

# GraphNeT: Graph neural networks for neutrino telescope event reconstruction

Roberto Cappuccio

# Index of Arguments

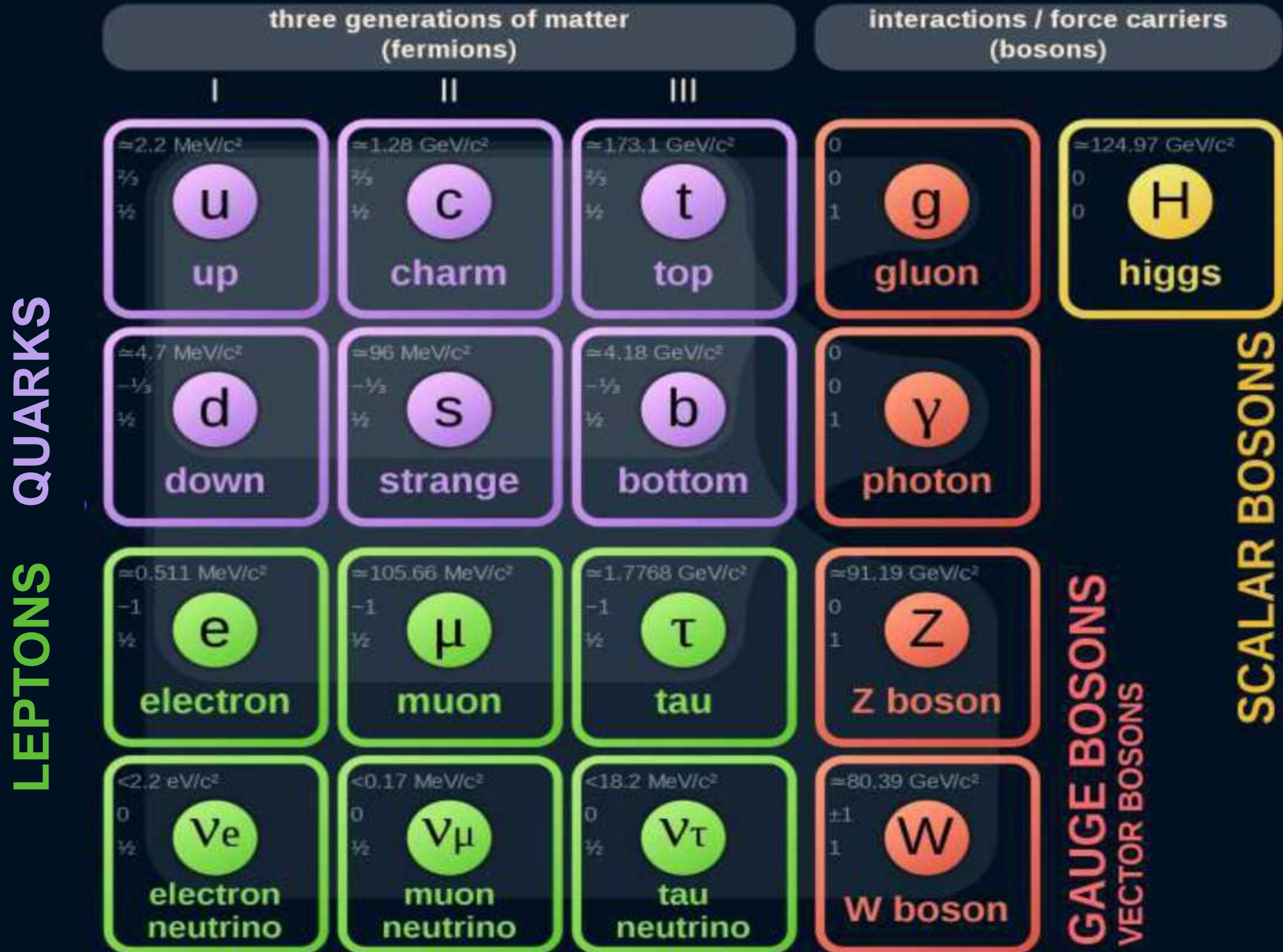
- Neutrino Physics
- Neutrino Detection
- Neutrino Experiments
- IceCube drilldown and data analysis
- GrapNet neural networks

# Introduction to Neutrinos

## Introduction

- Neutrinos are subatomic particles that play a crucial role in our understanding of the universe.
- Studying neutrinos can help us learn more about fundamental forces, particle physics, and astrophysics.
- This presentation will cover the basic properties of neutrinos, their detections and the experiment IceCube. Then a new neural-network-based methodology will be presented for the event classification tasks.

# Standard Model of Elementary Particles



# Charged Current Interaction Lagrangians

## quarks

$$\mathcal{L}_{\text{cc}} = \frac{g}{\sqrt{2}} \overline{(u \ c \ t)}_{\text{L}} \gamma^{\mu} U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\text{L}} W_{\mu}^{+} + \text{h.c.}$$

Cabibbo-Kobayashi-  
Maskawa Matrix

## leptons

$$\mathcal{L}_{\text{cc}} = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)}_{\text{L}} \gamma^{\mu} V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\text{L}} W_{\mu}^{-} + \text{h.c.}$$

Maki-Nakagawa-  
Sakata-Pontecorvo  
Matrix

# V-Matrix Parametrization

$$O_1(\theta_1, \alpha_1, \beta_1, \gamma_1) = \begin{pmatrix} c_1 e^{i\alpha_1} & s_1 e^{-i\beta_1} & 0 \\ -s_1 e^{i\beta_1} & c_1 e^{-i\alpha_1} & 0 \\ 0 & 0 & e^{i\gamma_1} \end{pmatrix}$$
$$O_2(\theta_2, \alpha_2, \beta_2, \gamma_2) = \begin{pmatrix} e^{i\gamma_2} & 0 & 0 \\ 0 & c_2 e^{i\alpha_2} & s_2 e^{-i\beta_2} \\ 0 & -s_2 e^{i\beta_2} & c_2 e^{-i\alpha_2} \end{pmatrix}$$
$$O_3(\theta_3, \alpha_3, \beta_3, \gamma_3) = \begin{pmatrix} c_3 e^{i\alpha_3} & 0 & s_3 e^{-i\beta_3} \\ 0 & e^{i\gamma_3} & 0 \\ -s_3 e^{i\beta_3} & 0 & c_3 e^{-i\alpha_3} \end{pmatrix}$$

where  $s_i \equiv \sin \theta_i$  and  $c_i \equiv \cos \theta_i$  (for  $i = 1, 2, 3$ )

## Dirac Neutrinos

$$V = O_i O_j O_i \quad (i \neq j)$$

## Majorana Neutrinos

$$V = O_i O_j O_k \quad (i \neq j \neq k)$$

# Dirac or Majorana Neutrinos?

If neutrinos are **Dirac** particles, the phases  **$x$** ,  **$y$**  and  **$z$**  can be removed. Then the neutrino mixing matrix is

## Dirac neutrino mixing matrix

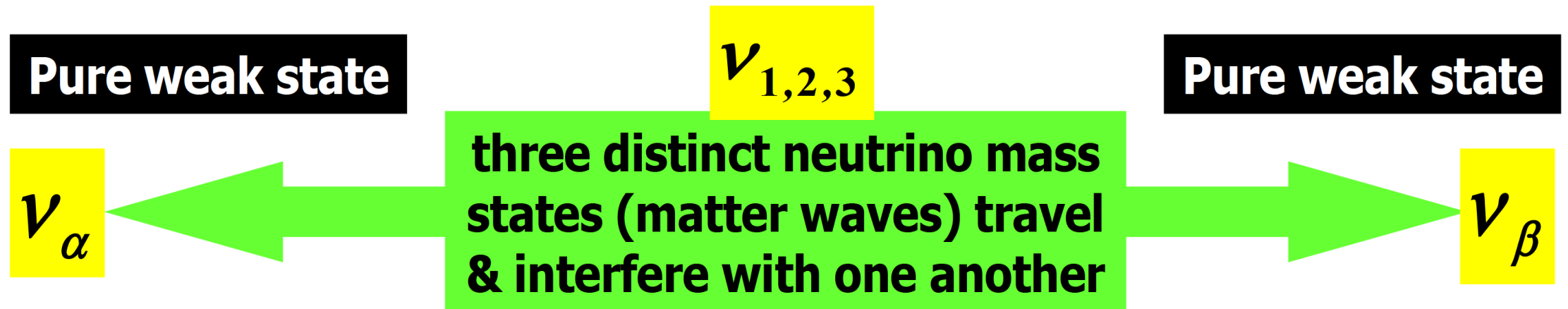
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

If neutrinos are **Majorana** particles, left- and right-handed fields are correlated. Hence only a common phase of three left-handed fields can be redefined (e.g.,  **$z = 0$** ). Then

