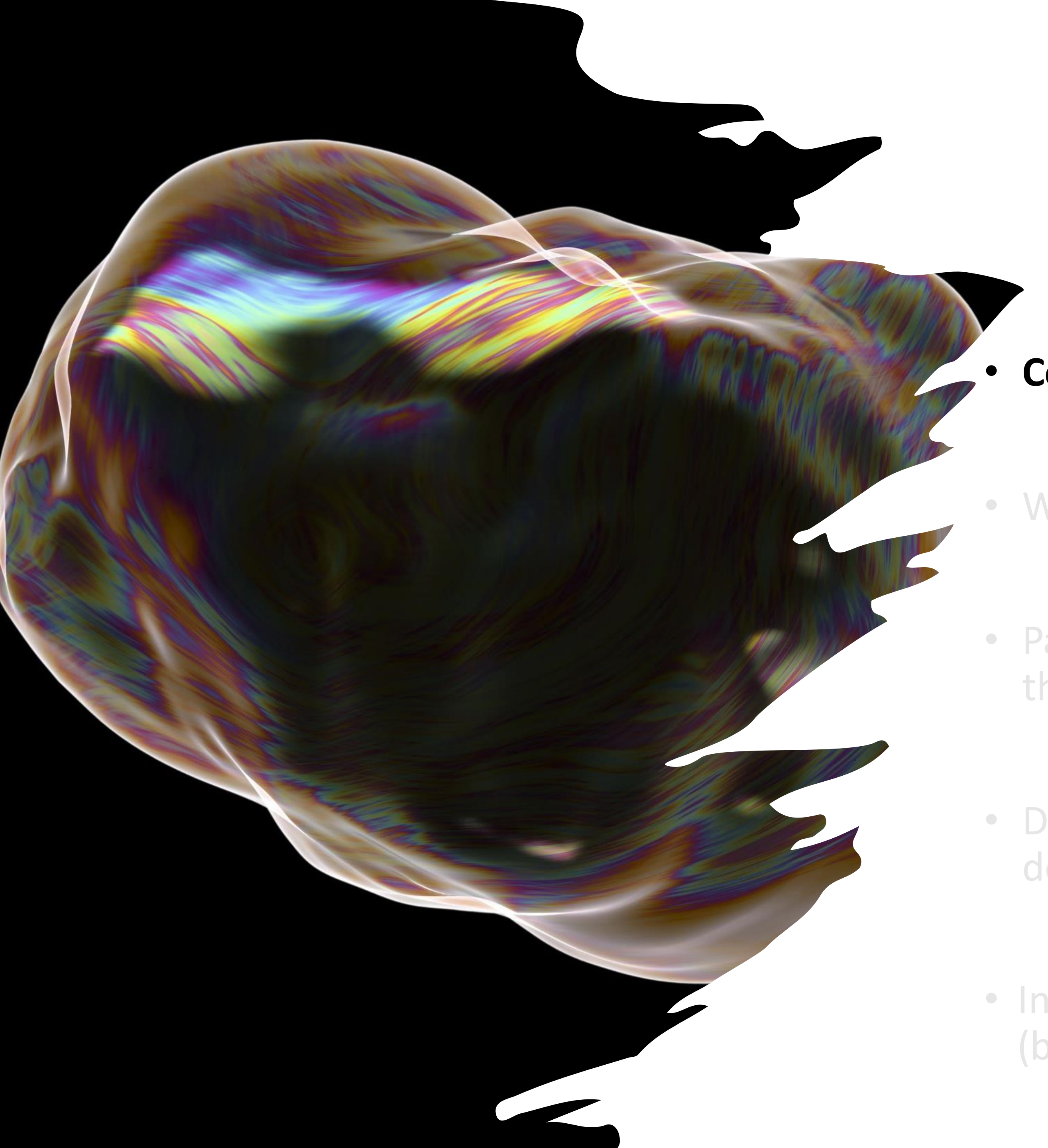
Statistical analysis to set upper limits on Dark Matter (DM) parameters

