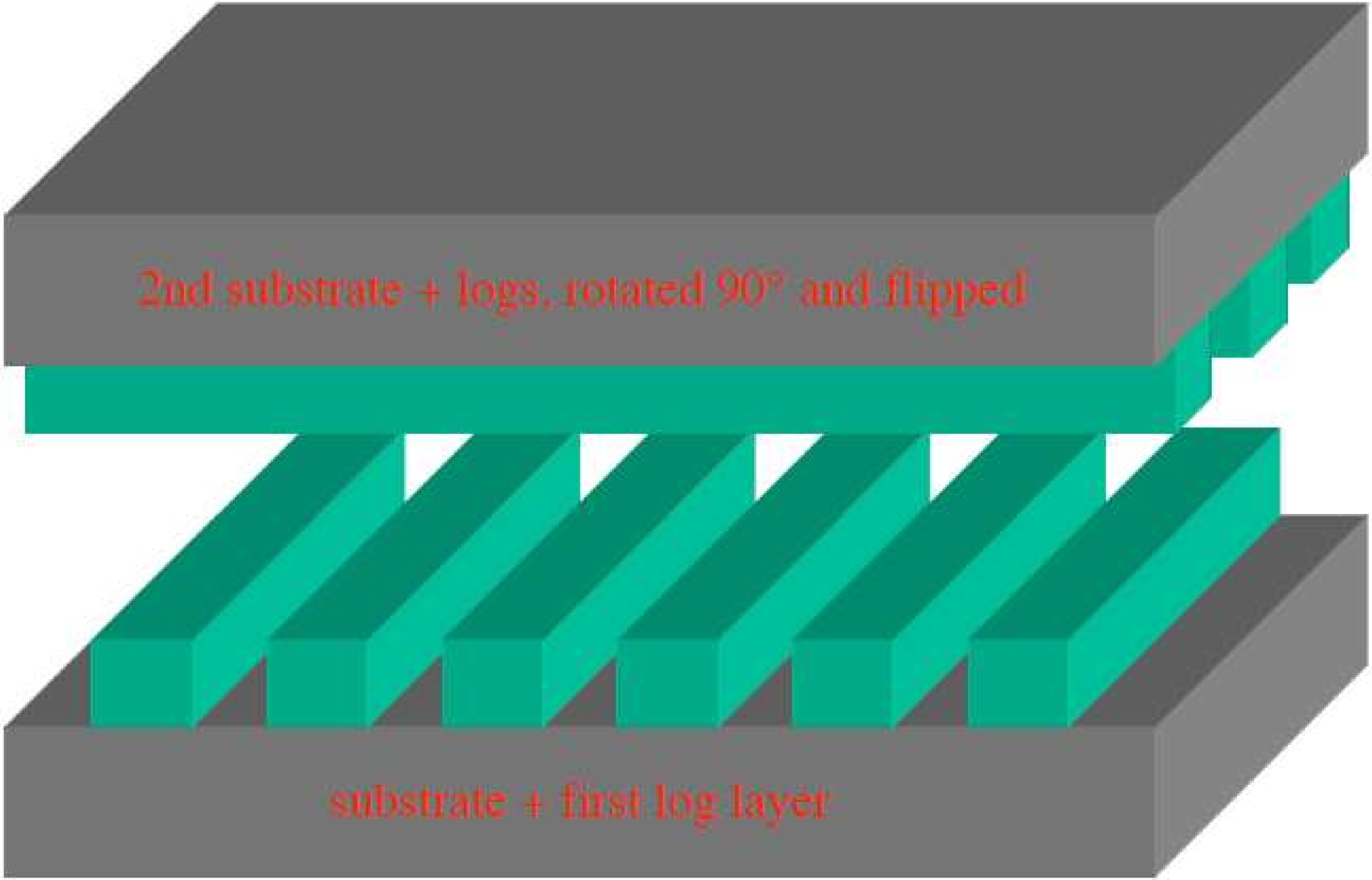








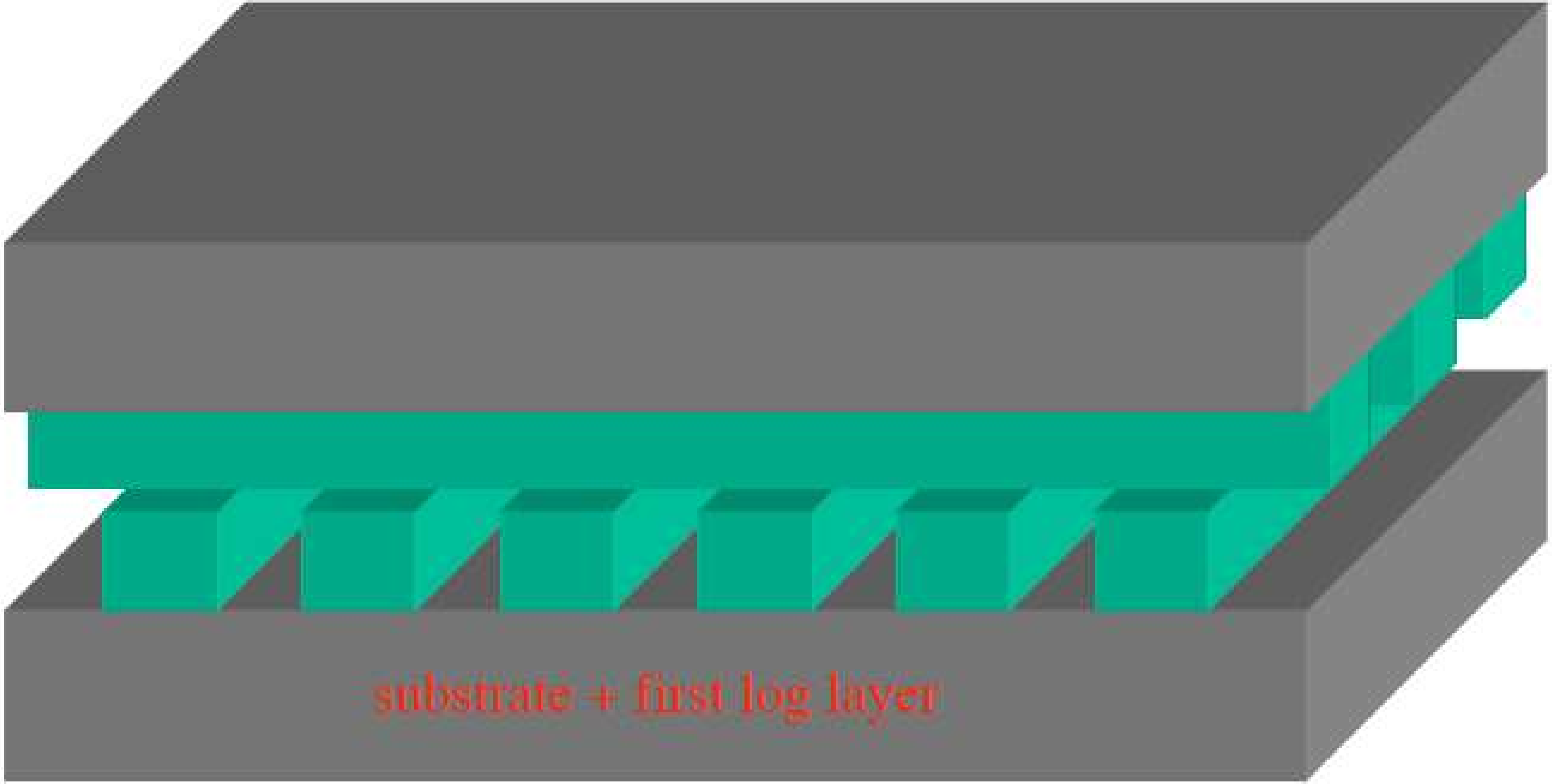
Woodpile by Wafer Fusion



[S. Noda *et al.*, *Science* **289**, 604 (2000)]

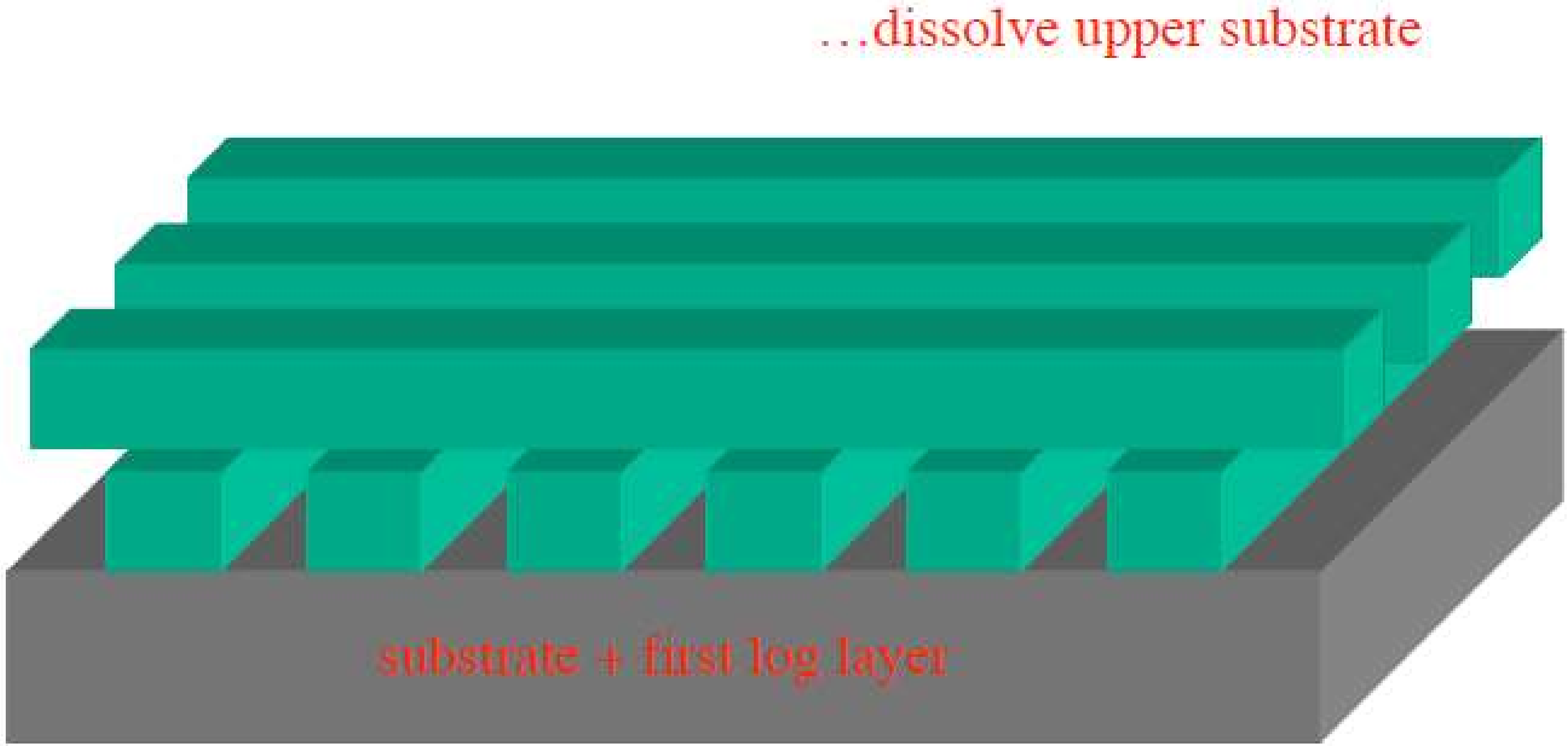
Woodpile by Wafer Fusion

fuse wafers together…



[S. Noda *et al.*, *Science* **289**, 604 (2000)]

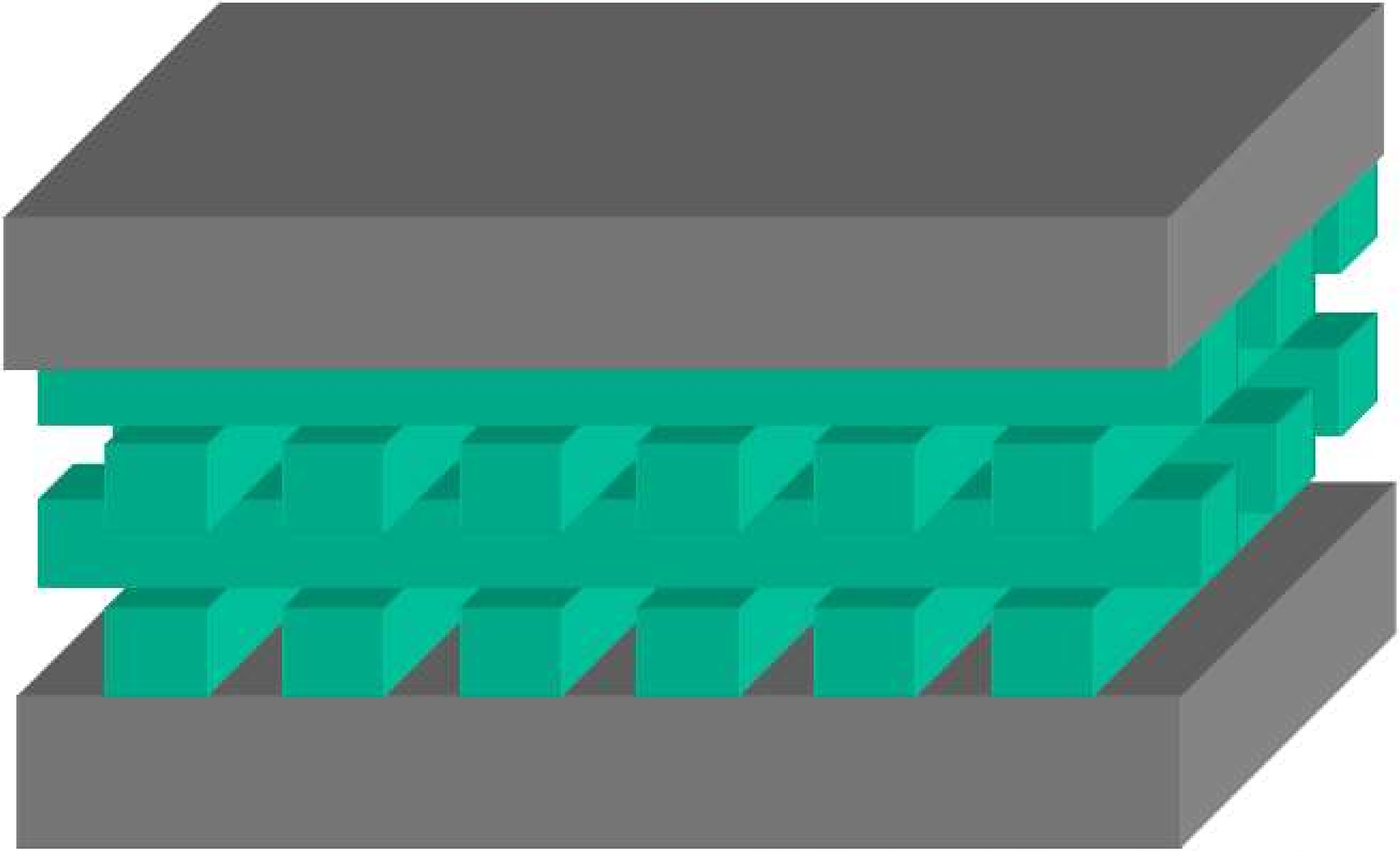
Woodpile by Wafer Fusion



[S. Noda *et al.*, *Science* **289**, 604 (2000)]