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# Neutrino Oscillations

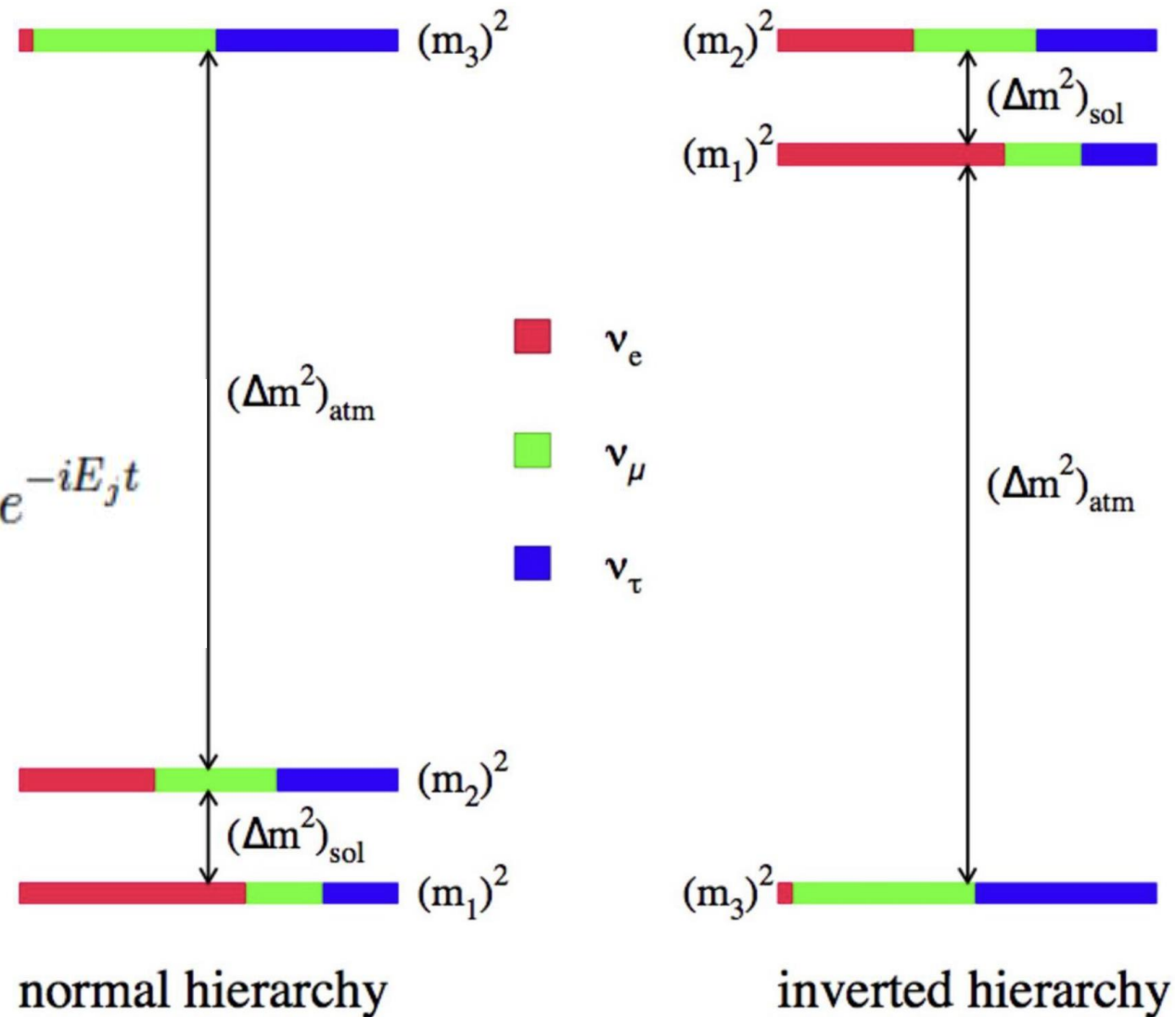
- Neutrino oscillation is a spontaneous periodic change from one neutrino flavor state to another.
- It is a specific quantum phenomenon and it occurs as a natural consequence of neutrino mixing.





# Neutrino Mass Hierarchy

$$|\nu_\ell, t\rangle = \sum_{j=1}^n U_{\ell j}^* |\nu_j, t\rangle \quad \text{where} \quad |\nu_j, t\rangle = |\nu_j, 0\rangle e^{-iE_j t}$$



# 2-Flavor Oscillation

The oscillation probability for **appearance**  $\nu$  experiments:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= |\langle \nu_e | \nu_\mu(t) \rangle|^2 = |(\cos\theta \langle \nu_1 | + \sin\theta \langle \nu_2 |) (-\sin\theta |\nu_1\rangle + \cos\theta e^{-i\Delta Et} |\nu_2\rangle)|^2 \\ &= |\sin\theta \cos\theta (1 - e^{-i\Delta Et})|^2 = 2(\sin\theta \cos\theta)^2 \left(1 - \cos \frac{\Delta m^2 t}{2E}\right) \\ &= \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E} \end{aligned}$$

The **conversion** and **survival** probabilities in realistic units:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E} \\ P(\nu_\mu \rightarrow \nu_\mu) &= 1 - \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E} \end{aligned}$$

Due to the smallness of (1,3) mixing, both **solar** & **atmospheric** neutrino oscillations are roughly the 2-flavor oscillation.

$\Delta m^2$  in unit of  $\text{eV}^2$ ,  $L$  in unit of  $\text{km}$ ,  $E$  in unit of  $\text{GeV}$

# 1.27 ?

	Natural units	Realistic units
Phase factors	$\exp(-iE_{1,2}t)$	$\exp\left(-i\frac{E_{1,2}}{\hbar}t\right)$
Energies and momentum	$E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$	$E_{1,2} = \sqrt{p^2c^2 + m_{1,2}^2c^4}$
Energy difference	$\Delta E = \frac{\Delta m^2}{2E}$	$\Delta E = \frac{\Delta m^2c^3}{2p} = \frac{\Delta m^2c^4}{2E}$
Time and distance	$t = L$	$t = \frac{L}{c}$
Oscillation argument	$\frac{1}{2}\Delta Et = \frac{\Delta m^2L}{4E}$	$\frac{1}{2}\frac{\Delta E}{\hbar}t = \frac{c^3}{\hbar} \cdot \frac{\Delta m^2L}{4E}$

$$c = 2.998 \times 10^5 \text{ km s}^{-1}$$

$$\hbar = 6.582 \times 10^{-25} \text{ GeV s}$$

$$\frac{c^3}{4\hbar} \Rightarrow \frac{1}{4 \times 0.1973} = 1.267 \approx 1.27$$

$$c = 1 \Rightarrow \hbar = 6.582 \times 10^{-25} \text{ GeV} \times 2.998 \times 10^5 \text{ km}$$

$$= 1.973 \times 10^{-19} \text{ GeV km} = 0.1973 \text{ eV}^2 \text{ GeV}^{-1} \text{ km}$$

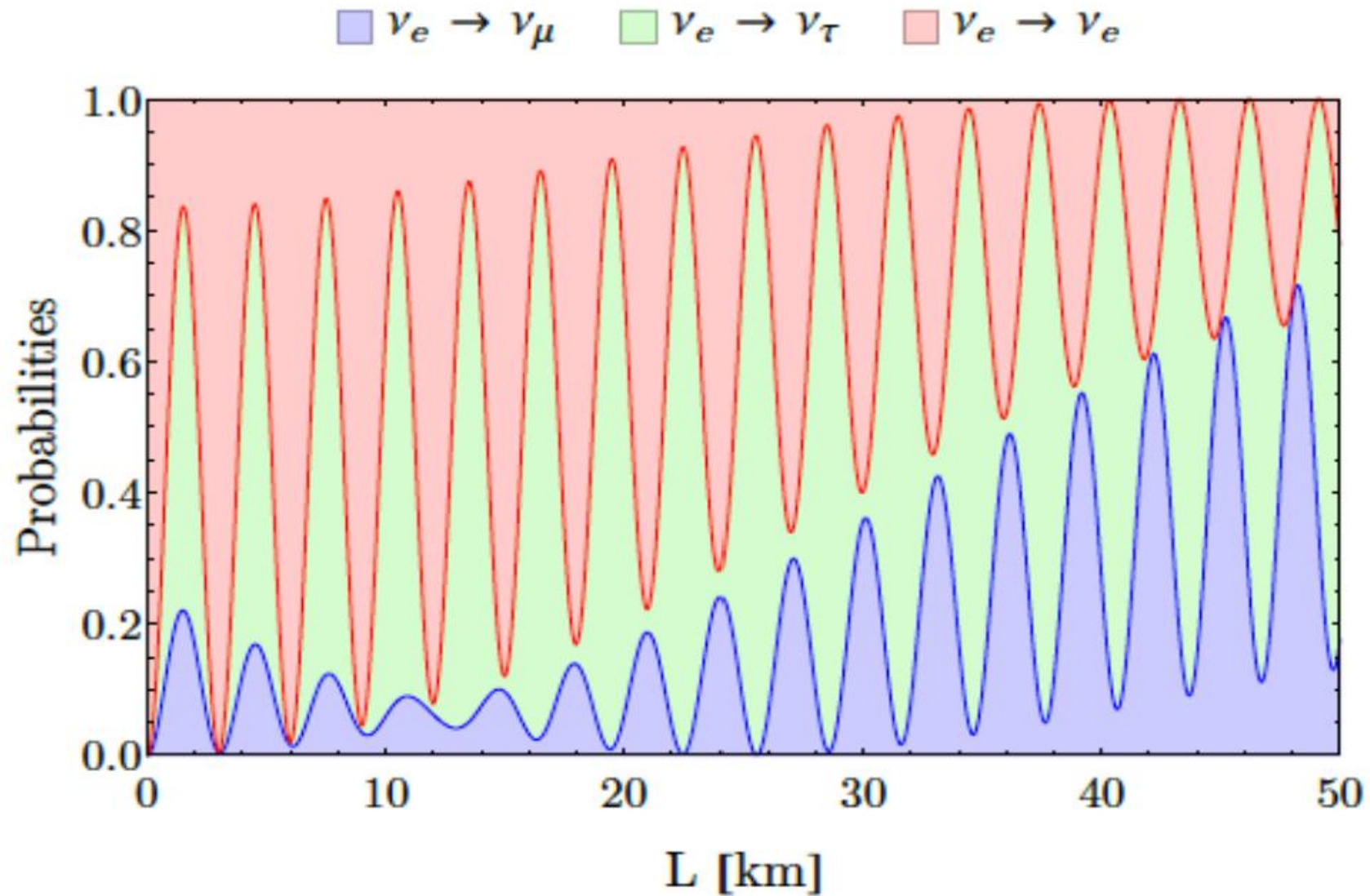
# 3-Flavor Oscillation

The **final** formula of 3-flavor oscillation probabilities with **CP** violation:

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j}^3 \operatorname{Re} \left( V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^* \right) \sin^2 \frac{\Delta m_{ji}^2 L}{4E} \\ + 8\mathcal{J} \sum_{\gamma} \epsilon_{\alpha\beta\gamma} \sin \frac{\Delta m_{21}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E} \sin \frac{\Delta m_{32}^2 L}{4E}$$

The **1<sup>st</sup>** oscillating term: **CP** conserving; and the **2<sup>nd</sup>** term: **CP** violating!