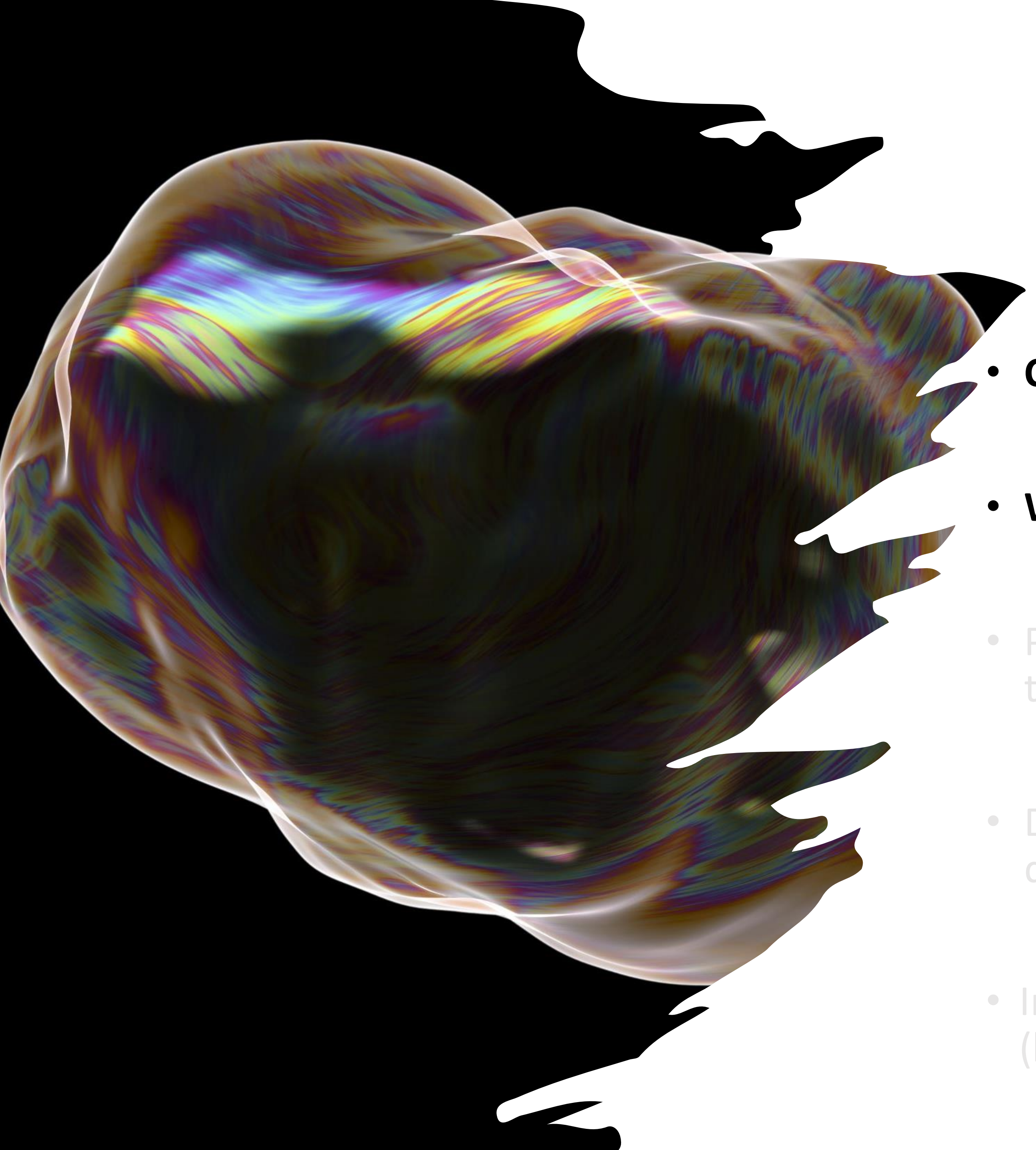
Abhishek

Università degli Studi di Siena



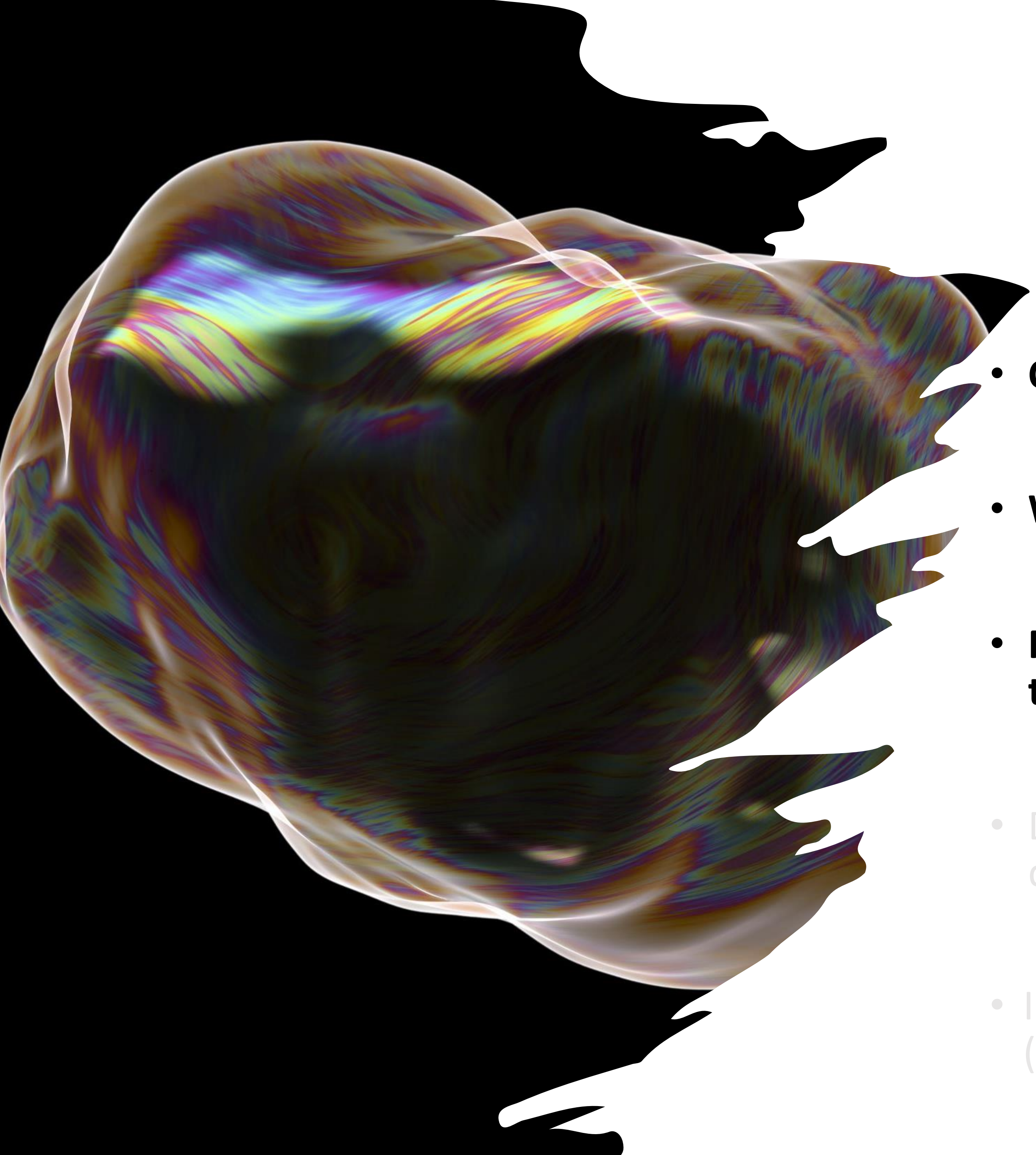
Search for DM

- **Cosmological ΛCDM model**
- Weakly Interacting Massive Particles (WIMPs)
- Particles are stable, massive, and interacts only through weak and gravitational interactions.
- Different approaches: Collider search, Direct detection and Indirect detection.
- Indirect: WIMPs annihilate into SM particles (bosons, quarks, leptons)