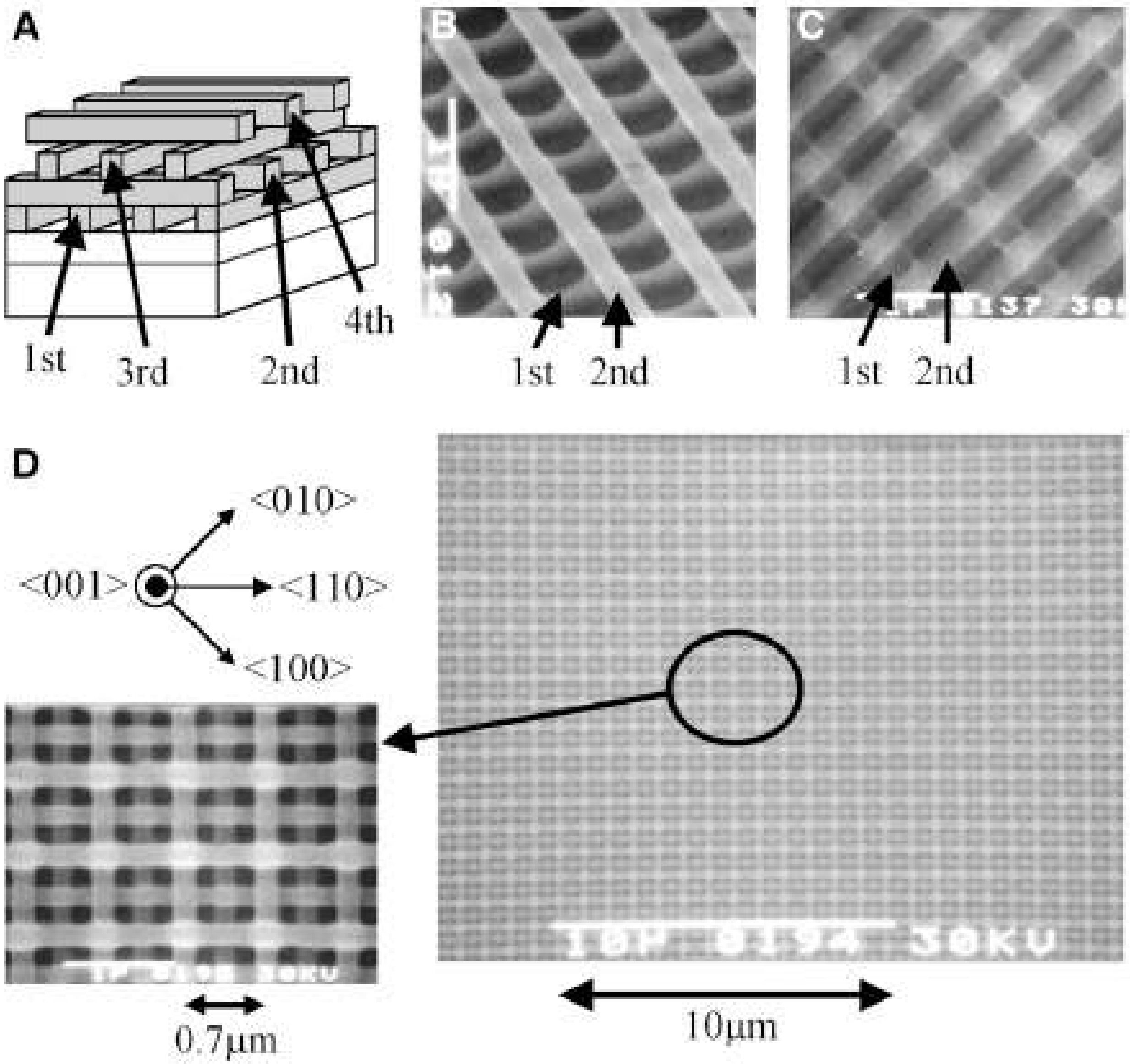
Woodpile by Wafer Fusion

double, double, toil and trouble…



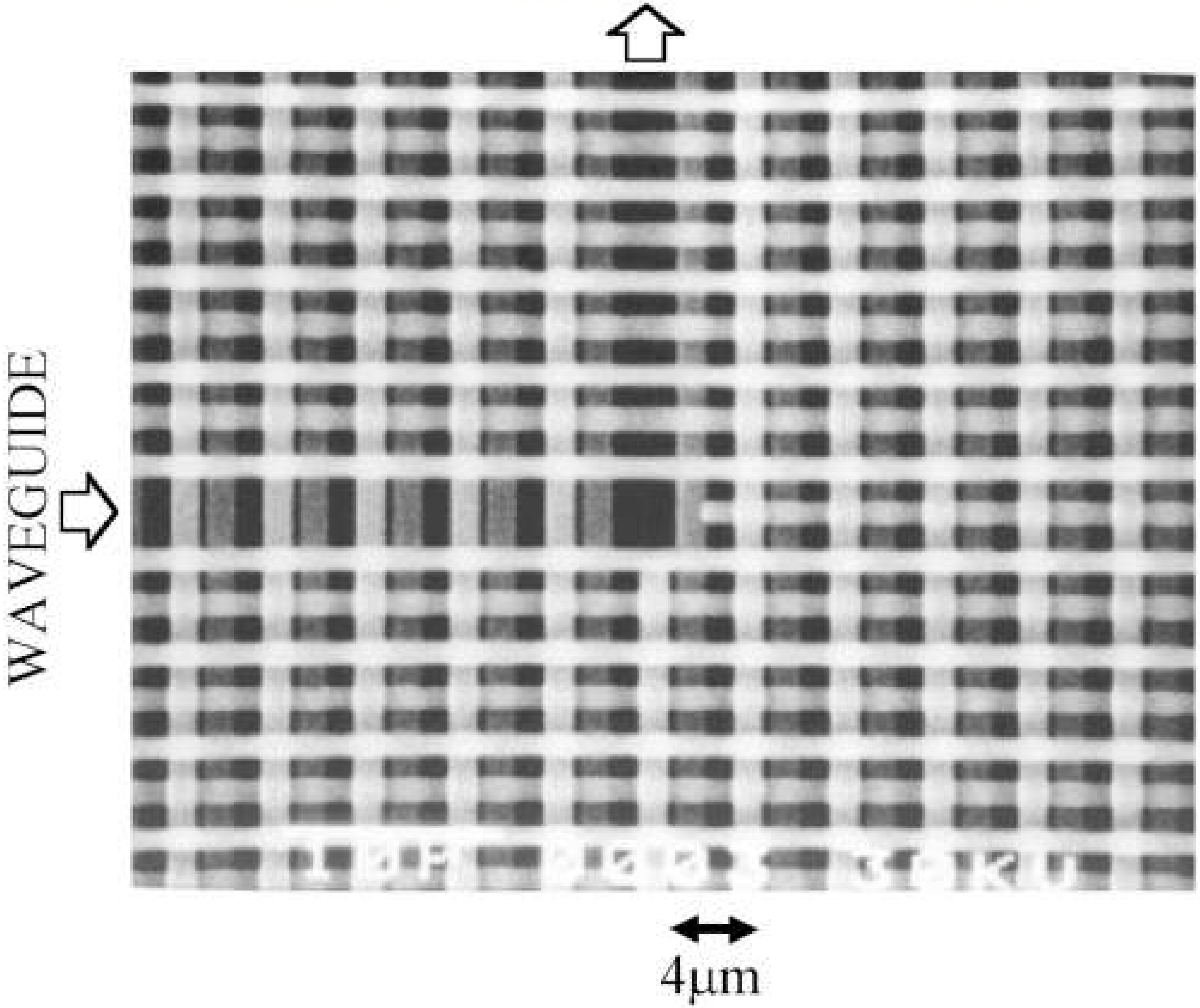
[S. Noda *et al.*, *Science* **289**, 604 (2000)]

"It's only wafer-thin." [M. Python]



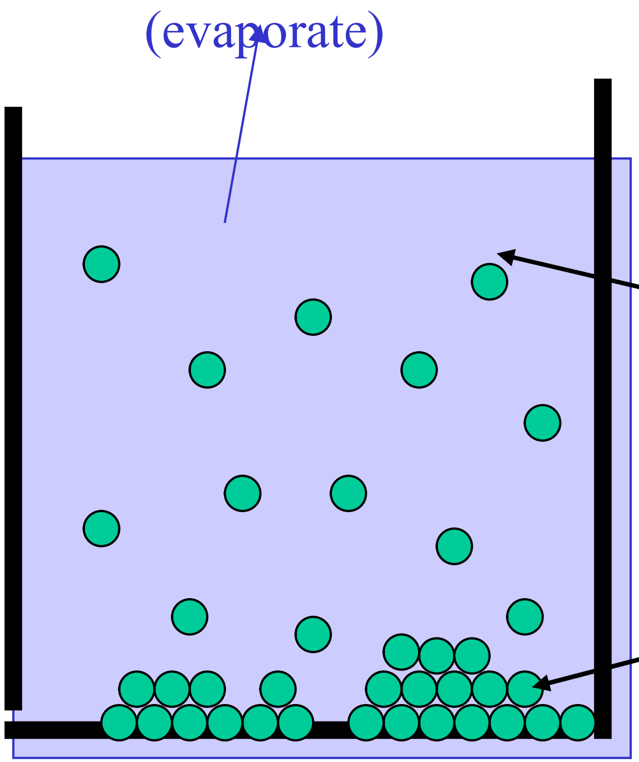
[S. Noda *et al.*, *Science* **289**, 604 (2000)]

Finally, a Defect!



[S. Noda *et al.*, *Science* **289**, 604 (2000)]

Mass-production II: Colloids

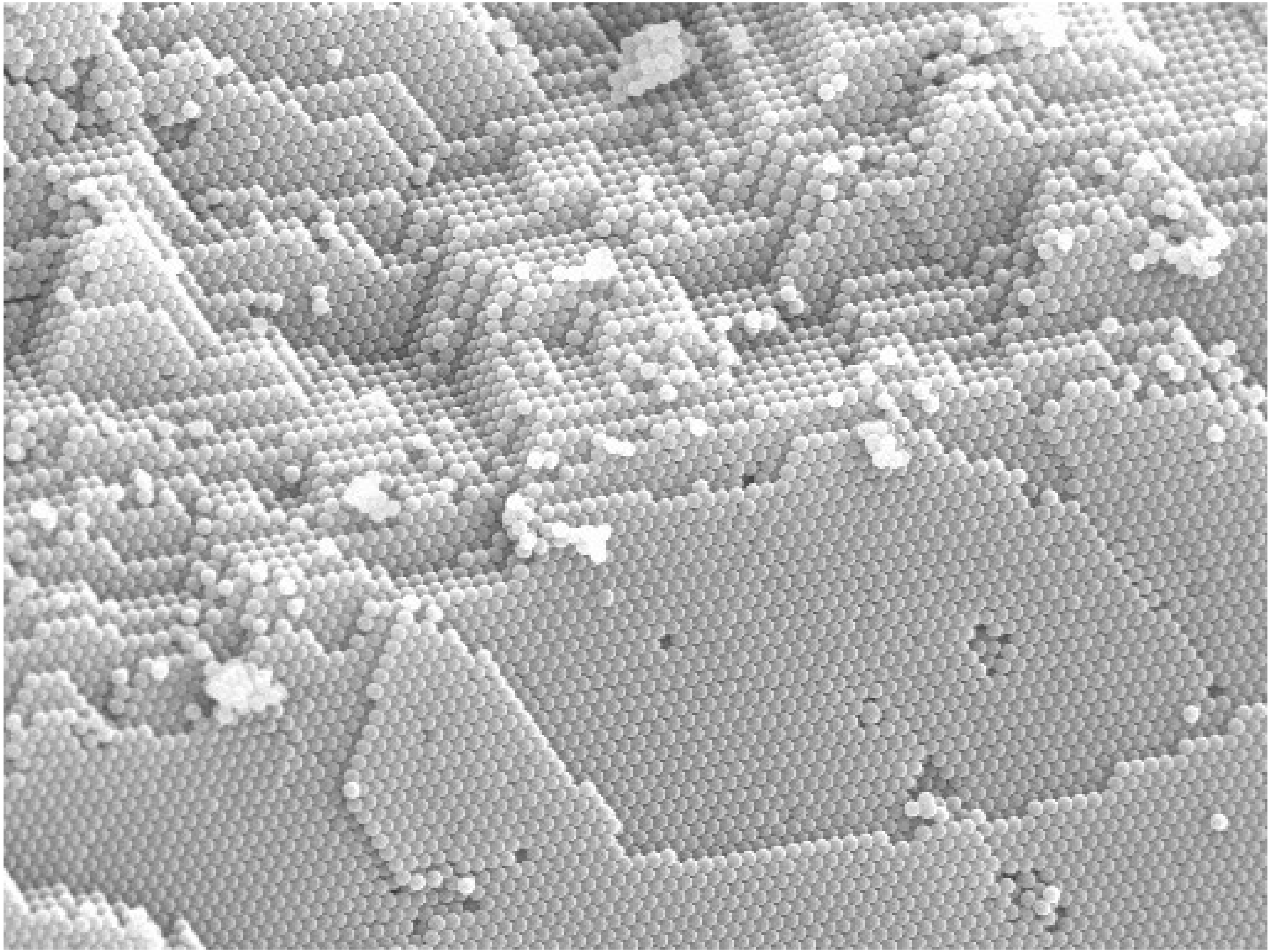


silica (SiO_2) microspheres (diameter < 1μm)

sediment by gravity into close-packed fcc lattice!