# Neutrino Oscillation Probabilities



# Neutrino Oscillation Studies

$\nu_\mu \rightarrow \nu_\tau$  oscillations ( $\Delta m_{23}^2, \theta_{23}$ )

Atmospheric: Super-K, Soudan-2,  
MACRO IceCube/Deepcore, ...

LBL: K2K, MINOS, OPERA, T2K, NOvA, ...

$\nu_e \rightarrow (\nu_\mu + \nu_\tau)$  oscillations ( $\Delta m_{12}^2, \theta_{12}$ )

Solar: SNO, Super-K, Borexino, ...

Reactor: KamLAND

$\theta_{13}$  experiments

LBL: MINOS, T2K, NOvA, ...

Reactor: Daya Bay, Reno, Double Chooz

## Status (before Neutrino 2016)

Parameter	best-fit ( $\pm 1\sigma$ )
$\Delta m_{21}^2$ [ $10^{-5}$ eV <sup>2</sup> ]	$7.54^{+0.26}_{-0.22}$
$ \Delta m^2 $ [ $10^{-3}$ eV <sup>2</sup> ]	$2.43 \pm 0.06$ ( $2.38 \pm 0.06$ )
$\sin^2 \theta_{12}$	$0.308 \pm 0.017$
$\sin^2 \theta_{23}, \Delta m^2 > 0$	$0.437^{+0.033}_{-0.023}$
$\sin^2 \theta_{23}, \Delta m^2 < 0$	$0.455^{+0.039}_{-0.031}$
$\sin^2 \theta_{13}, \Delta m^2 > 0$	$0.0234^{+0.0020}_{-0.0019}$
$\sin^2 \theta_{13}, \Delta m^2 < 0$	$0.0240^{+0.0019}_{-0.0022}$
$\delta/\pi$ ( $2\sigma$ range quoted)	$1.39^{+0.38}_{-0.27}$ ( $1.31^{+0.29}_{-0.33}$ )

K. Nakamura and S.T. Petcov, "14. Neutrino mass, mixing and oscillations"

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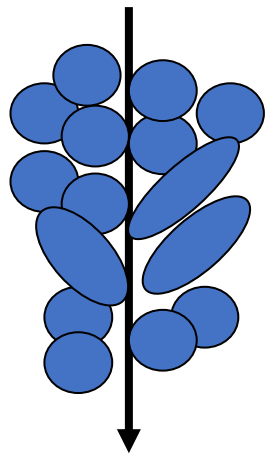
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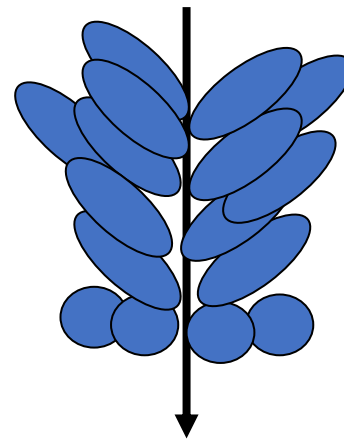
# How to Detect Neutrinos: Cerenkov Light

- Cerenkov radiation is emitted whenever a charged particle passes through a (dielectric) medium with velocity  $\beta c = v > c/n$ , where  $v$  is the velocity of the particle and  $n$  the refractive index of the medium.
- Incoming particle polarizes atoms in the medium which, in turn, become electric dipoles:

No radiation  
Emission



$\beta < 1/n$



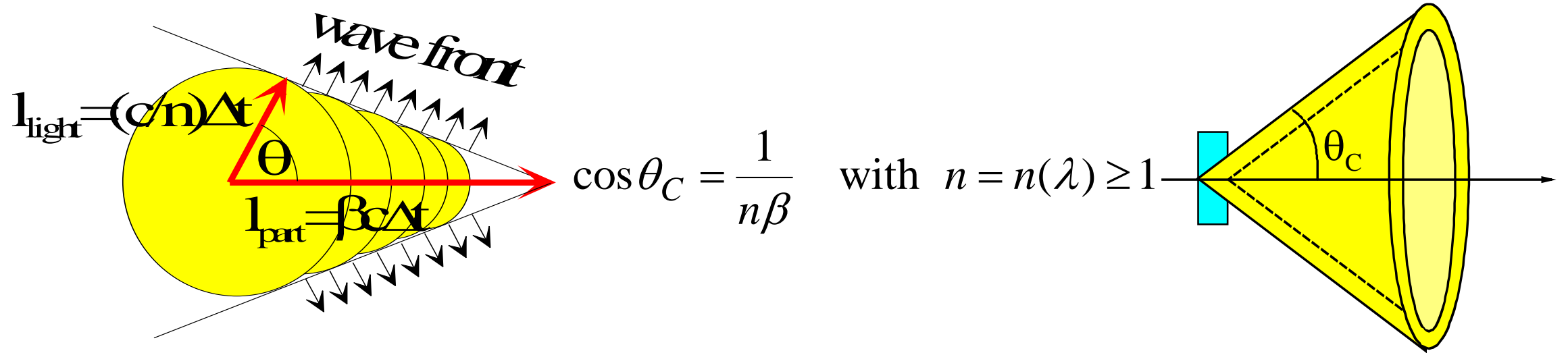
$\beta > 1/n$

Radiation  
Emission



# Cerenkon Light

- The emission angle  $\theta_c$  can be qualitatively interpreted as a shock wave as happens for a boat or a supersonic plane.



- Threshold velocity:  $\beta_s = 1/n \rightarrow \theta_c \sim 0$
- Maximum angle:  $\theta_{\text{max}} = \arccos(1/n)$
- In water  $n = 1.33$  and  $\theta_c = 41^\circ$

The above equation are valid in  $L \gg \lambda$  where  $L$  is the length of the medium and  $\lambda$  is the wavelength of emitted light

# Cerenkov Spectrum

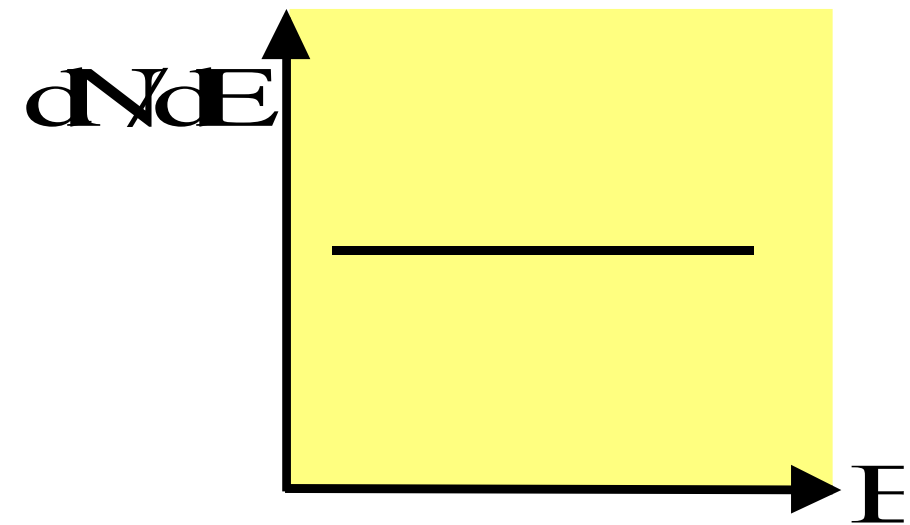
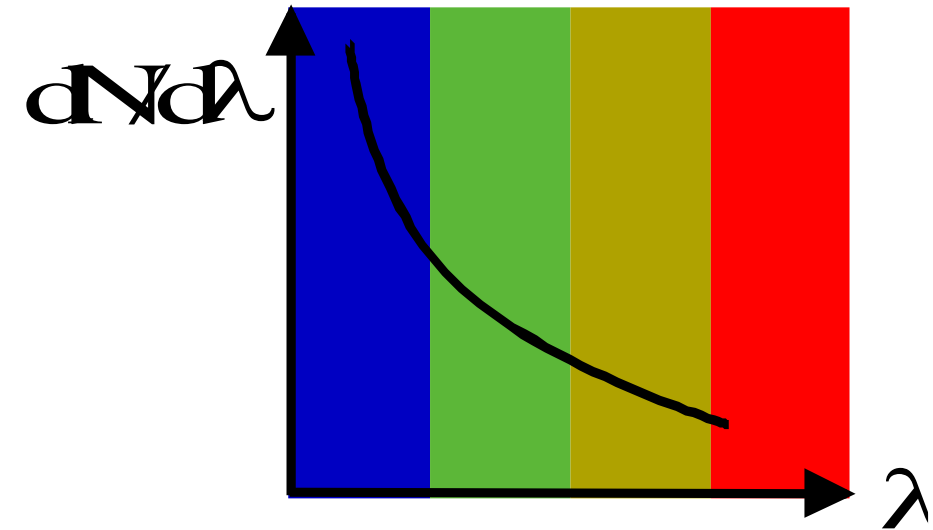
Number of photons emitted per unit length and unit wavelength range. We observe that it decreases as  $\lambda$  increases.

$$\frac{d^2 N}{dx d\lambda} = \frac{2\pi z^2 \alpha}{\lambda^2} \left( 1 - \frac{1}{\beta^2 n^2} \right) = \frac{2\pi z^2 \alpha}{\lambda^2} \sin^2 \theta_C$$

$$\frac{d^2 N}{dx d\lambda} \propto \frac{1}{\lambda^2} \quad \text{with } \lambda = \frac{c}{\nu} = \frac{hc}{E} \quad \frac{d^2 N}{dx dE} = \text{const.}$$

$$\alpha = \frac{2\pi e^2}{hc} = \frac{1}{137} \frac{1}{4\pi\epsilon_0} = 1$$

The number of photons emitted per unit of Length does not depend on energy.