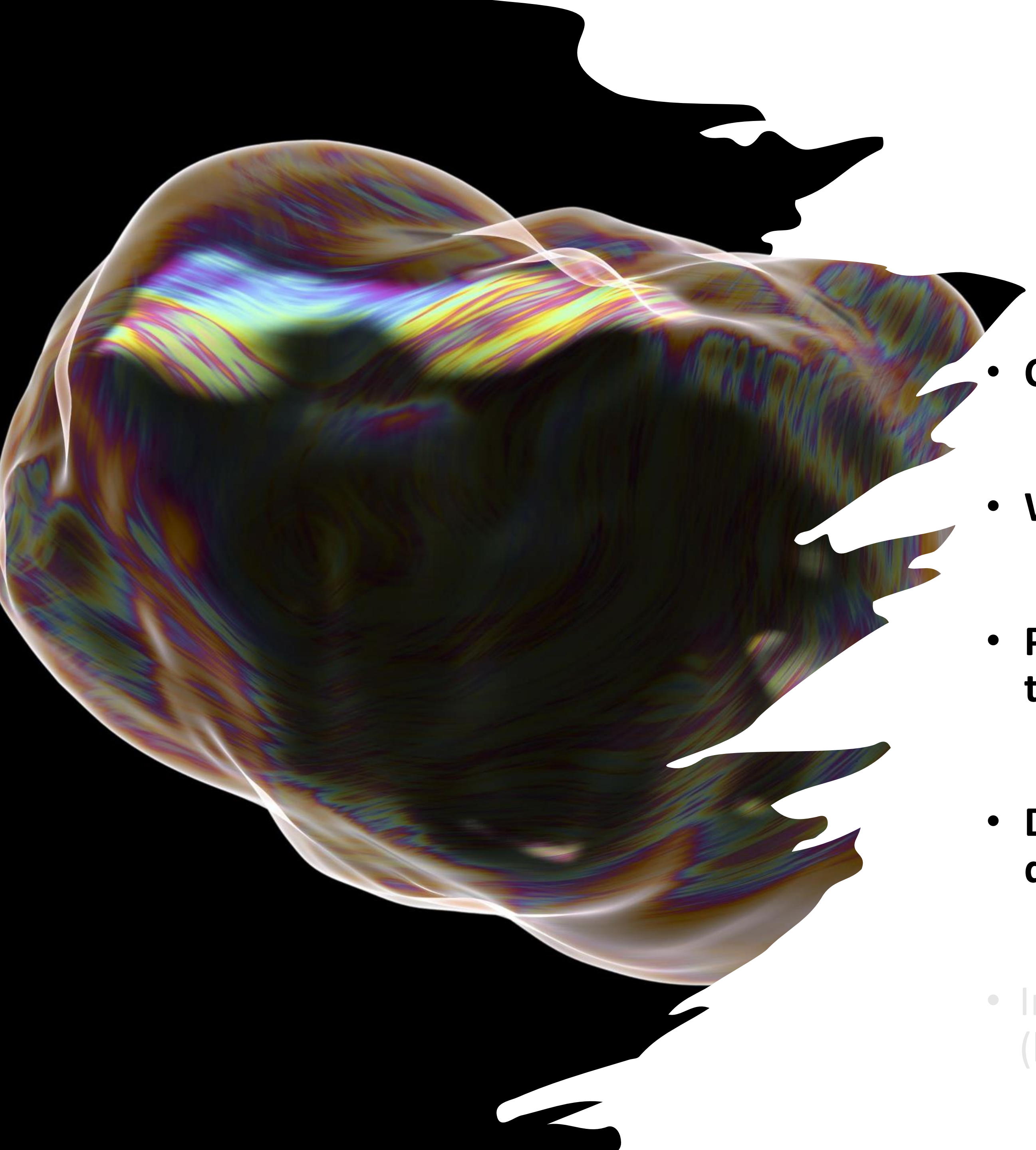
Search for dark matter

- **Cosmological ΛCDM model**
- **Weakly Interacting Massive Particles (WIMPs)**
- Particles are stable, massive, and interacts only through weak and gravitational interactions.
- Different approaches: Collider search, Direct detection and Indirect detection.
- Indirect: WIMPs annihilate into SM particles (bosons, quarks, leptons)