On creating nanoparticles, from:
http://www.icmm.csic.es/cefe/
W. Stöber, A. Fink, and E. Bohn.1968. J. Colloid Interface Sci., 26, 62.

Mass-production II: Colloids



http://www.icmm.csic.es/cefe/

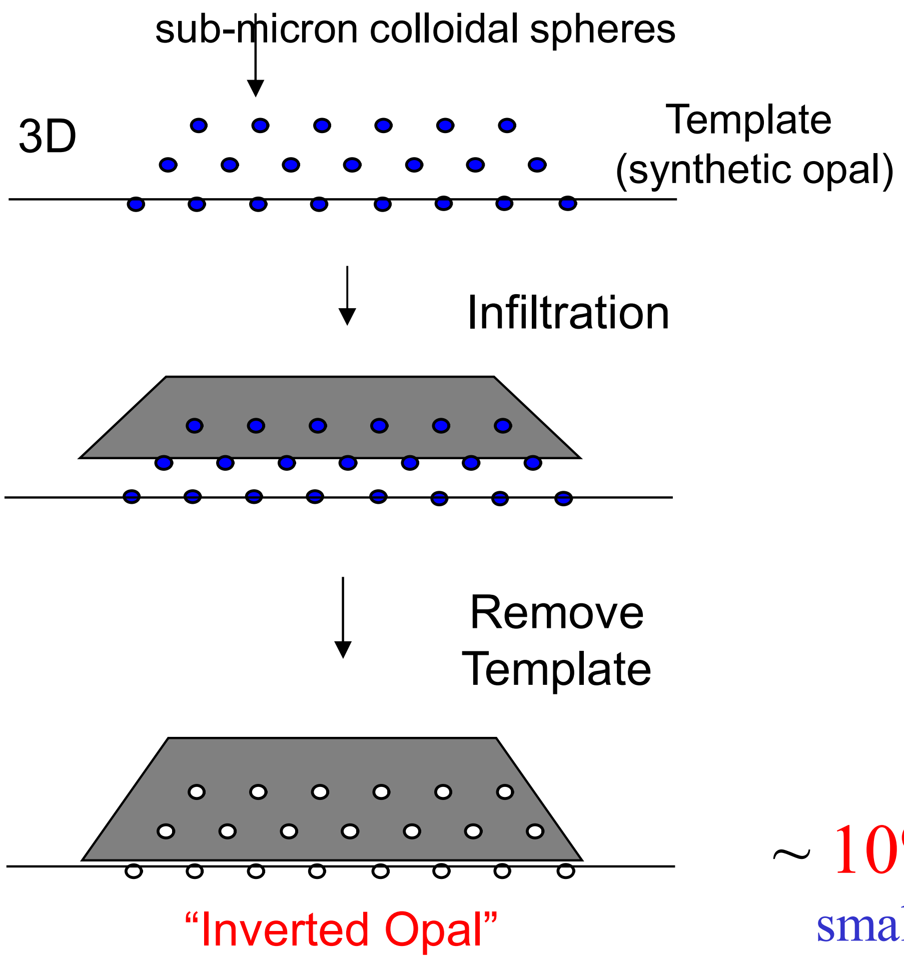
Inverse Opals

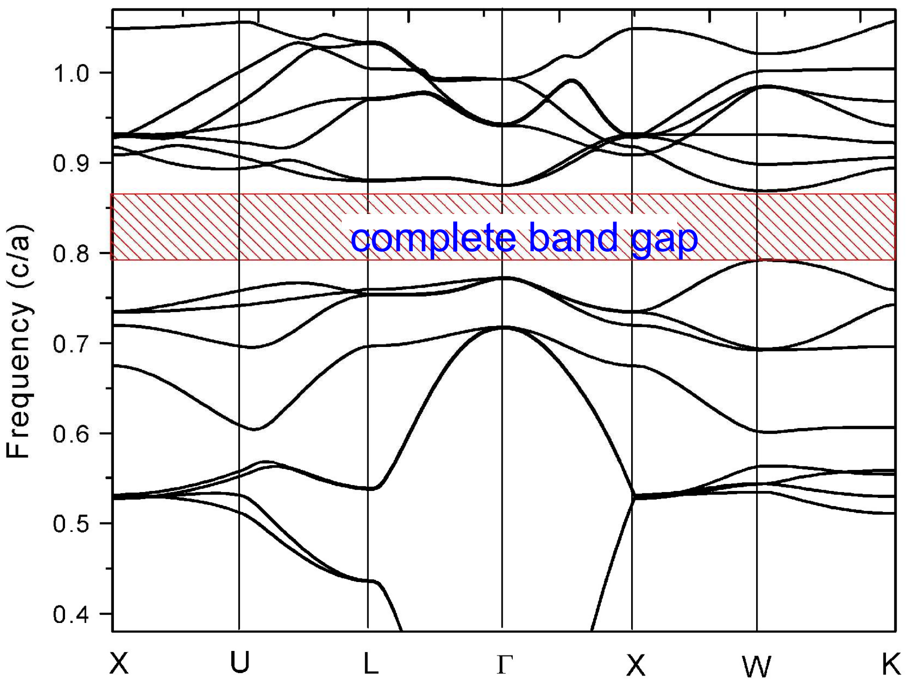
[figs courtesy
D. Norris, UMN]

[H. S. Sözüer, *PRB* **45**, 13962 (1992)]

fcc solid spheres do not have a gap…

…but fcc spherical holes in Si *do* have a gap



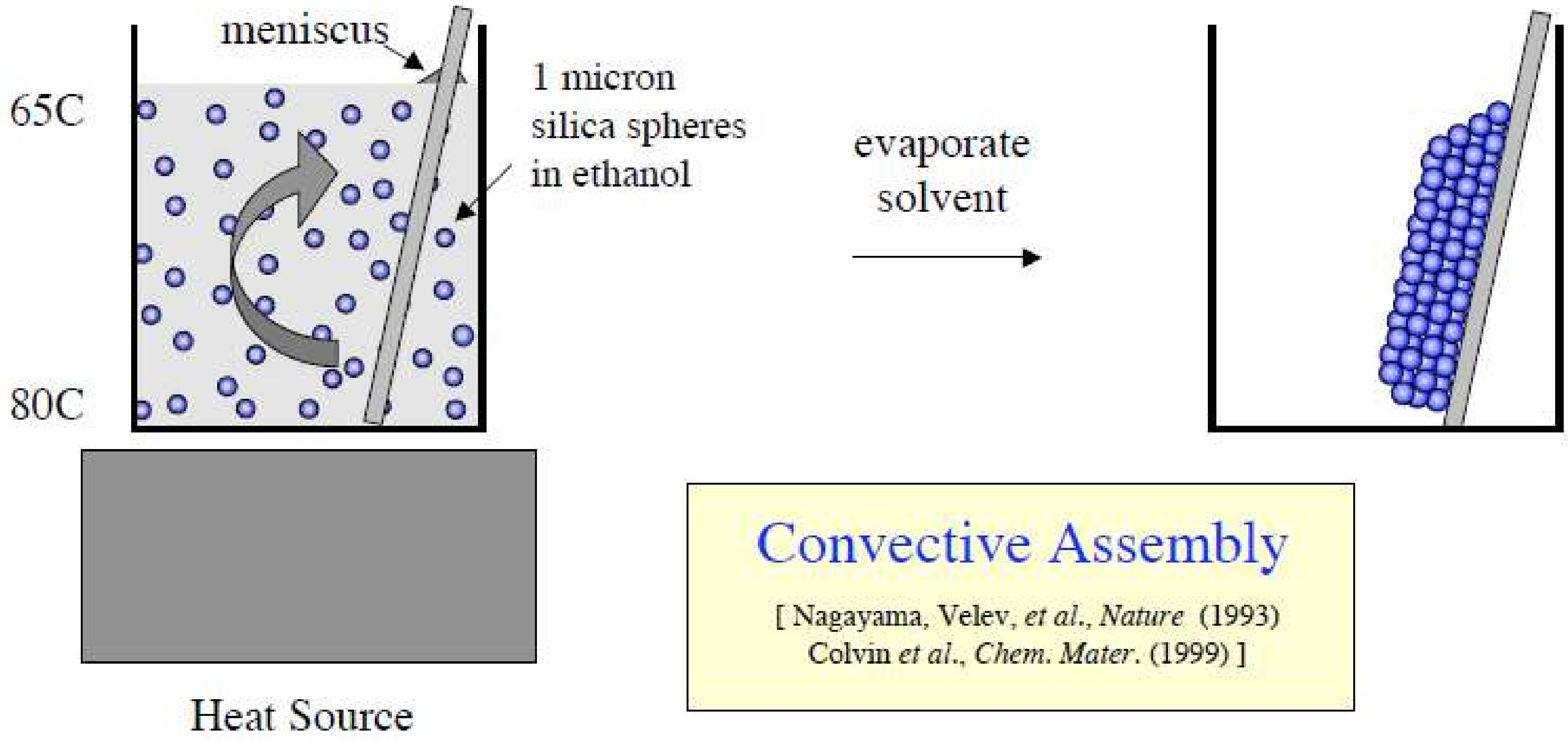


~ 10% gap between 8th & 9th bands

small gap, upper bands: sensitive to disorder

In Order To Form a More Perfect Crystal…

[figs courtesy D. Norris, UMN]



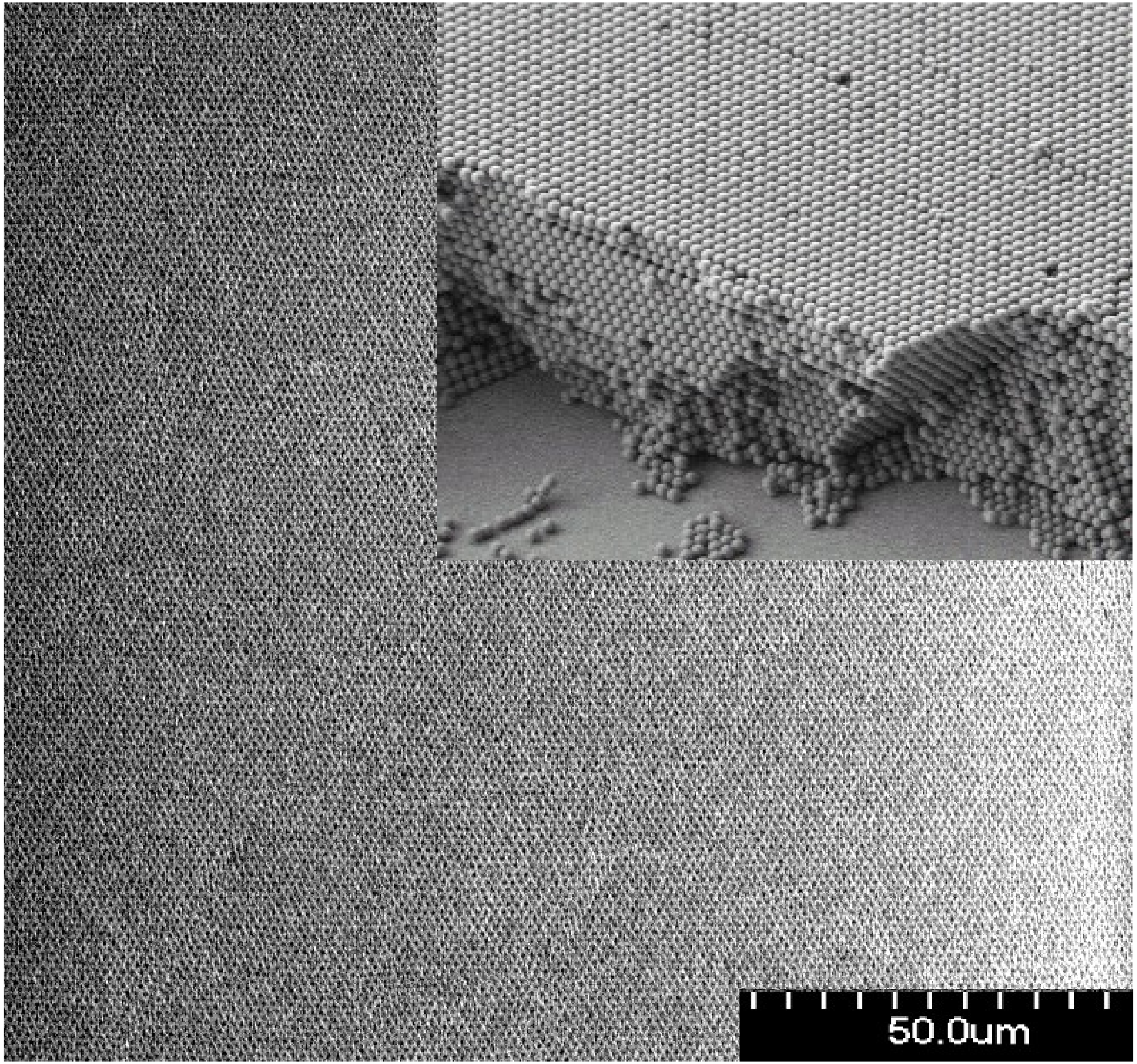
Convective Assembly

[Nagayama, Velev, *et al.*, *Nature* (1993)
Colvin *et al.*, *Chem. Mater.* (1999)]