# Cerenkov Light Energy Loss

- The energy lost by Cerenkov radiation increases with  $\beta$ . However even with  $\beta \rightarrow 1$  it is very small, and generally smaller than Bethe Block energy loss:

$$-\frac{dE}{dx} = z^2 \alpha \frac{\hbar}{c} \int \omega \left( 1 - \frac{1}{\beta^2 n^2(\omega)} \right) d\omega$$

medium	n	$\theta_{\max} (\beta=1)$	$N_{\text{ph}} (\text{eV}^{-1} \text{cm}^{-1})$
air	1.000283	1.36	0.208
isobutane	1.00127	2.89	0.941
water	1.33	41.2	160.8
quartz	1.46	46.7	196.4

# How much light?

Let us consider:

- a 1 cm thick radiator
- an angle  $\theta_c = 30^\circ$
- a DE = 1 eV
- a particle of charge = 1

$$\frac{dN}{dEdx} = \frac{z^2 \alpha}{\hbar c} \sin^2 \vartheta_c$$

$$\Rightarrow N_{ph} = 370 \cdot \sin^2 \vartheta_c \cdot L \cdot \Delta E = 370 \times 0.25 = 92.5$$

Considering, also, that the quantum efficiency of a photomultiplier is around 20% then:

$$\Rightarrow N_{ph} = 18$$

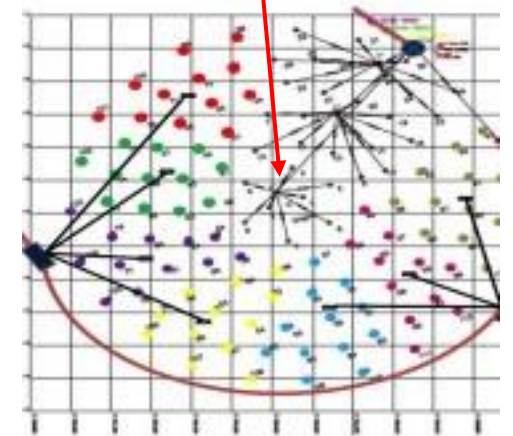
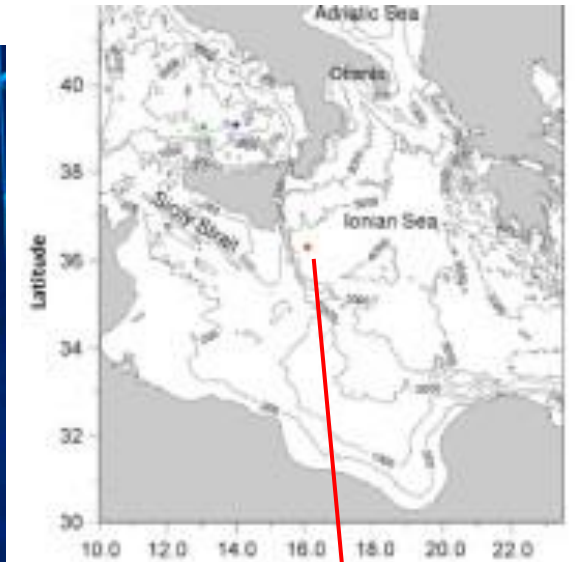
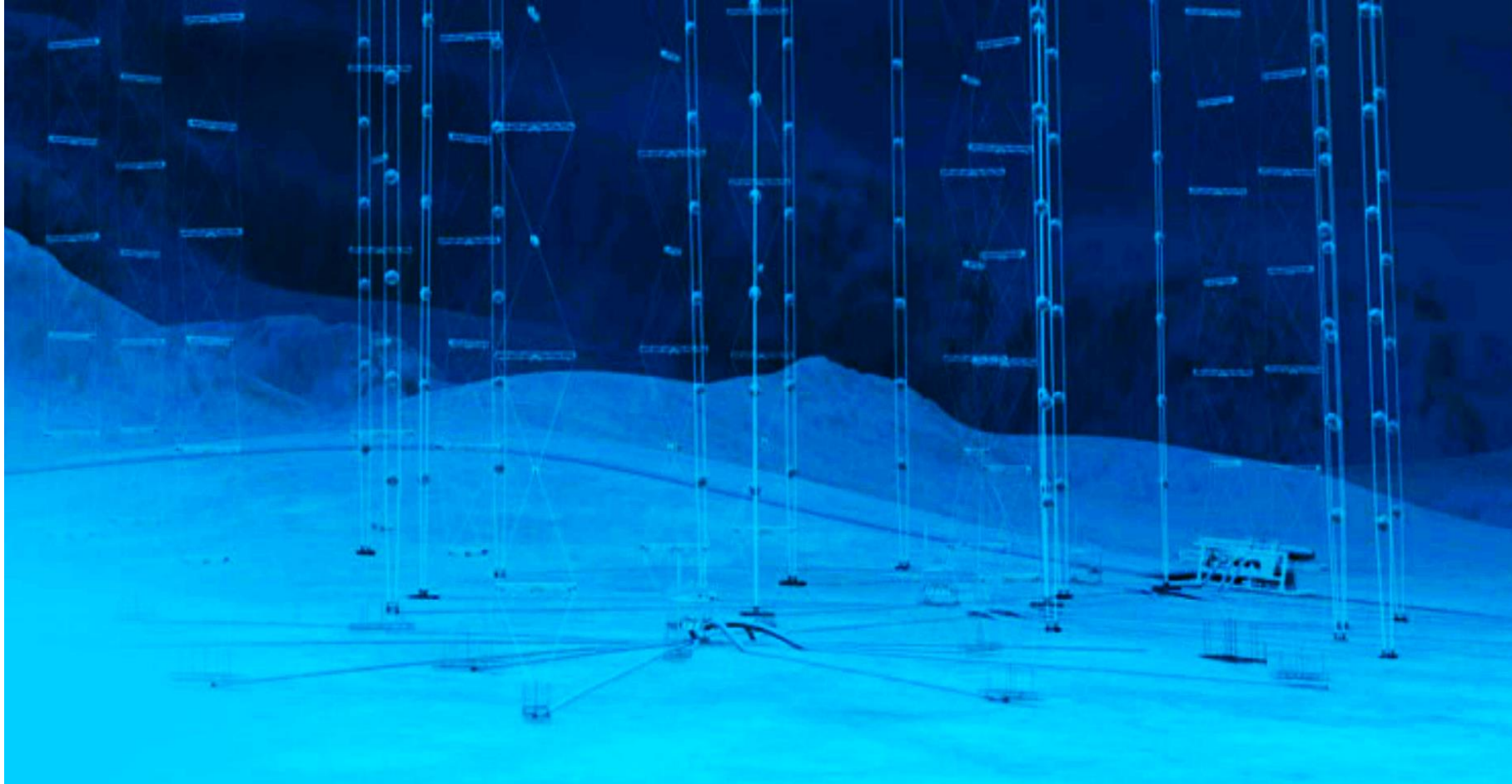
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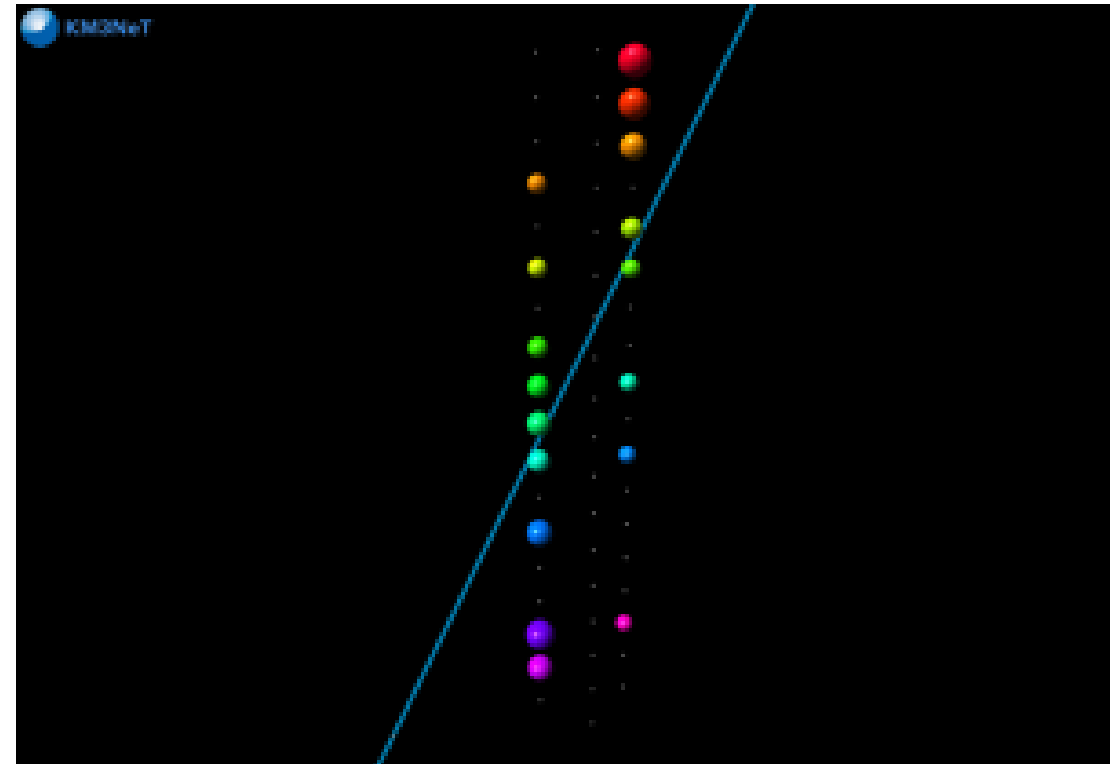
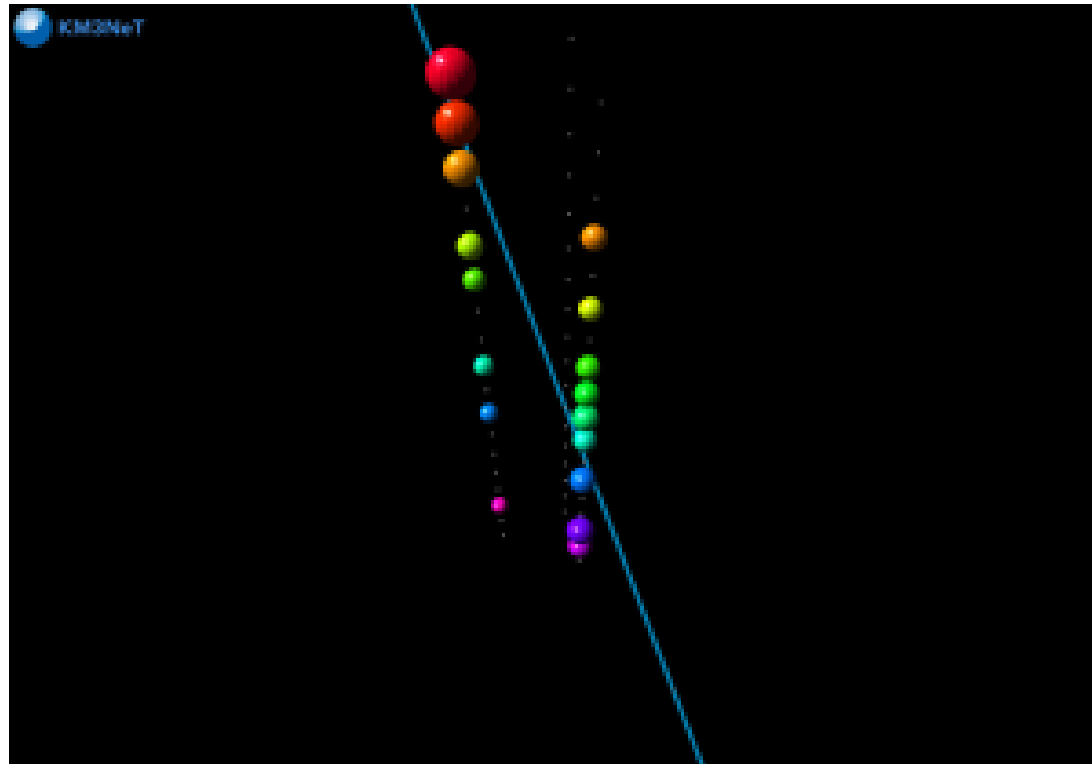
# Cherenkov Neutrino Experiment Timeline