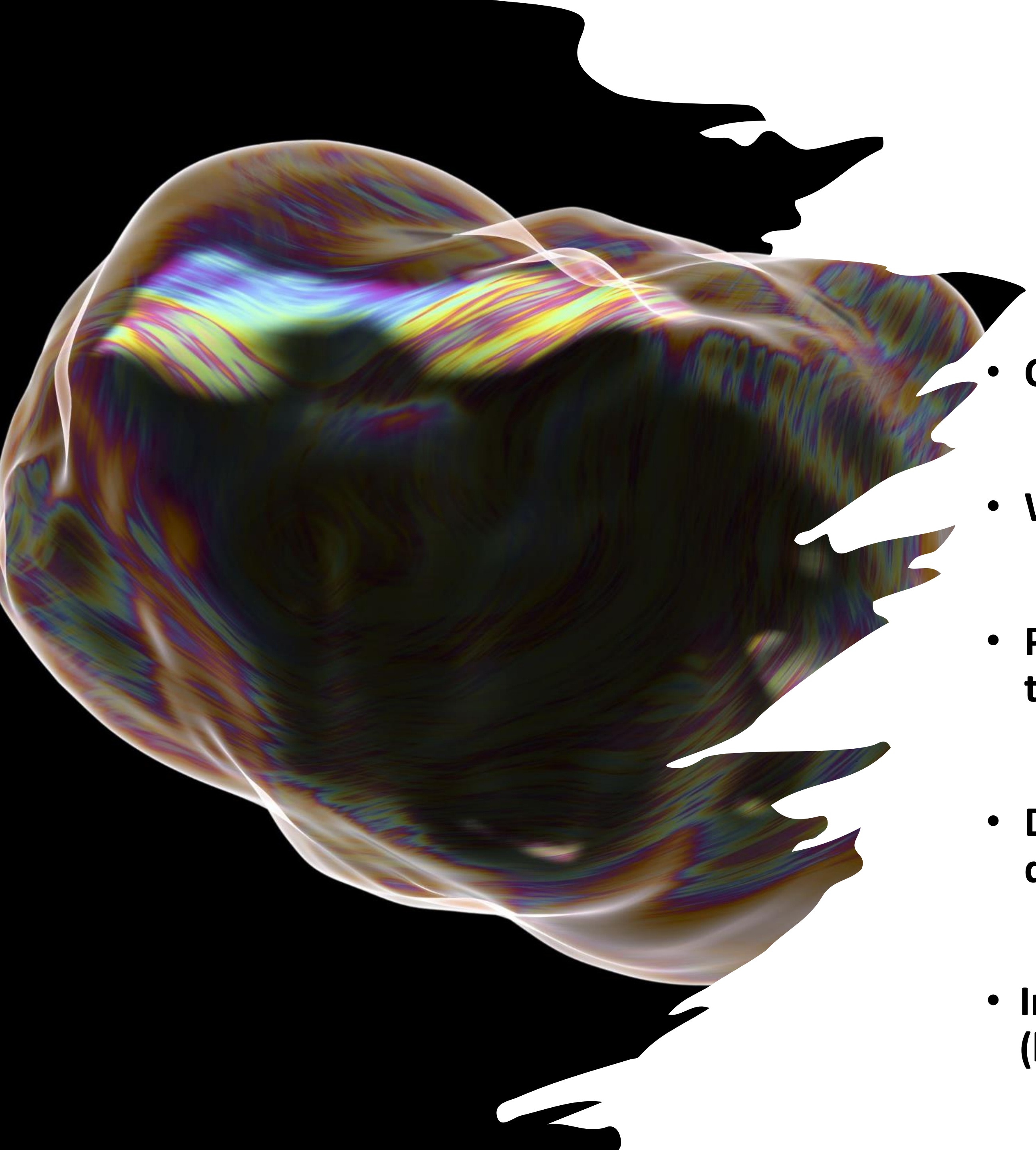
Search for dark matter

- **Cosmological ΛCDM model**
- **Weakly Interacting Massive Particles (WIMPs)**
- **Particles are stable, massive, and interacts only through weak and gravitational interactions**.
- Different approaches: Collider search, Direct detection and Indirect detection.
- Indirect: WIMPs annihilate into SM particles (bosons, quarks, leptons)



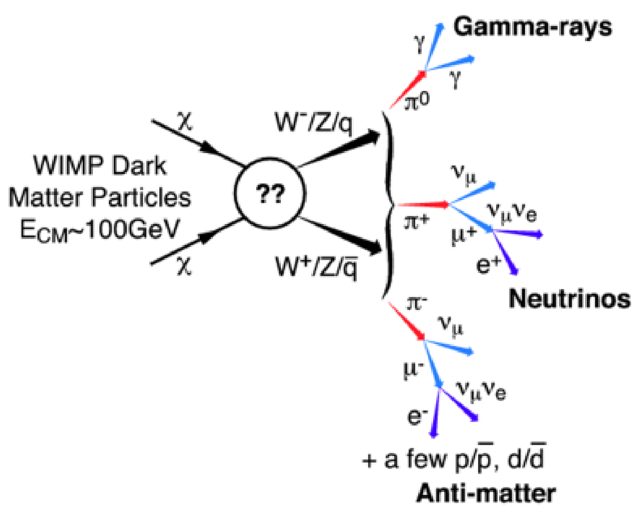
Search for dark matter

- **Cosmological ΛCDM model**
- **Weakly Interacting Massive Particles (WIMPs)**
- **Particles are stable, massive, and interacts only through weak and gravitational interactions**.
- **Different approaches: Collider search, Direct detection and Indirect detection.**
- Indirect: WIMPs annihilate into SM particles (bosons, quarks, leptons)