- Capillary forces during drying cause assembly in the meniscus
- Extremely flat, large-area opals of controllable thickness

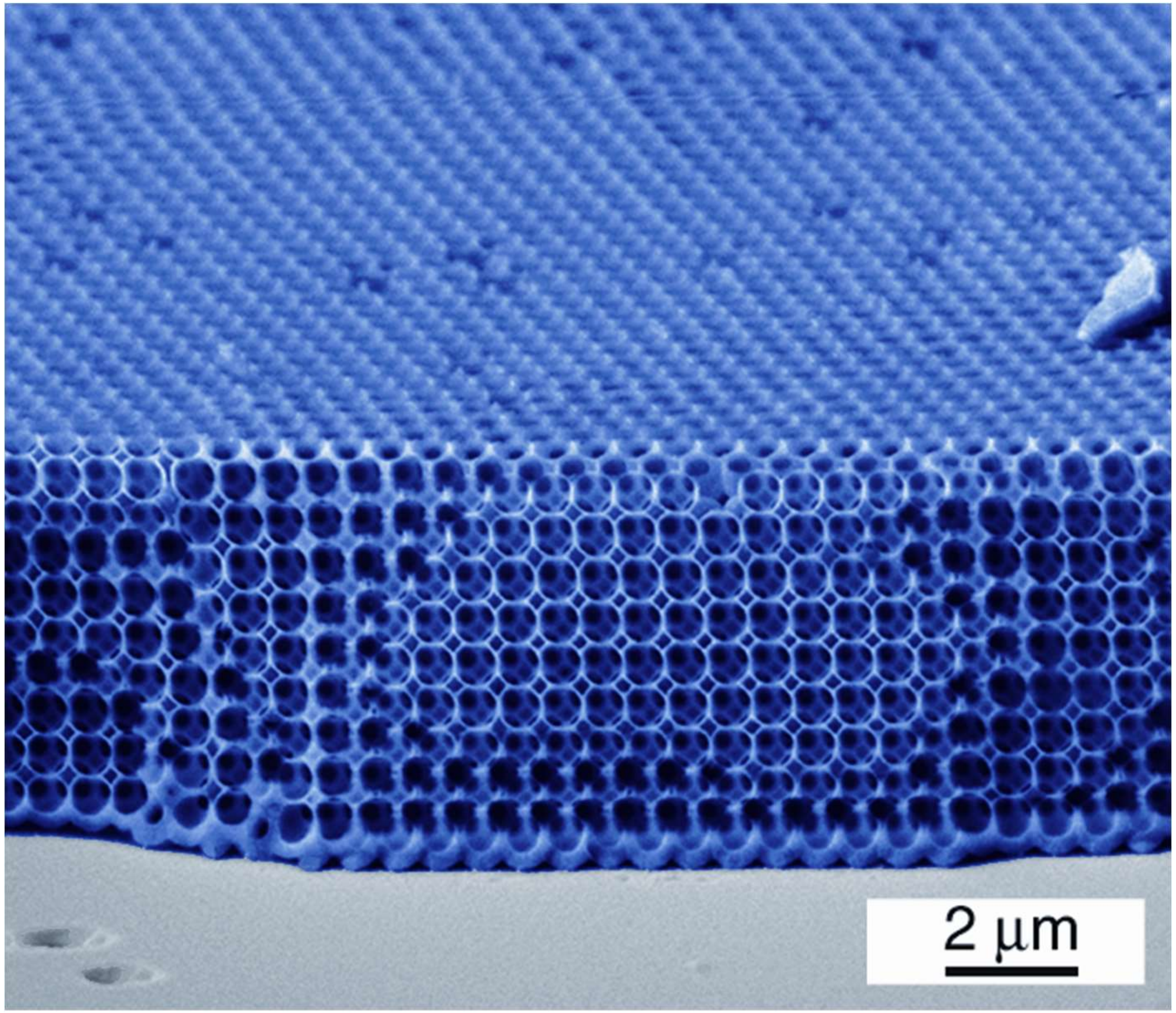
A Better Opal

[fig courtesy
D. Norris, UMN]



Inverse-Opal Photonic Crystal

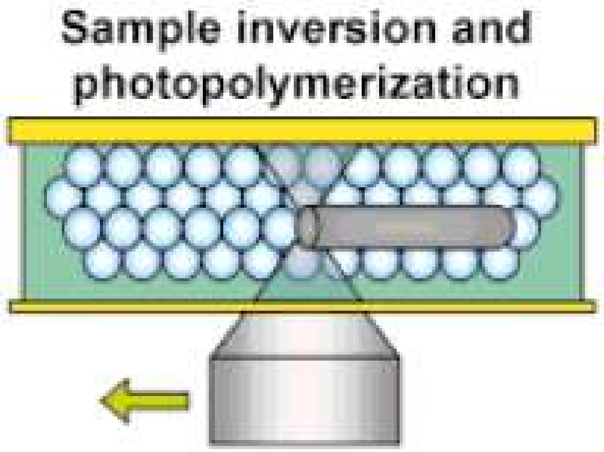
[fig courtesy
D. Norris, UMN]



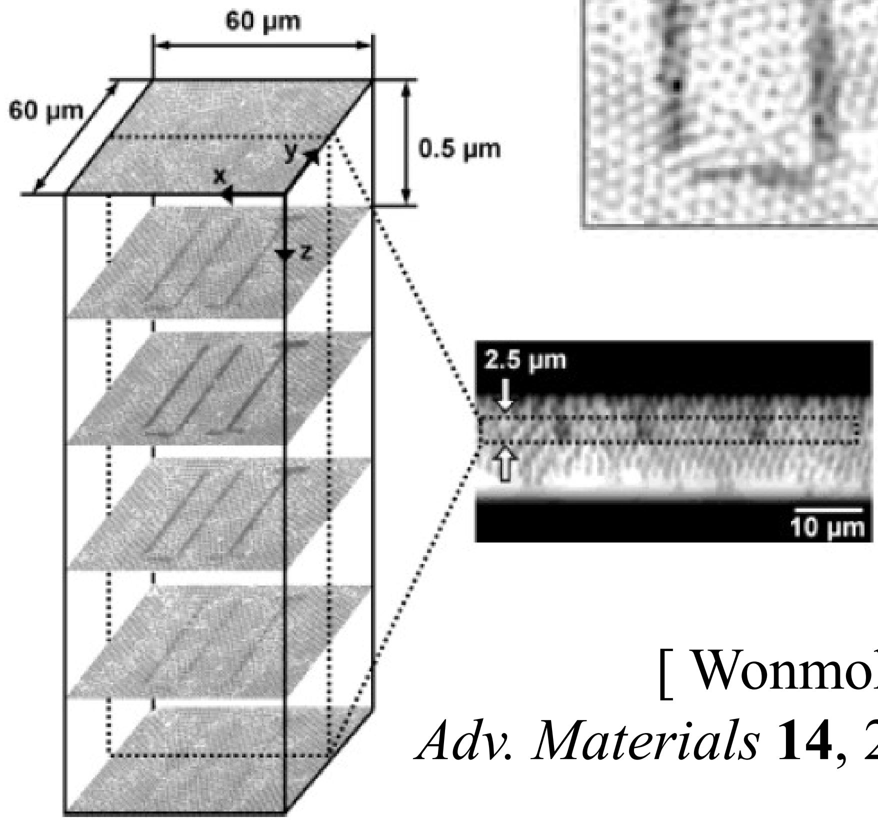
[Y. A. Vlasov *et al.*, *Nature* **414**, 289 (2001).]

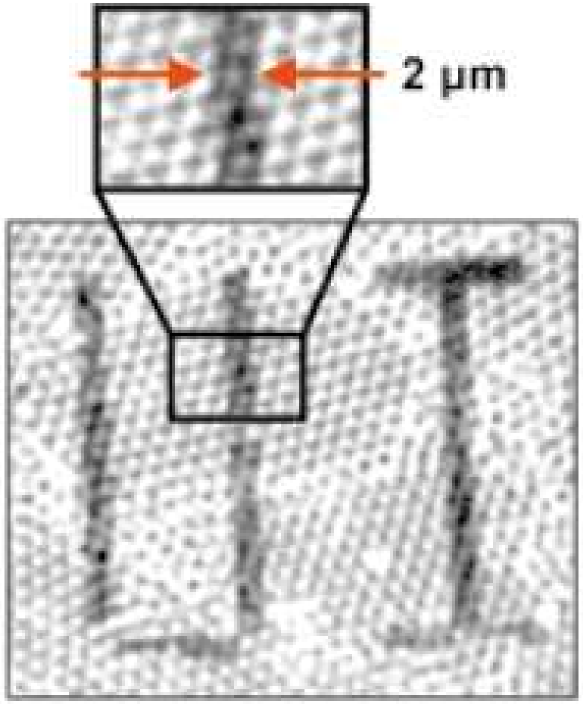
Inserting Defects in Inverse Opals

e.g., Waveguides



Three-photon lithography
with
laser scanning
confocal microscope
(LSCM)





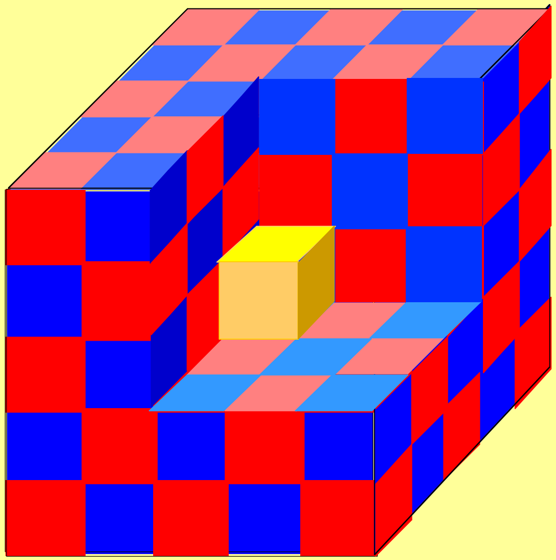
[Wonmok,
Adv. Materials **14**, 271 (2002)]

Outline

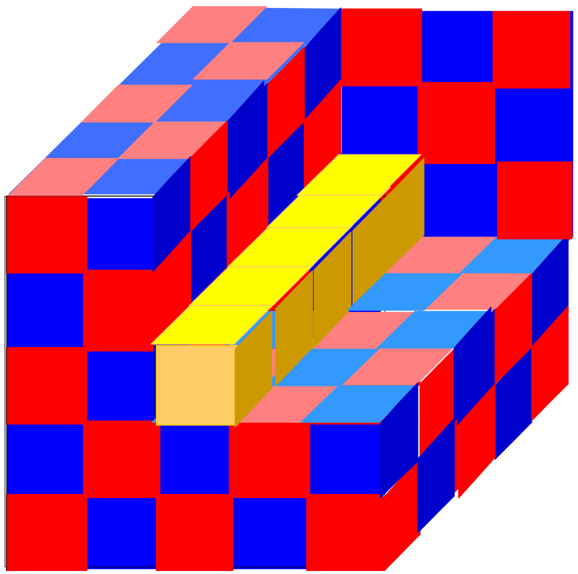
- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Photonic-crystal fibers
- Perturbations, tuning, and disorder

Intentional "defects" are good

microcavities



waveguides ("wires")



Resonance

an oscillating mode trapped for a long time in some volume
(of light, sound, …)

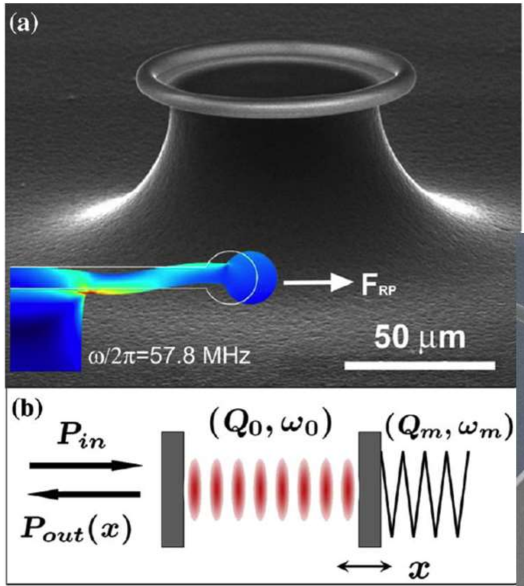
lifetime $\tau \gg 2\pi/\omega_0$

frequency ω_0

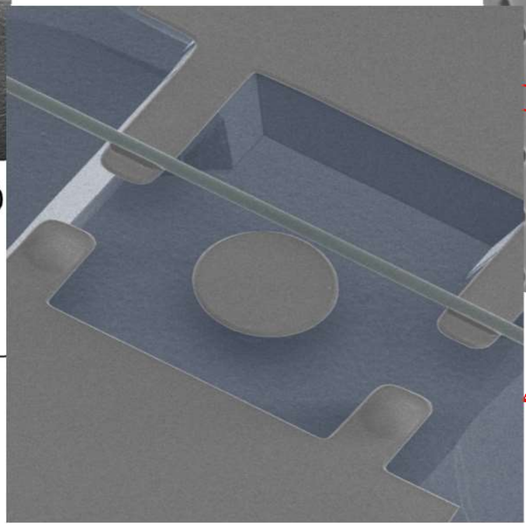
quality factor $Q = \omega_0\tau/2$

energy $\sim e^{-\omega_0\tau/Q}$

modal volume V

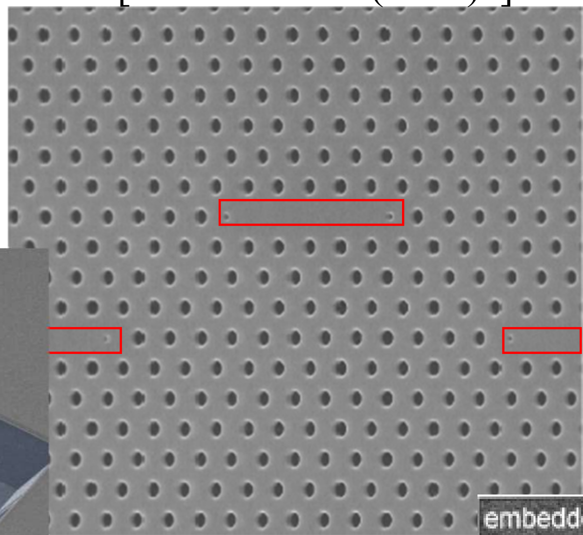


[Schliesser et al., *PRL* **97**, 243905 (2006)]



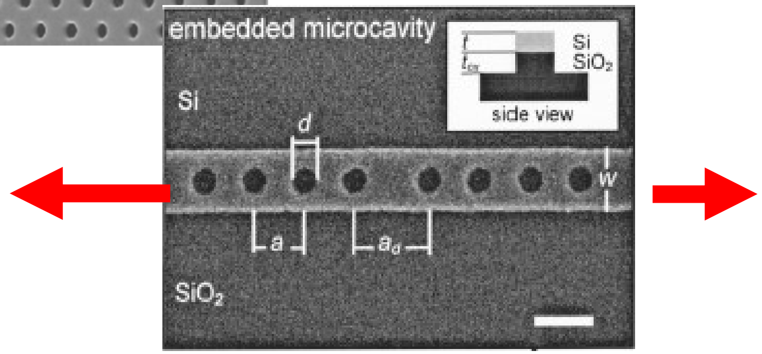
[Eichenfield et al. *Nature Photonics* **1**, 416 (2007)]

[Notomi *et al.* (2005).]