- DUMAND: Deep Underwater Muon and Neutrino Detector - proposed by F. Reynes on 1975 (9 strings and 216 OMs, 4.6 Km deep of bottom level).
- BAIKAL - Bezrukov, Domogatsky, Berezinsky, Zatsepin - 1981 First site exploration and R&D, 1.3 Km depth, Baikal lake, from NT36 (36 OMs) to NT200 (200 OMs)
- NESTOR - Pylos island, Greece - 3.8 Km deep. Survey started in the 90's (12 floors with 14 OMs per floor, 168 in total)
- **AMANDA** - South Pole - 1996 started (19 strings and 677 OMs)
- **ANTARES** - R&D started in 1997, off-shore of Toulon, 2.5 Km deep (12 strings, 75 OMs
- each string)
- NEMO - R&D start in 1998 - first site exploration in 2002, off-shore of Capopassero, 33.5 Km deep, phase 1 (4 floors 16 OMs), phase 2 (8 floors 32 OMs)
- **IceCube** - construction 2005-2010, South Pole, 86 strings, 5160 OMs
- **KM3NeT/ARCA** - construction started on Dec 2015, off-shore Capopassero, 2 building blocks with 115 strings per block and 18 Doms each string.

# KM3NeT/ARCA Overview



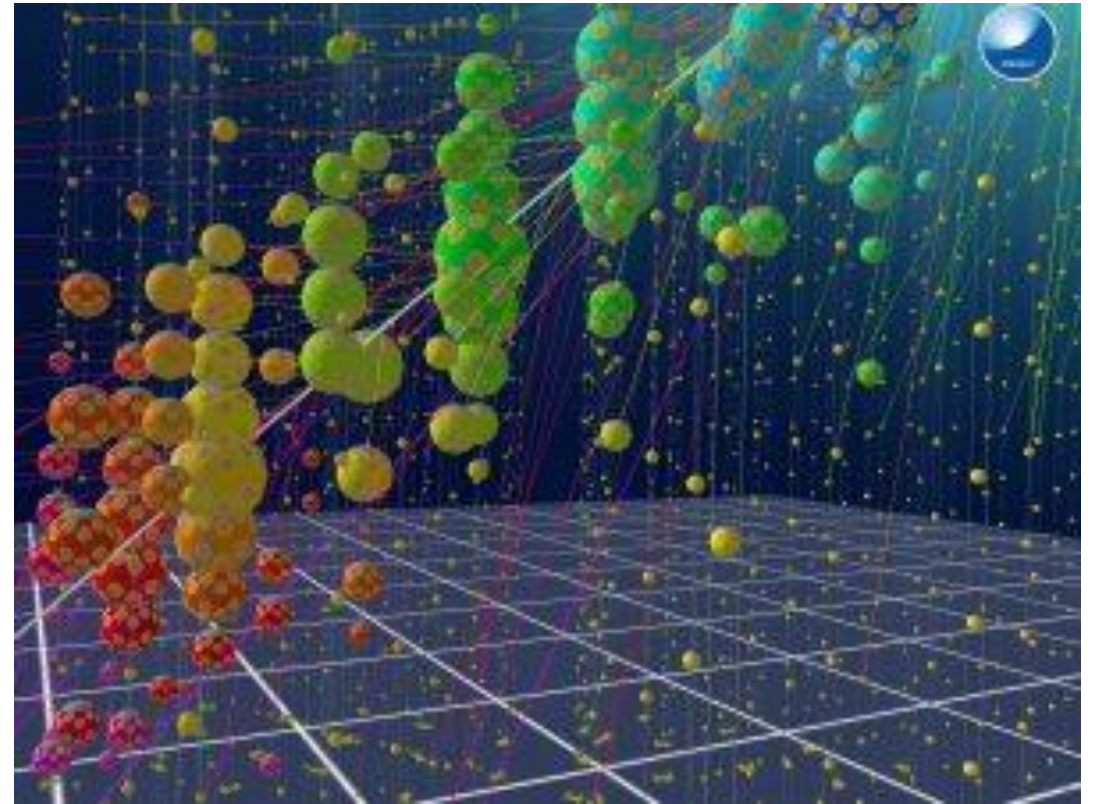
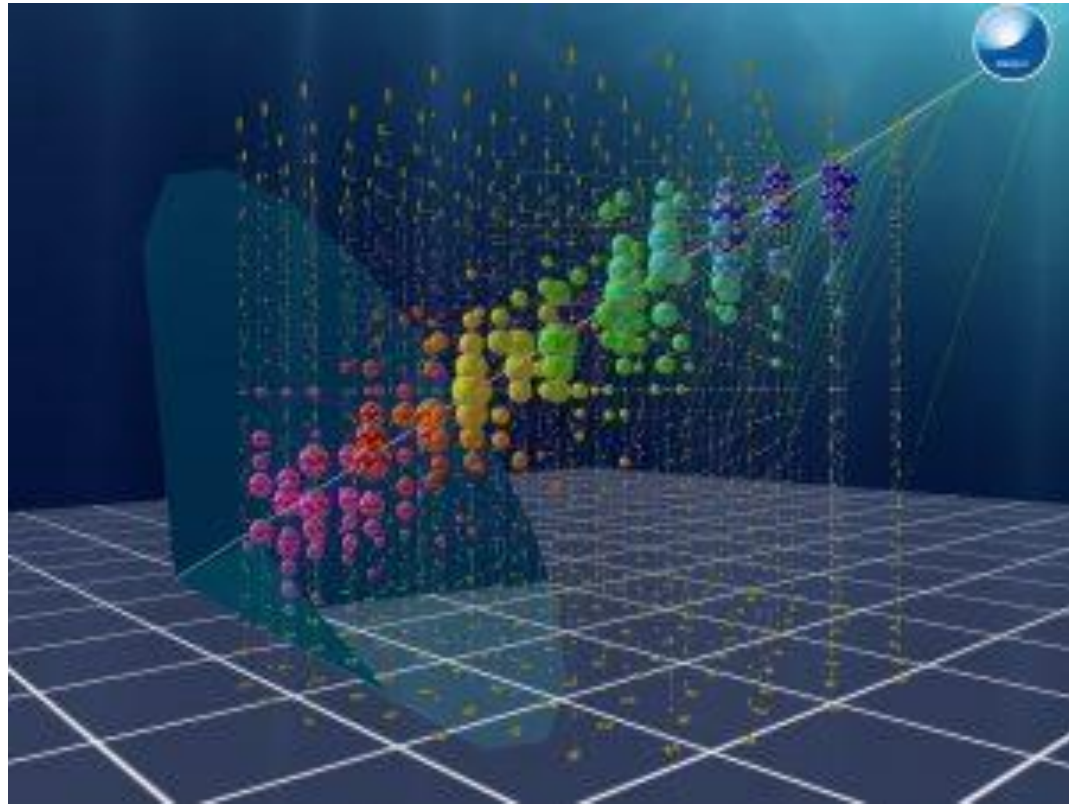
# ARCA Reconstructed Events (muon)



- The size of the spheres reflects the amount of light collected and their colour represents the arrival time of the light on the module: red is earlier than blue.
- The path of the muon particle that travels from above is reconstructed from this information and is made visible as the blue line.