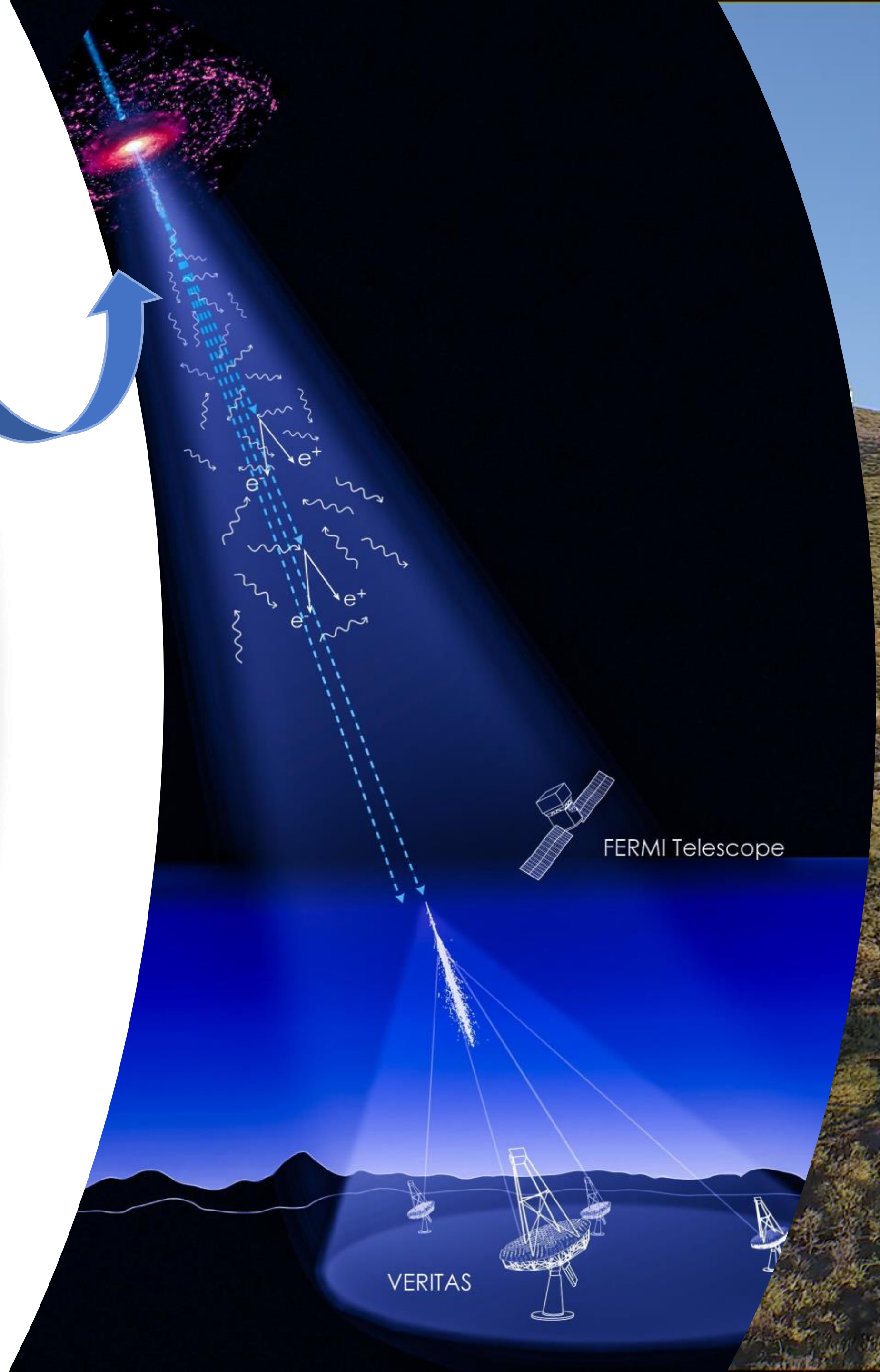


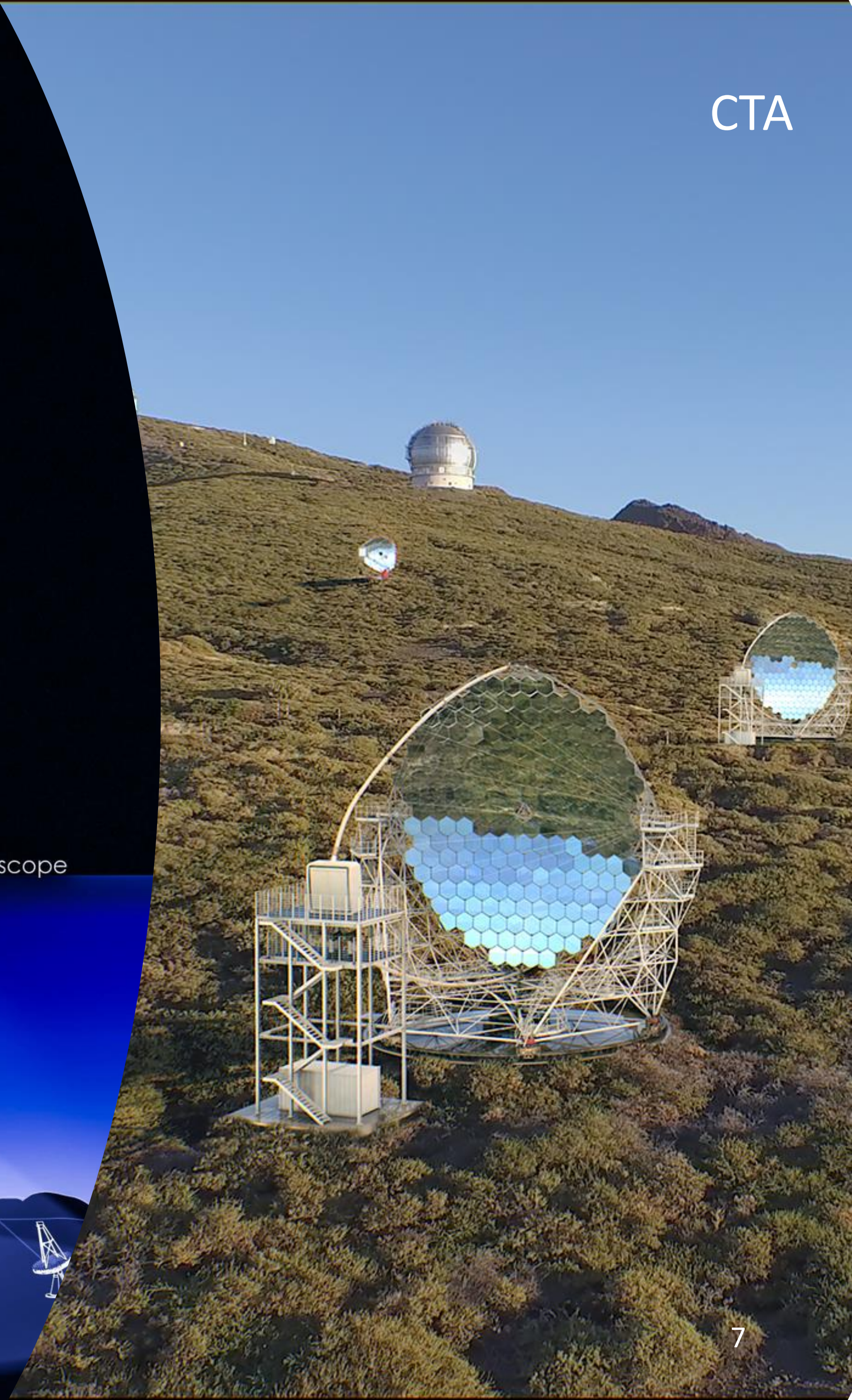
Search for dark matter

- **Cosmological ΛCDM model**
- **Weakly Interacting Massive Particles (WIMPs)**
- **Particles are stable, massive, and interacts only through weak and gravitational interactions.**
- **Different approaches: Collider search, Direct detection and Indirect detection.**
- **Indirect: WIMPs annihilate into SM particles (bosons, quarks, leptons)**



Detection of γ-ray flux





Expected γ-ray flux

$$\frac{d\phi_\gamma}{dE_\gamma} = \frac{1}{\xi}\frac{\langle\sigma v\rangle}{4\pi m_\chi^2}\sum_f B_f \frac{dN_\gamma^f}{dE_\gamma} \times J$$

- $1/\xi$depends on whether the DM particle is Majorana fermion or Dirac fermion
- DM annihilation cross-section averaged over the velocity distribution $\langle\sigma v\rangle$
- DM particle mass m_χ
- J-factor describes the DM distribution and the amount of DM annihilations within the source

Statistical analysis

- A widely used procedure to establish discovery is based on a frequentist significance test using a likelihood ratio as a test statistic.
- We use log-likelihood ratio statistical test to constrain the DM annihilation cross-section setting upper limits.

Formalism

- Suppose, we are measuring x, and compute a histogram for all the events in the signal sample.
- The expectation value of number of events in each bin will be:

$$E[n_i] = \mu s_i + b_i$$

Formalism

- Suppose, we are measuring x, and compute a histogram for all the events in the signal sample.
- The expectation value of number of events in each bin will be:

$$E[n_i] = \mu s_i + b_i$$

- The mean number of entries in the i_{th} bin from signal and background are:

$$s_i = s_{tot} \int_{bin\ i} f_s(x; \boldsymbol{\theta}_s) dx$$

$$b_i = b_{tot} \int_{bin\ i} f_s(x; \boldsymbol{\theta}_b) dx$$

Formalism

- Now we compute the likelihood function which is the product of Poisson probabilities for all bins.

$$L(\mu, \boldsymbol{\theta}) = \prod_{j=1}^{N} \frac{\left(\mu s_j + b_j\right)^{n_j}}{n_j!} e^{-(\mu s_j + b_j)}$$

Formalism

- Now we compute the likelihood function which is the product of Poisson probabilities for all bins.

$$L(\mu, \boldsymbol{\theta}) = \prod_{j=1}^{N} \frac{(\mu s_j + b_j)^{n_j}}{n_j!} e^{-(\mu s_j + b_j)}$$

- Further measurements to constrain the nuisance parameters:

$$L(\mu, \boldsymbol{\theta}) = \prod_{j=1}^{N} \frac{(\mu s_j + b_j)^{n_j}}{n_j!} e^{-(\mu s_j + b_j)} \prod_{k=1}^{M} \frac{u_k^{m_k}}{m_k!} e^{-u_k}$$

Background

Formalism

Profile likelihood ratio

$$\lambda(\mu) = \frac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}\right)}{L\left(\hat{\mu}, \hat{\boldsymbol{\theta}}\right)}$$

Formalism

Profile likelihood ratio

$$\lambda(\mu) = \frac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}\right)}{L\left(\hat{\mu}, \hat{\boldsymbol{\theta}}\right)}$$

Denotes the value of θ that maximizes likelihood for the specified μ (conditional)

Formalism

Profile likelihood ratio

$$\lambda(\mu) = \frac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}\right)}{L\left(\hat{\mu}, \hat{\boldsymbol{\theta}}\right)}$$

Denotes the value of θ that maximizes likelihood for the specified μ (conditional)

Unconditional ML estimators

Formalism

- [G. Casella, R.L. Berger, Statistical Inference, Duxbury Press, 1990] - $-2ln\lambda$ converges in distribution to a χ^2 random variable with k degrees of freedom.