420 nm

[C.-W. Wong, *APL* **84**, 1242 (2004).]



Why Resonance?

an oscillating mode trapped for a long time in some volume

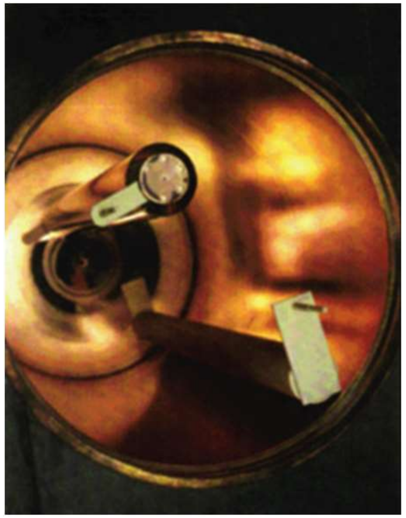
- long time = narrow bandwidth … filters (WDM, etc.)
  — $1/Q$ = fractional bandwidth

- resonant processes allow one to "impedance match" hard-to-couple inputs/outputs

- long time, small V … enhanced wave/matter interaction
  — lasers, nonlinear optics, opto-mechanical coupling, sensors, LEDs, thermal sources, …

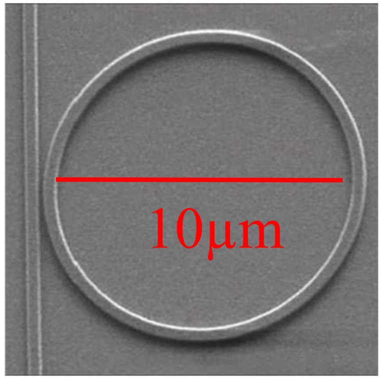
How Resonance?

need mechanism to trap light for long time



[llnl.gov]

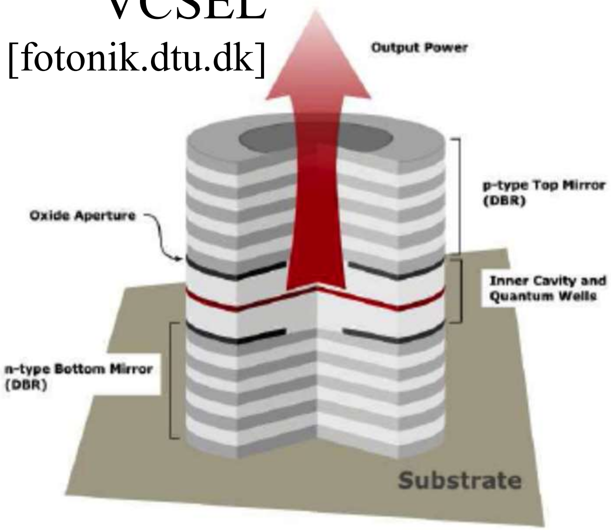
metallic cavities:
good for microwave,
dissipative for infrared

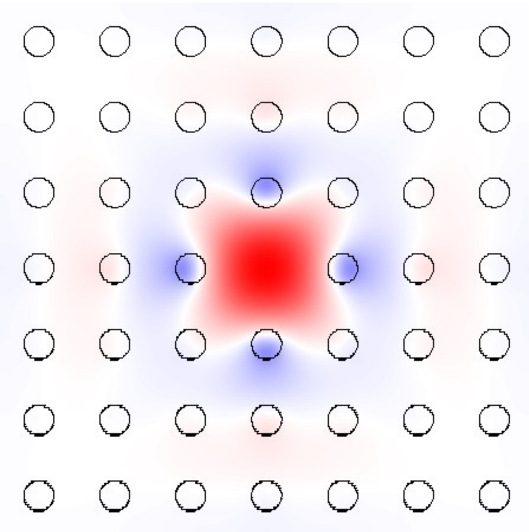


[Xu & Lipson (2005)]

ring/disc/sphere resonators:
a waveguide bent in circle,
bending loss ~ exp(–radius)

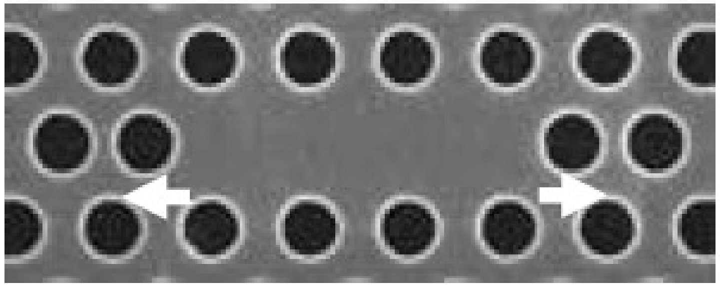
VCSEL
[fotonik.dtu.dk]





photonic bandgaps
(complete or partial
+ index-guiding)

[Akahane, *Nature* **425**, 944 (2003)]

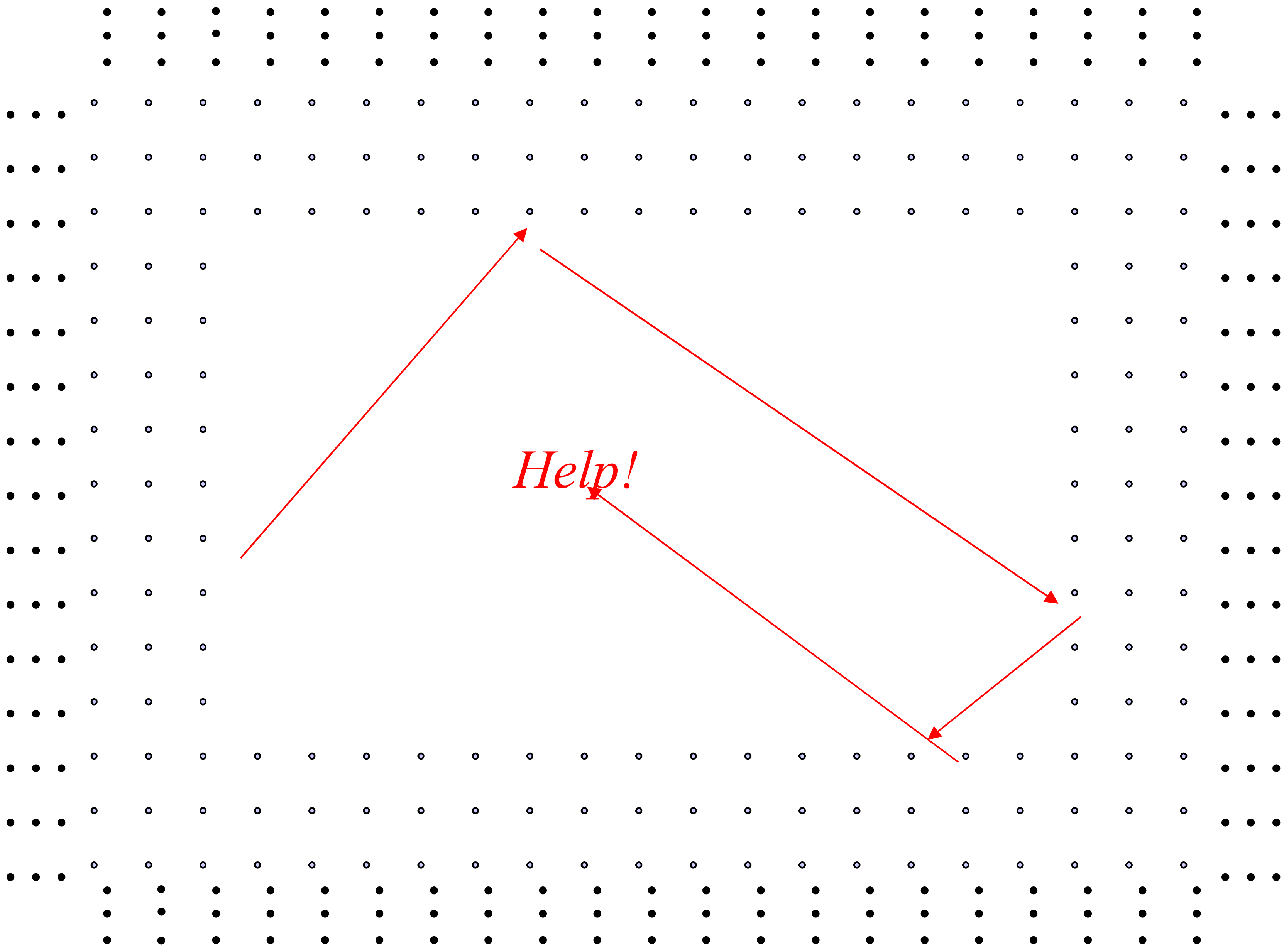


(planar Si slab)

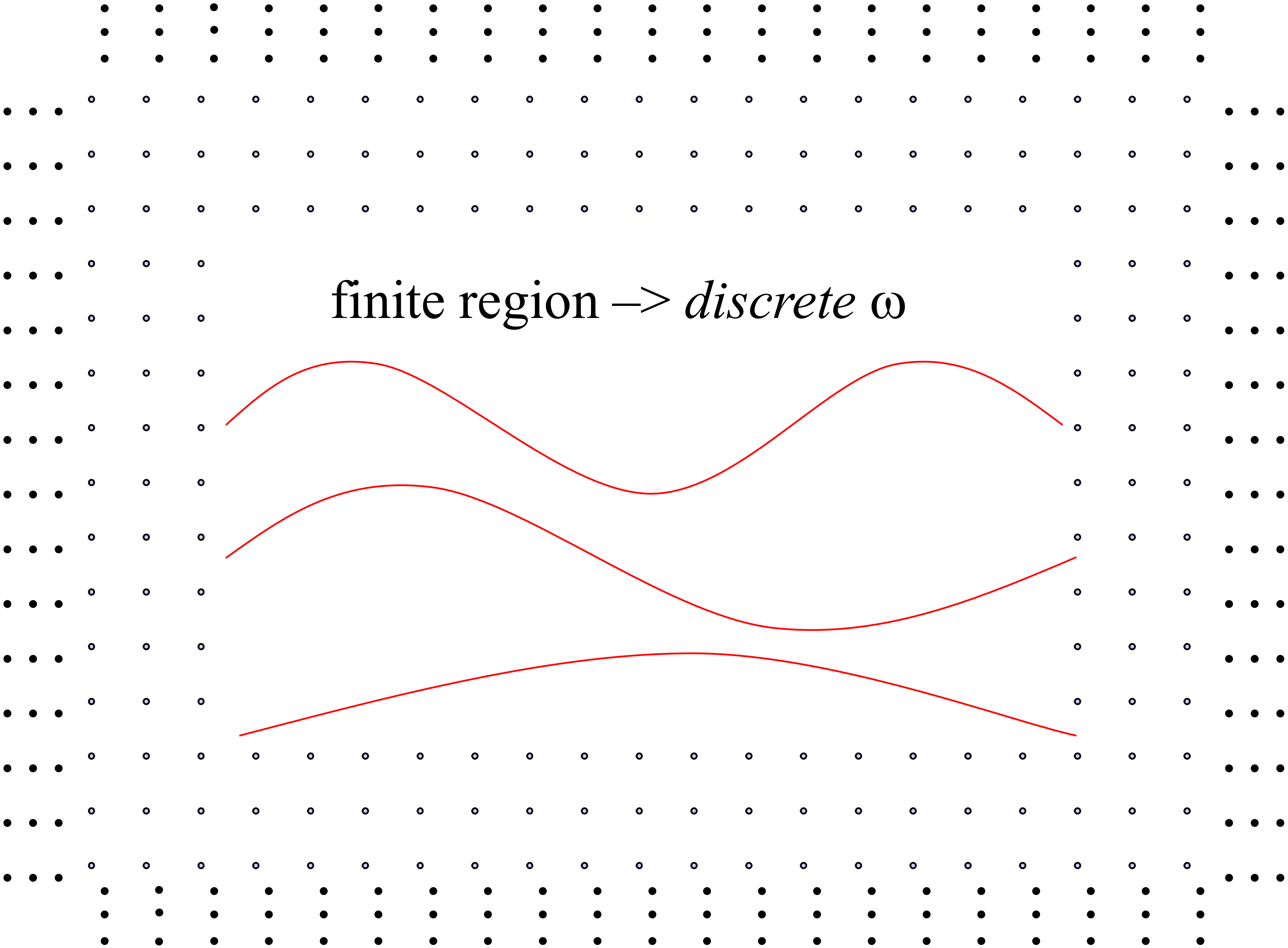
Why do defects in crystals trap resonant modes?