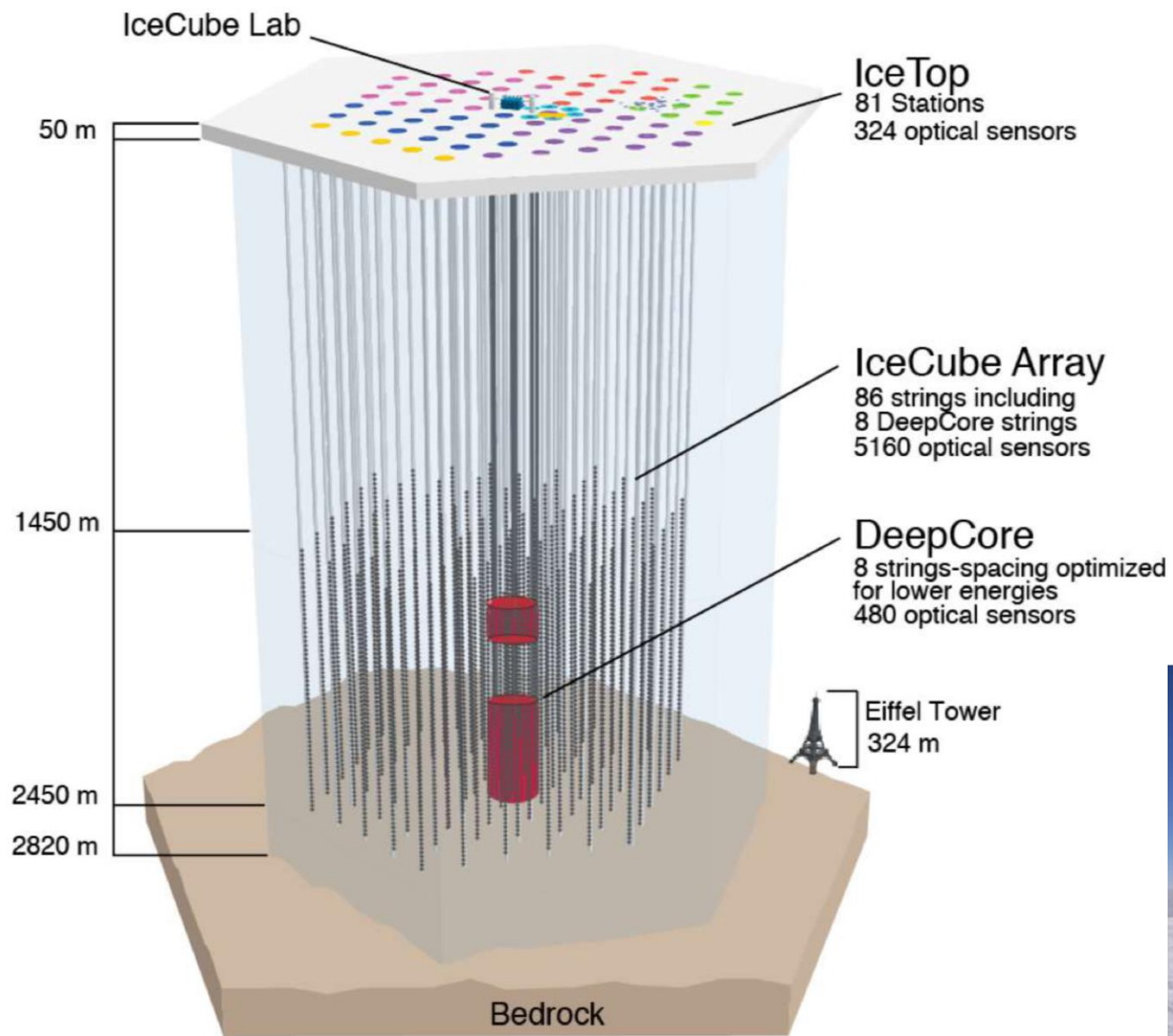
# ARCA Simulated Events



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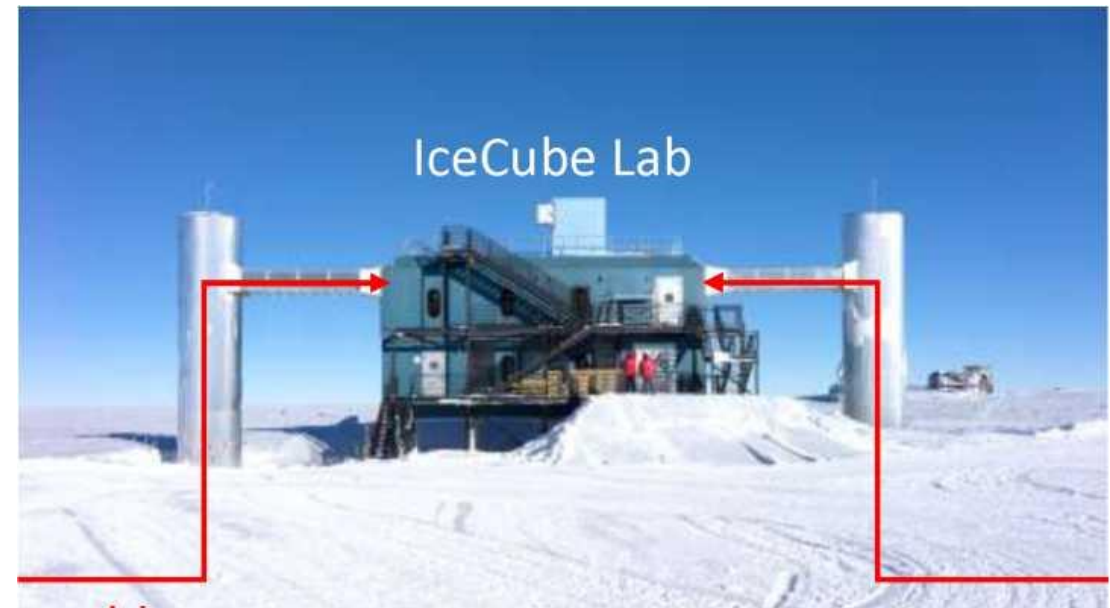
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# IceCube Overview



# IceCube Overview

- The cables of all 86 strings run together in the IceCube Lab
- On-line processing and filtering
- Detector uptime of 99.8% (!)
- Data transfer to North:
  - High priority data (e.g. alerts) can be sent 24/7 over IRIDIUM connections (very low bandwidth), Starlink in testing phase
  - Usually, a couple hours per day satellites with higher bandwidth are in reach, can transfer up to ~100 GB/day
- Rest of data is literally “shipped” out on disk

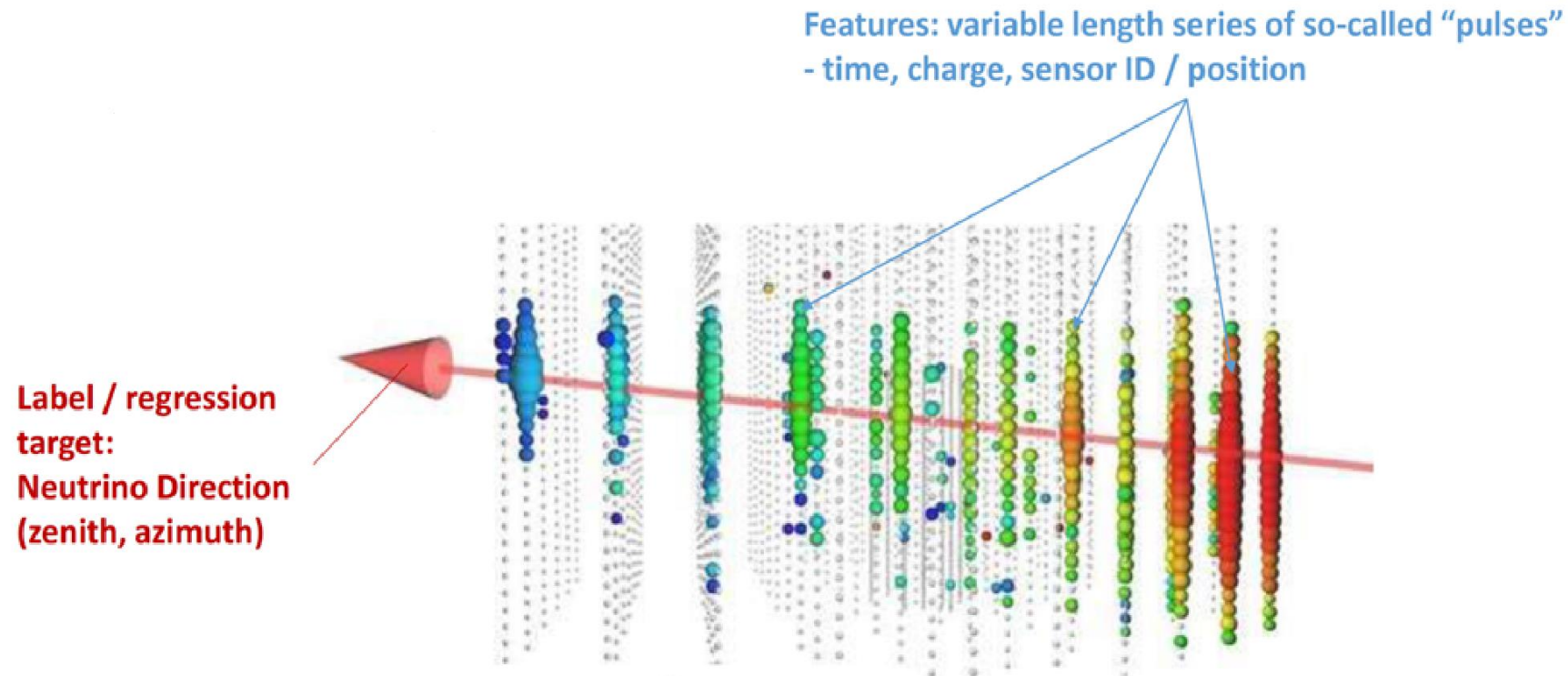




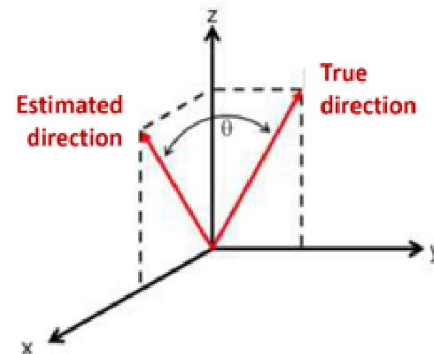
# Events in IceCube

- Every DOM gets around ~500-800 hits per second, mainly from dark noise
- Hits from physics events are ~1 order of magnitude fewer
- Most of this is suppressed by trigger conditions
- Per year, we read out roughly:
  - $10^{10}$  events caused by atmospheric muons
  - $10^9$  events caused by noise
  - 100.000 events from atmospheric neutrinos
- “A handful of very high energy events likely to be of astrophysical origin”
- Special triggers exist for example looking for supernovae, they monitor the overall hit rate, where a correlated increase could indicate a nearby supernova