Formalism

- [G. Casella, R.L. Berger, Statistical Inference, Duxburry Press, 1990] - $-2ln\lambda$ converges in distribution to a χ^2 random variable with k degrees of freedom.

$$t_\mu = -2 \ln \lambda(\mu)$$

$$p_\mu = \int_{t_{\mu,\mathrm{obs}}}^{\infty} f(t_\mu|\mu)\, dt_\mu$$

Formalism

$$t_\mu = -2 \ln \lambda(\mu)$$

- No condition on values of μ.
- As $\mu = 0$: background-only events, presence of a new signal can only increase the mean event rate beyond what is expected from background alone.

Formalism

$$t_\mu = -2 \ln \lambda(\mu)$$

- No condition on values of μ.
- As $\mu = 0$: background-only events, presence of a new signal can only increase the mean event rate beyond what is expected from background alone.
- $\boldsymbol{\mu \geq 0}$

Formalism

$$\tilde{\lambda}(\mu) = \begin{cases} \dfrac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}(\mu)\right)}{L\left(\hat{\mu}, \hat{\boldsymbol{\theta}}\right)} & \hat{\mu} \geq 0 \\ \dfrac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}(\mu)\right)}{L\left(0, \hat{\hat{\boldsymbol{\theta}}}(0)\right)} & \hat{\mu} < 0 \end{cases}$$

- For $\hat{\mu} < 0$, the best level of agreement between the data and any physical value of μ occurs for $\mu = 0$.

Formalism

$$\tilde{\lambda}(\mu) = \begin{cases} \dfrac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}(\mu)\right)}{L(\hat{\mu}, \hat{\boldsymbol{\theta}})} & \hat{\mu} \geq 0 \\ \dfrac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}(\mu)\right)}{L\left(0, \hat{\hat{\boldsymbol{\theta}}}(0)\right)} & \hat{\mu} < 0 \end{cases}$$

- For $\hat{\mu} < 0$, the best level of agreement between the data and any physical value of μ occurs for $\mu = 0$.

$$\tilde{t}_\mu = -2\ln\tilde{\lambda}(\mu) = \begin{cases} -2\ln\dfrac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}(\mu)\right)}{L(\hat{\mu}, \hat{\boldsymbol{\theta}})} & \hat{\mu} \geq 0 \\ -2\ln\dfrac{L\left(\mu, \hat{\hat{\boldsymbol{\theta}}}(\mu)\right)}{L\left(0, \hat{\hat{\boldsymbol{\theta}}}(0)\right)} & \hat{\mu} < 0 \end{cases}$$

Formalism : Upper limits

- To establish an upper limit on a parameter, let us define:

$$q_\mu = \begin{cases} -2\ln\lambda(\mu) & \hat{\mu} \leq \mu \\ 0 & \hat{\mu} > \mu \end{cases}$$

Formalism : Upper limits

- To establish an upper limit on a parameter, let us define:

$$q_\mu = \begin{cases} -2\ln\lambda(\mu) & \hat{\mu} \leq \mu \\ 0 & \hat{\mu} > \mu \end{cases}$$

- To set an upper limit, we are only interested in the data corresponding to μ > the ML estimator of μ.

Formalism : Upper limits

- But for the condition: $\mu \geq 0$

$$q_\mu = \begin{cases} -2\ln\lambda(\mu) & \hat{\mu} \leq \mu \\ 0 & \hat{\mu} > \mu \end{cases} = \begin{cases} -2\ln\frac{L\left(\mu,\hat{\hat{\boldsymbol{\theta}}}(\mu)\right)}{L\left(0,\hat{\hat{\boldsymbol{\theta}}}(0)\right)} & \hat{\mu} < 0 \\ -2\ln\frac{L\left(\mu,\hat{\hat{\boldsymbol{\theta}}}(\mu)\right)}{L\left(\hat{\mu},\hat{\boldsymbol{\theta}}\right)} & 0 \leq \hat{\mu} < \mu \\ 0 & \hat{\mu} > \mu \end{cases}$$

$$p_\mu = \int_{\mathrm{q}_{\mu,obs}}^{\infty} f\left(\mathrm{q}_\mu|\mu\right)dq_\mu$$

Examples: [C. Armand and B. Herrmann]

- 'Dark matter indirect detection limits from complete annihilation patterns'
- AIM: Studying simulated data of a dwarf galaxy to derive constraints on the DM annihilation cross-section within the singlet scalar dark matter model.
- Assumptions:
  - Candidate: real singlet scalar S
  - DM annihilation into fermions proceeds solely through s-channel Higgs exchange.
  - DM annihilation into bosonic states can proceed through direct four-vertex interactions.

https://iopscience.iop.org/article/10.1088/1475-7516/2022/11/055/pdf

Expected γ-ray flux

$$\frac{d\phi_\gamma}{dE_\gamma} = \frac{1}{\xi}\frac{\langle\sigma v\rangle}{4\pi m_\chi^2}\sum_f B_f \frac{dN_\gamma^f}{dE_\gamma} \times J$$

- $1/\xi$ depends on whether the DM particle is Majorana fermion or Dirac fermion
- DM annihilation cross-section averaged over the velocity distribution $\langle\sigma v\rangle$
- DM particle mass m_χ
- J-factor describes the DM distribution and the amount of DM annihilations within the source