What do the modes look like?

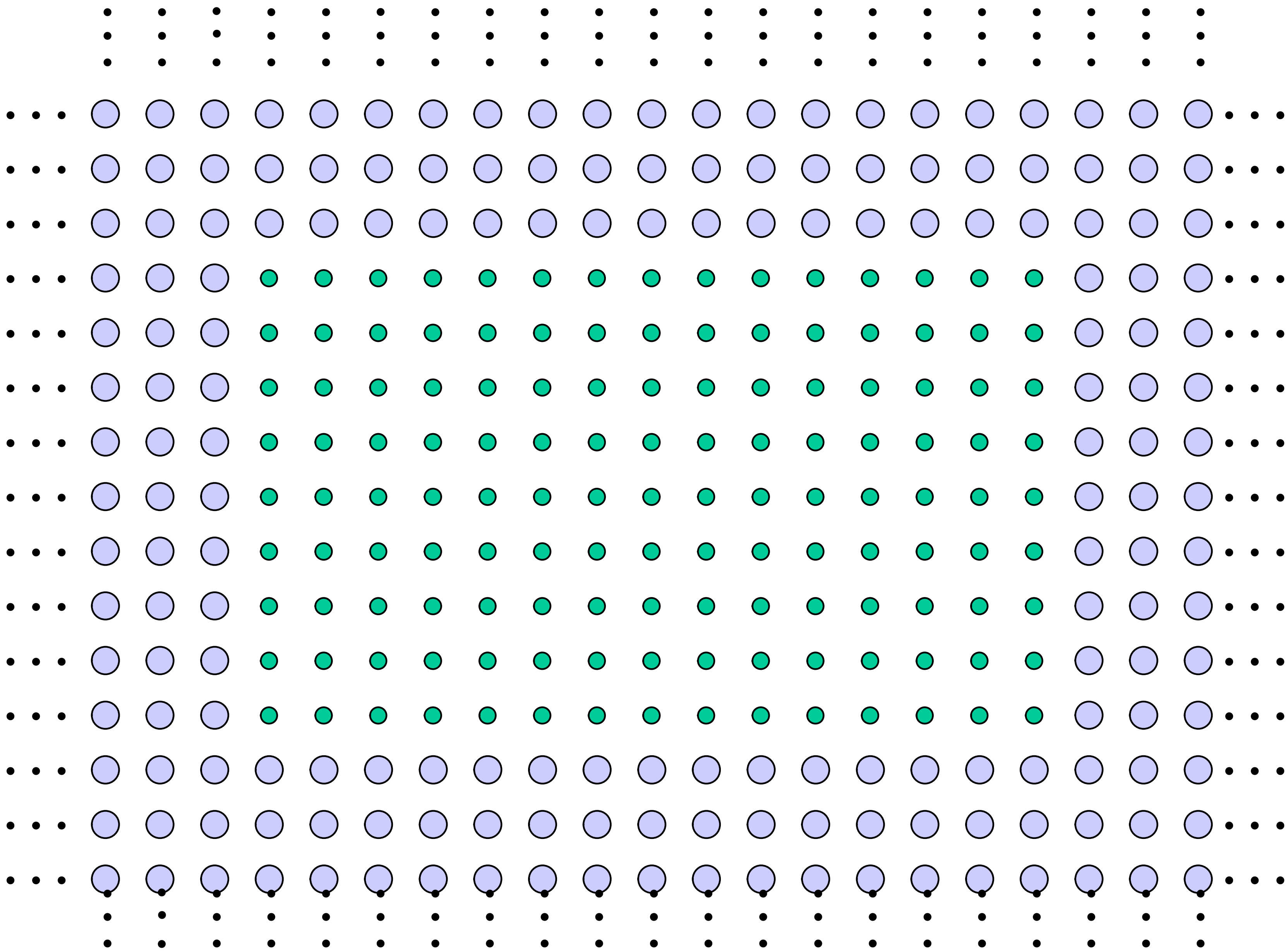
Cavity Modes



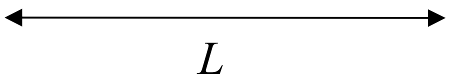
Cavity Modes



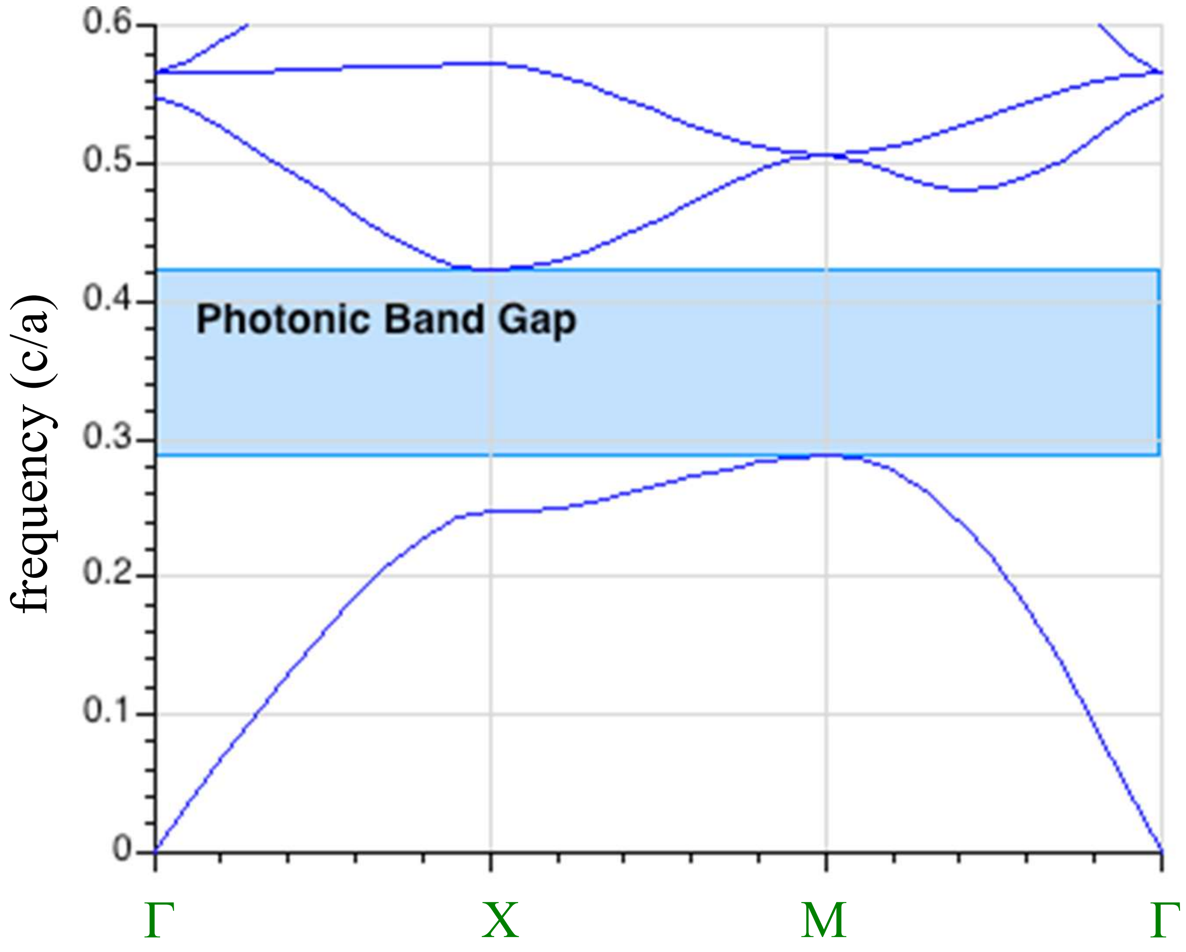
Cavity Modes: Smaller Change

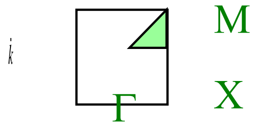


Cavity Modes: Smaller Change

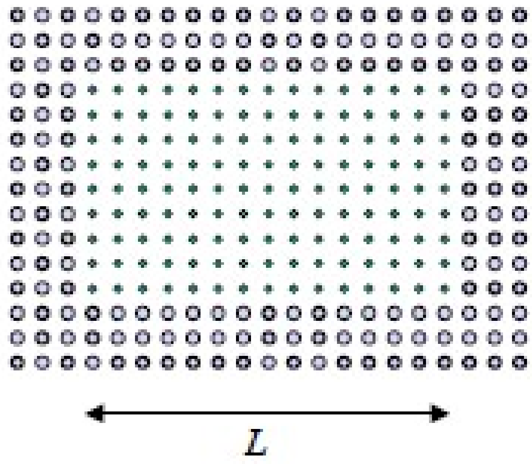


Bulk Crystal Band Diagram





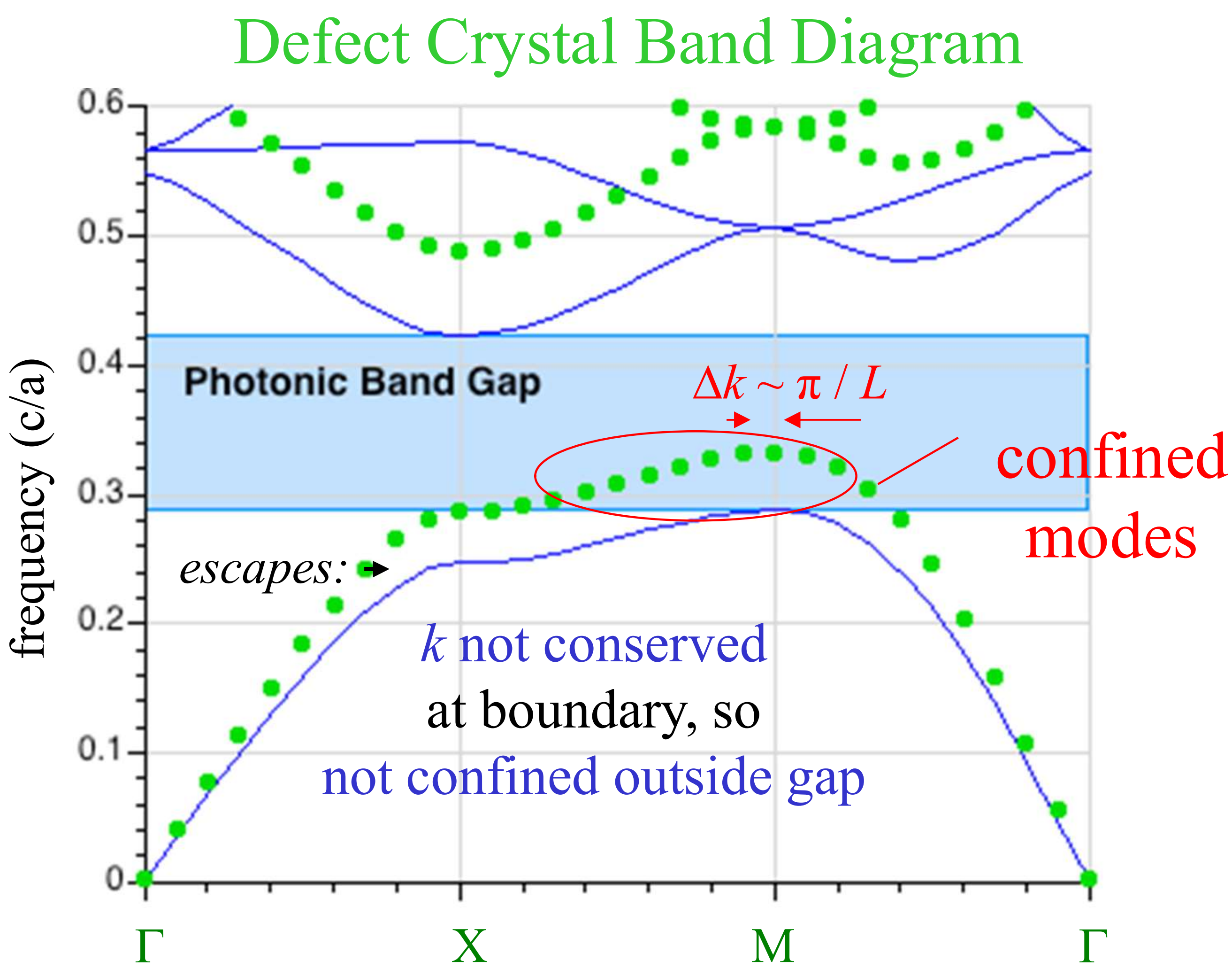
Cavity Modes: Smaller Change

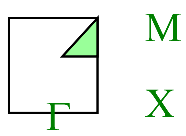


Defect bands are shifted *up* (less ε)