# IceCube Event Reconstruction



Scoring Metric: Angle  
between true and  
estimated direction:

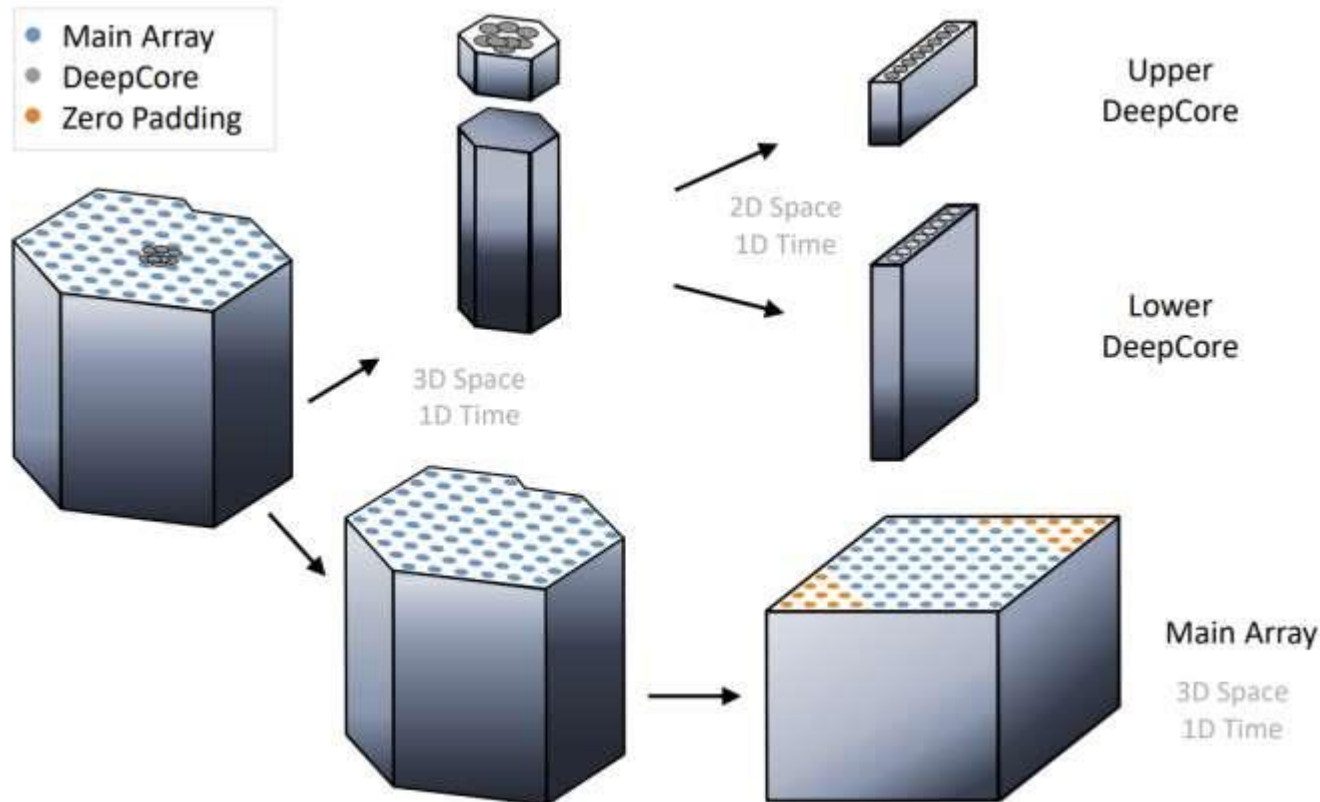


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# CNNs in Neutrino Telescopes

- Transform the raw data into 3d images



- A convolutional neural network based cascade reconstruction for the IceCube Neutrino Observatory

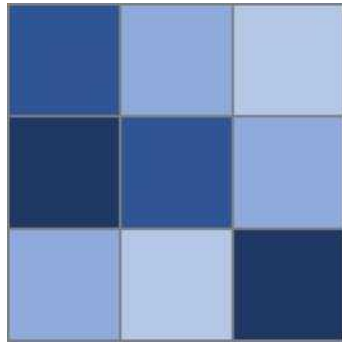
- <https://doi.org/10.1088/1748-0221/16/07/P07041>

→ **One big limitation:**  
transformation onto  
rigid 3d “image”  
→ **Especially for DeepCore**



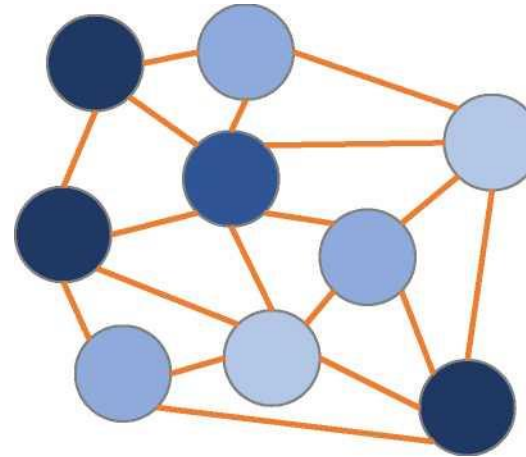
# Graph Neural Networks

**CNN:** fixed, rigid correlation structure:



But, the data / detector geometry may deviate from this regularity

**GGN:** Representation of data as a mathematical graph:



- **Node** features can be exactly the same as pixels in CNNs
- **Edges** representing the pair-wise relation between nodes (e.g. spatial distance)

- **Graph Neural Networks (GNNs):**
  - generalization of NNs to arbitrary geometries
  - can define “graph convolutions” similar to CNNs

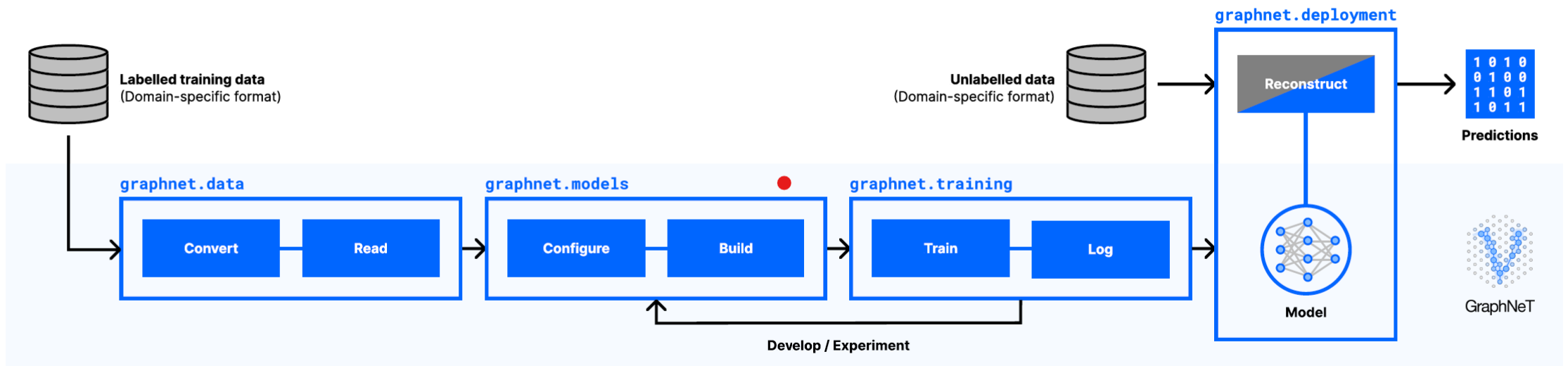


**GraphNeT**

Graph Neural Networks for  
Neutrino Telescope Event Reconstruction

[graphnet-team/graphnet](https://github.com/graphnet-team/graphnet): Graph neural networks for neutrino telescope event reconstruction ([github.com](https://github.com))

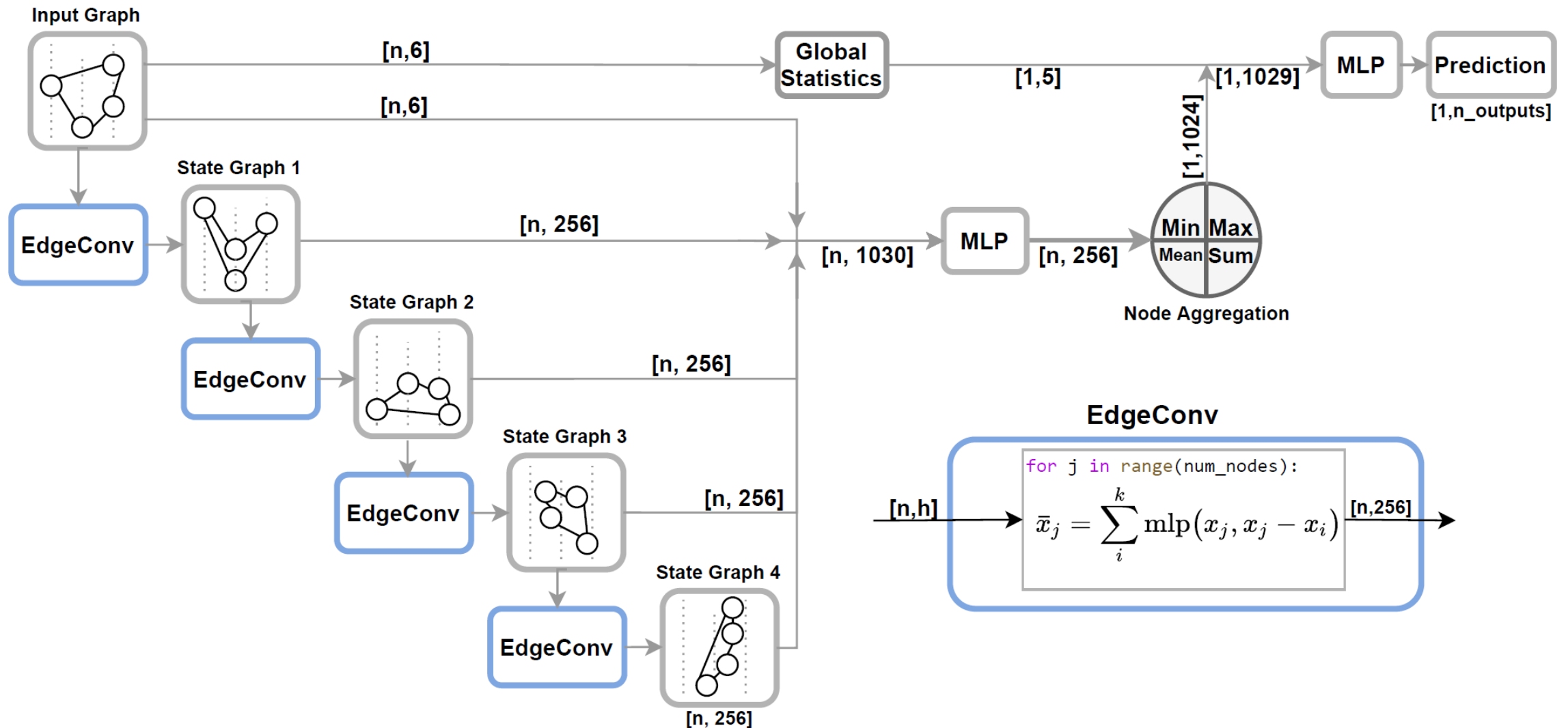
# GraphNet Workflow



High-level overview of a typical workflow using GraphNeT:

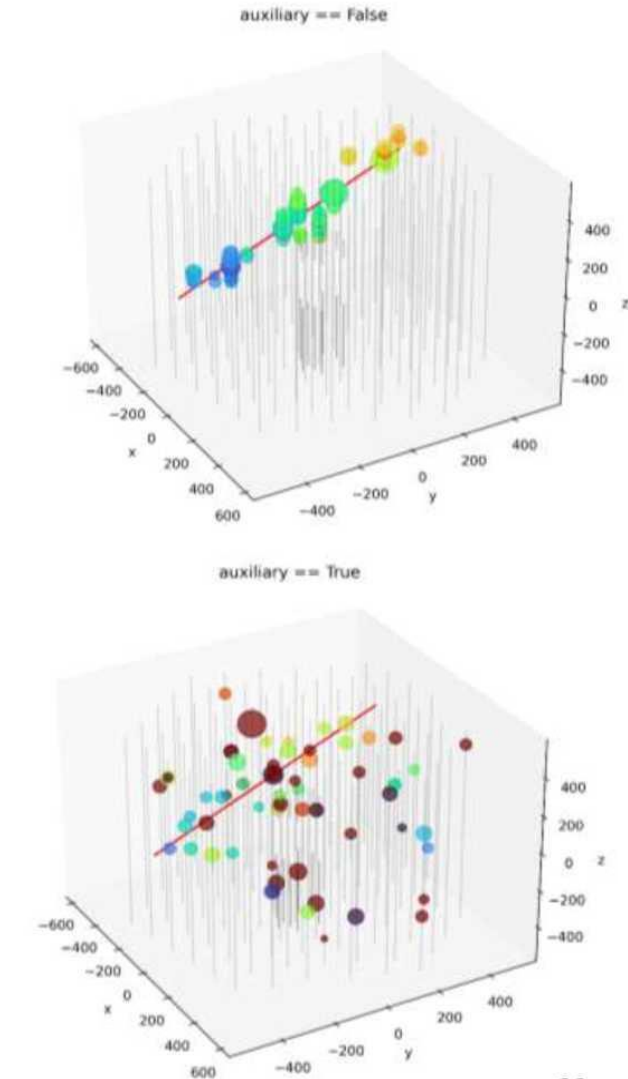
1. [graphnet.data](#) enables converting domain-specific data to industry-standard, intermediate file formats and reading this data;
2. [graphnet.models](#) allows for configuring and building complex GNN models using simple, physics-oriented components;
3. [graphnet.training](#) manages model training and experiment logging;
4. [graphnet.deployment](#) allows for using trained models for inference in domain-specific reconstruction chains.

# DynEdge: the Heart of GraphNet

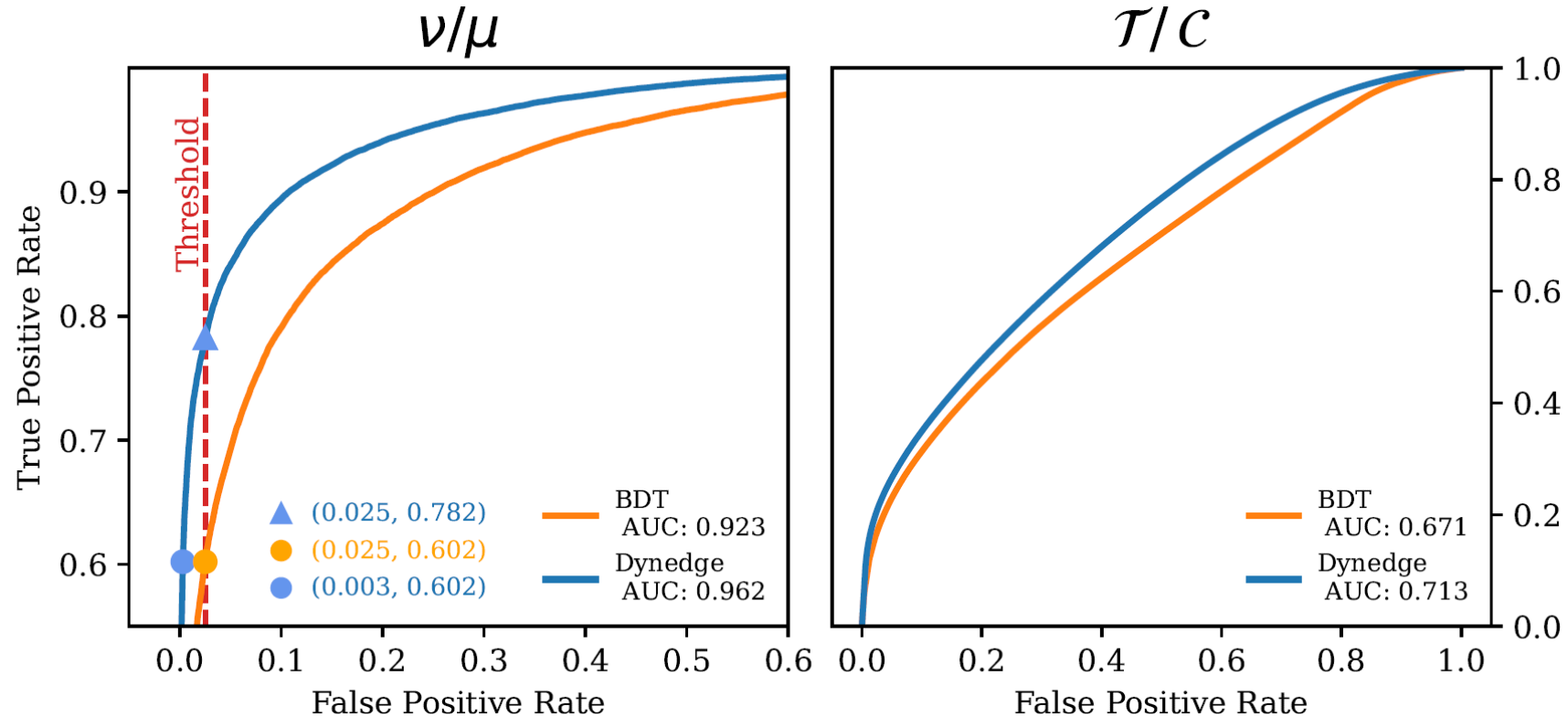


# DynEdge Train & Test

- Low energy neutrino dataset from 1GeV to 1000GeV simulated with GENIE  
<https://doi.org/10.1016/j.nima.2009.12.009>
- Training on the simulated dataset with separation of “track-like” events from “cascade-like” ones.
- IceCube detector has 5160 sensors at fixed, known locations
- Per event each sensor receives zero - many "pulses", i.e. variable-length time series
- Comparison of results with RETRO algorithm  
<https://doi.org/10.1140/epjc/s10052-022-10721-2>



# IceCube Event Reconstruction Sample



## DYNEDGE classifier vs Boosted Decision Tree performance

- 18% increase in  $\nu/\mu$  classification task
- 6% increase in AUC score in  $\tau/c$  classification task



# The End