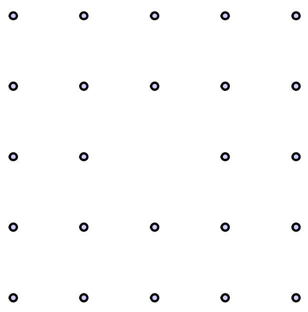
with *discrete k*

$(k \sim 2\pi / \lambda)$



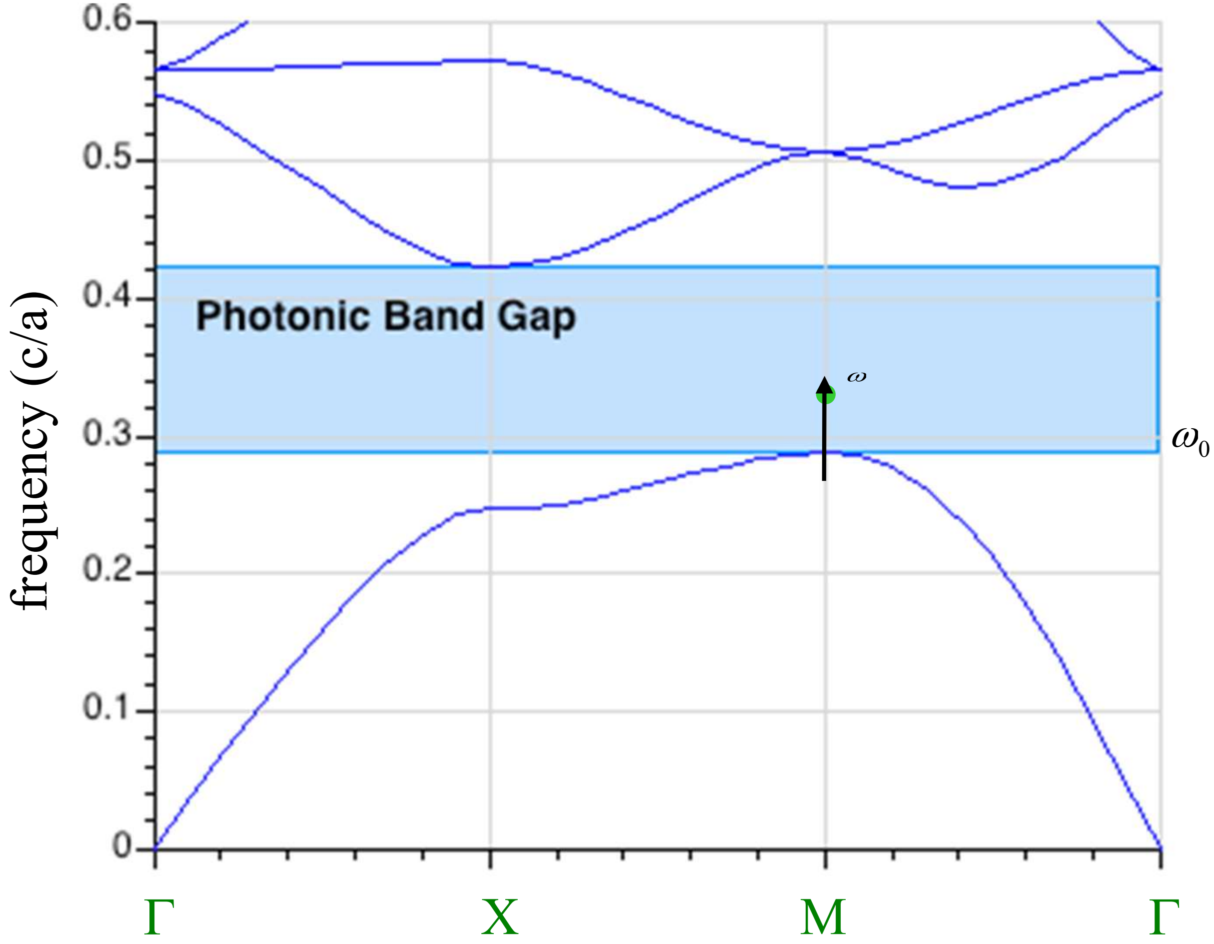


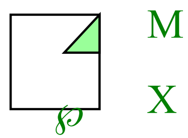
Single-Mode Cavity



A *point defect*
can push up
a single mode
from the band edge

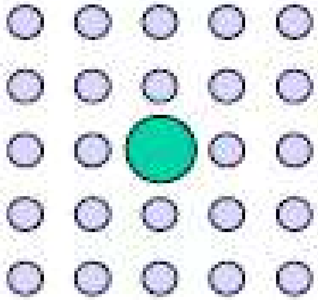
Bulk Crystal Band Diagram





(k not conserved)

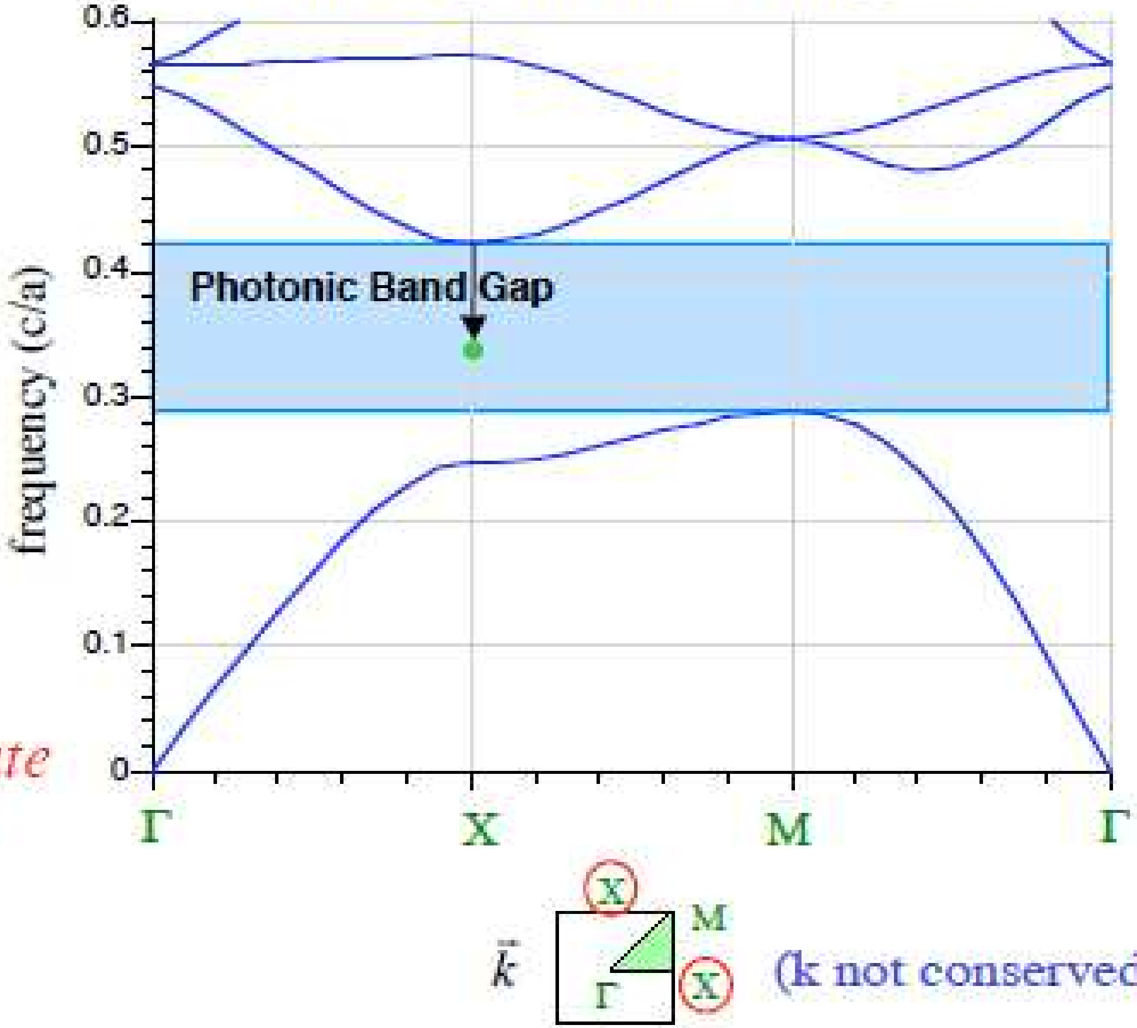
"Single"-Mode Cavity



A *point defect*
can pull down
a "single" mode

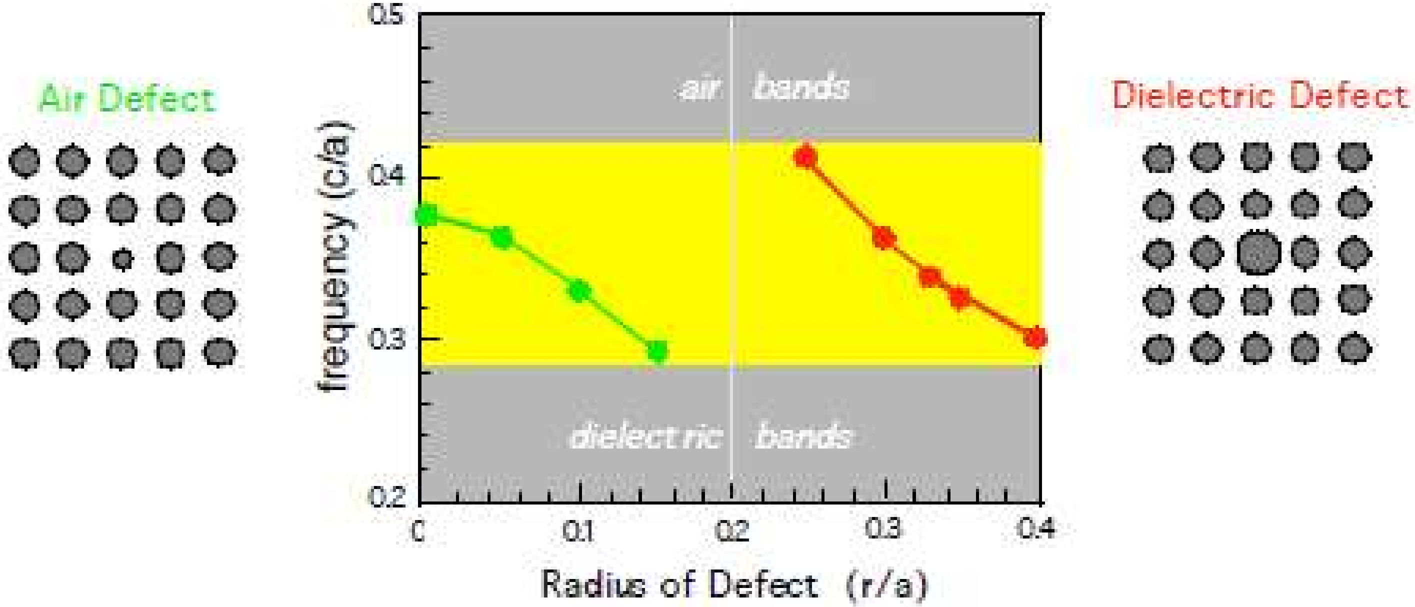
...here, doubly-*degenerate*
(*two* states at *same* ω)

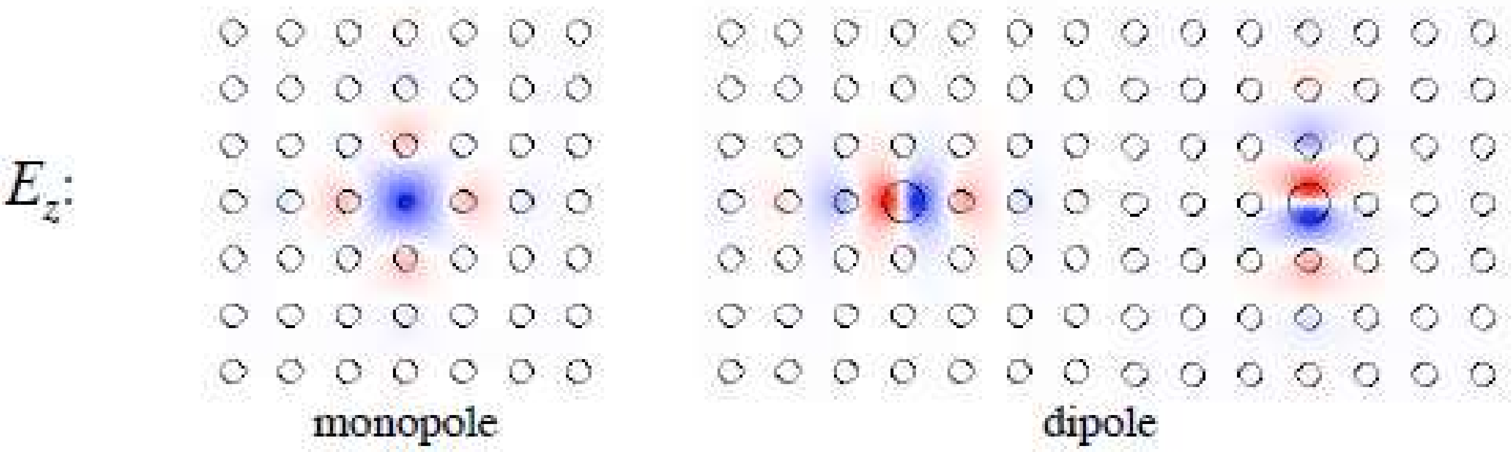
Bulk Crystal Band Diagram

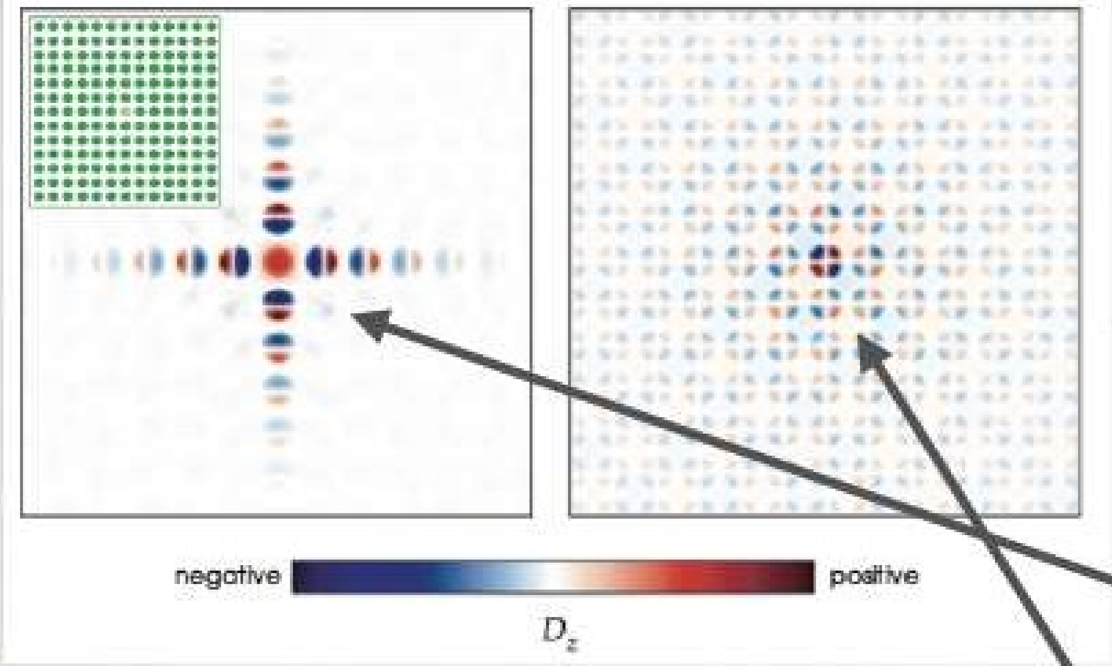


(k not conserved)

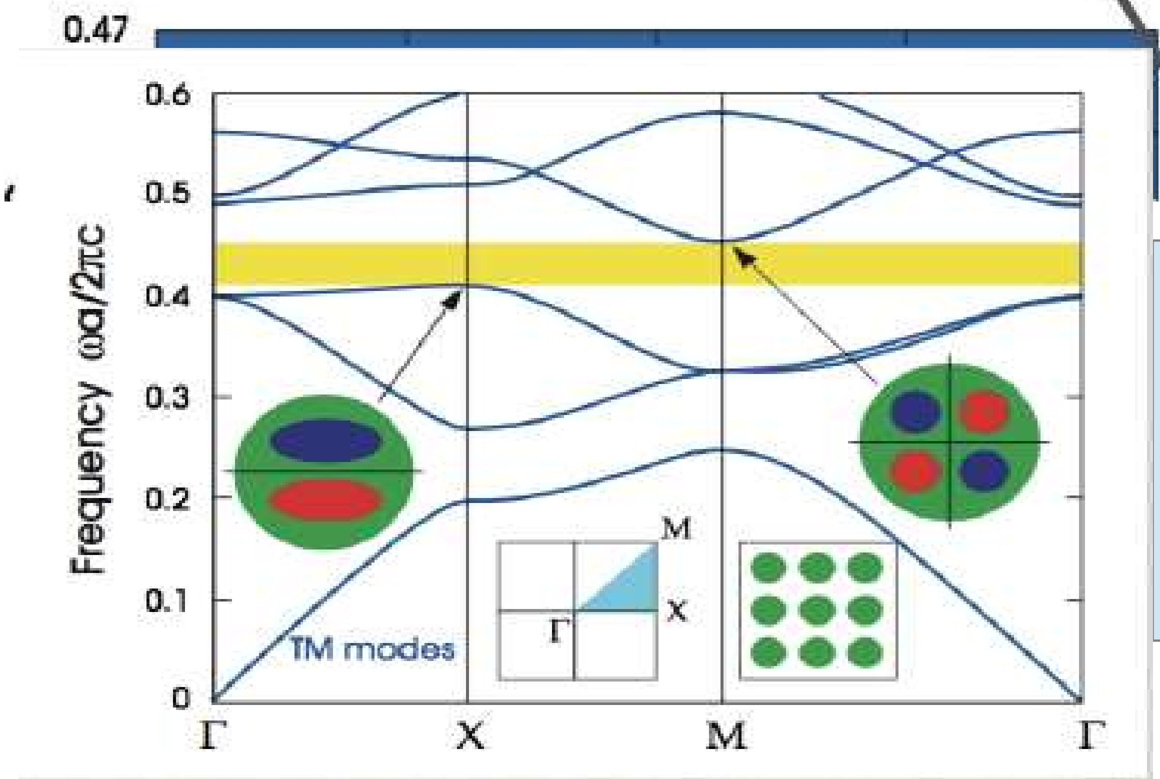
Tunable Cavity Modes





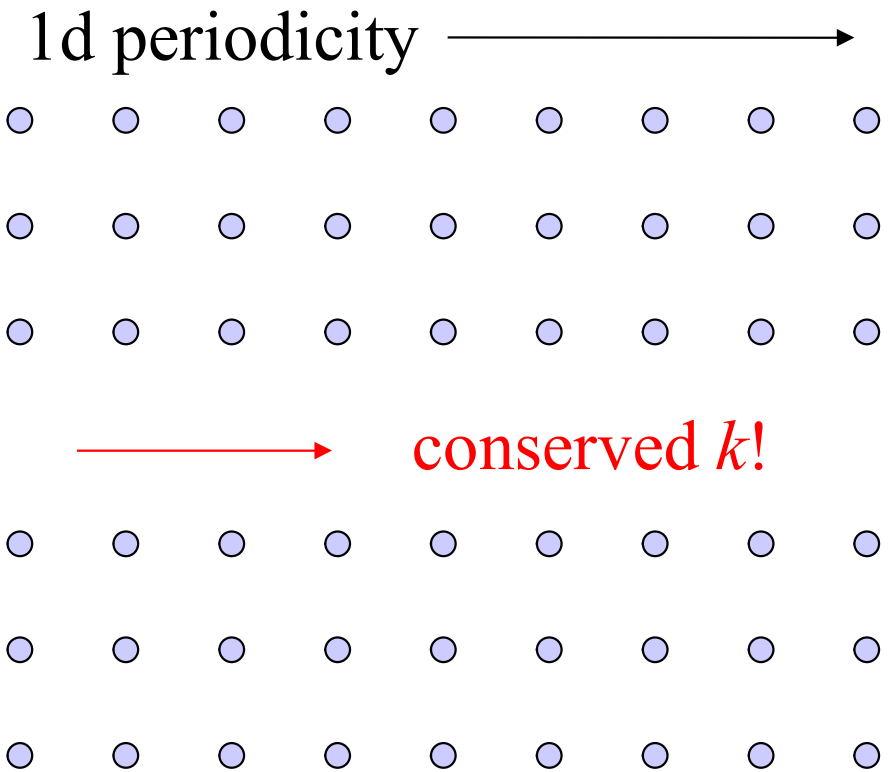


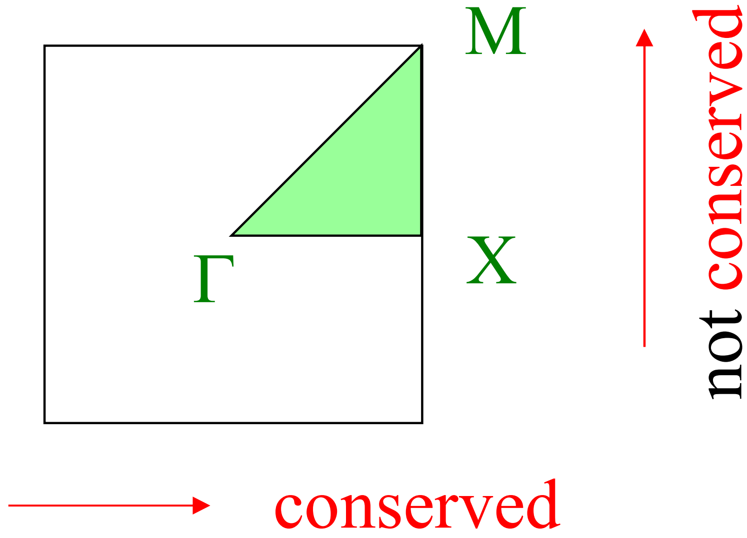
Δn=1.58, **monopole** mode, from the π band to the gap



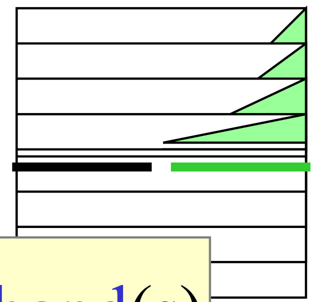
Quadrupole mode
from the δ band to the gap

Projected Band Diagrams



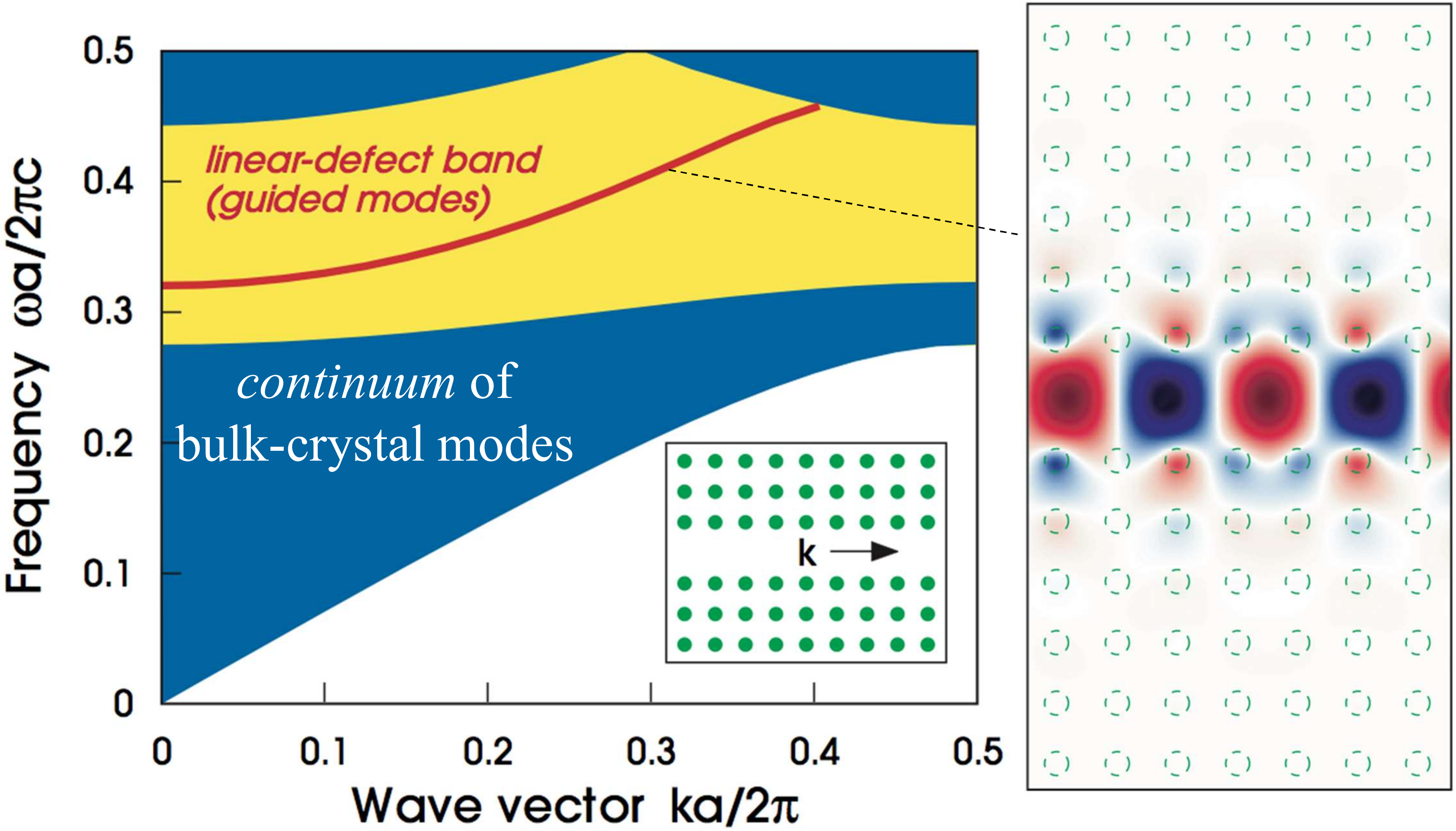


So, plot ω vs. k_x only…project Brillouin zone onto Γ–X:



gives continuum of bulk states + discrete guided band(s)

Air-waveguide Band Diagram



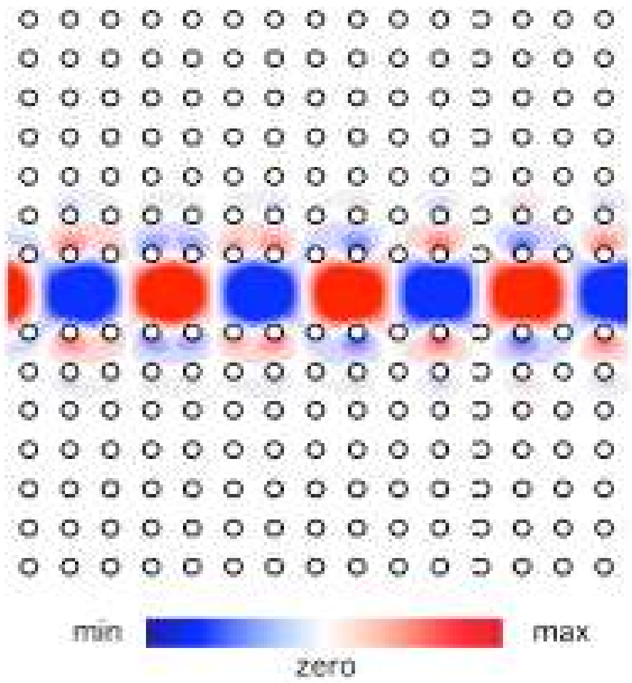
any state in the gap cannot couple to bulk crystal ⇒ localized

(Waveguides don't really need a *complete* gap)

Fabry-Perot waveguide:

This is exploited *e.g.* for photonic-crystal fibers…

So What?



Guiding Optical Light through Air

Reduction of Absorption Losses

Reduction of Non-Linearity Effects

High Power Transmission