Feynman diagrams for all relevant annihilation products

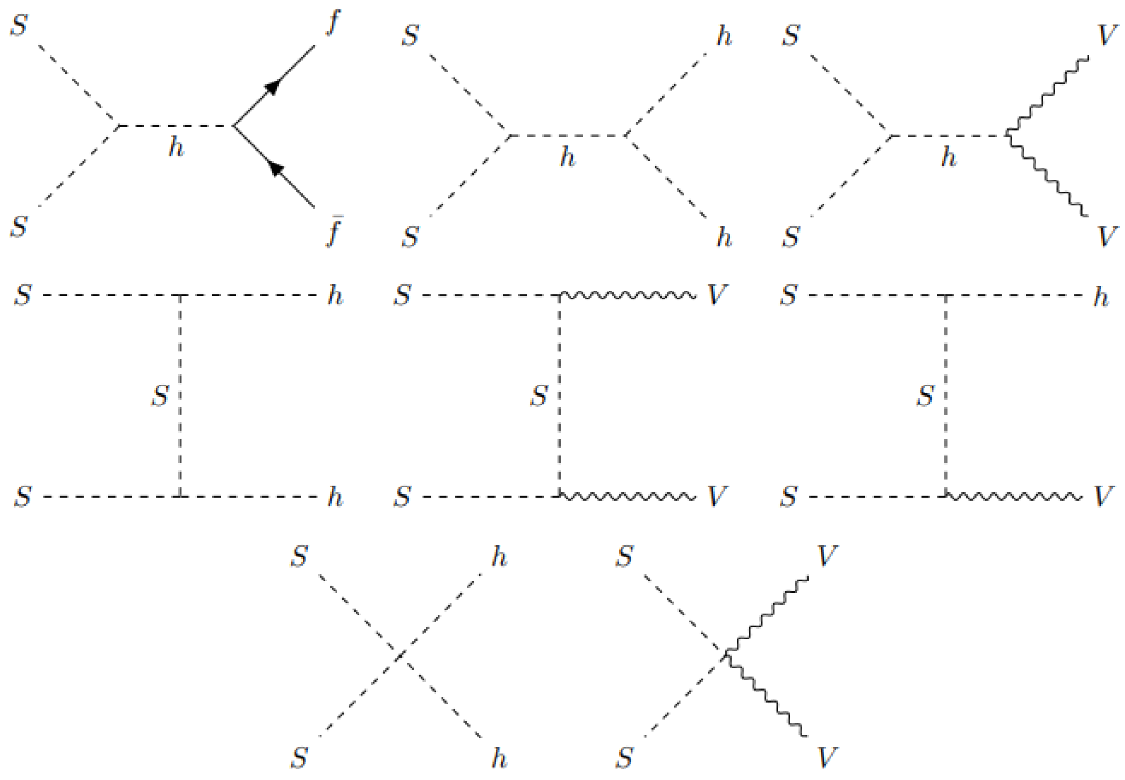


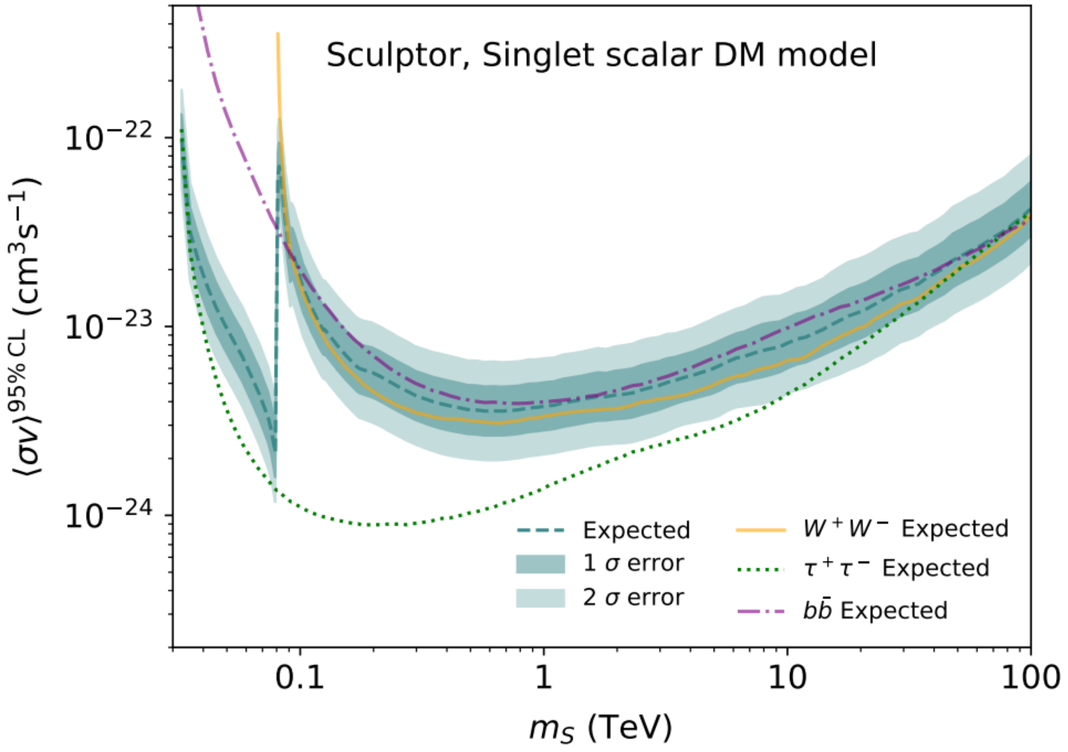
Figure 1. Tree-level Feynman diagrams for scalar pair annihilation into fermions ($f = e, \mu, \tau, u, d, c, s, t, b$), gauge ($V = W^{\pm}, Z^0$) and Higgs bosons ($h$). Diagrams corresponding to u-channels obtained through crossing are not separately depicted. Annihilation into $\gamma\gamma$ and gg final states proceeds through effective couplings to the Higgs boson h^0.

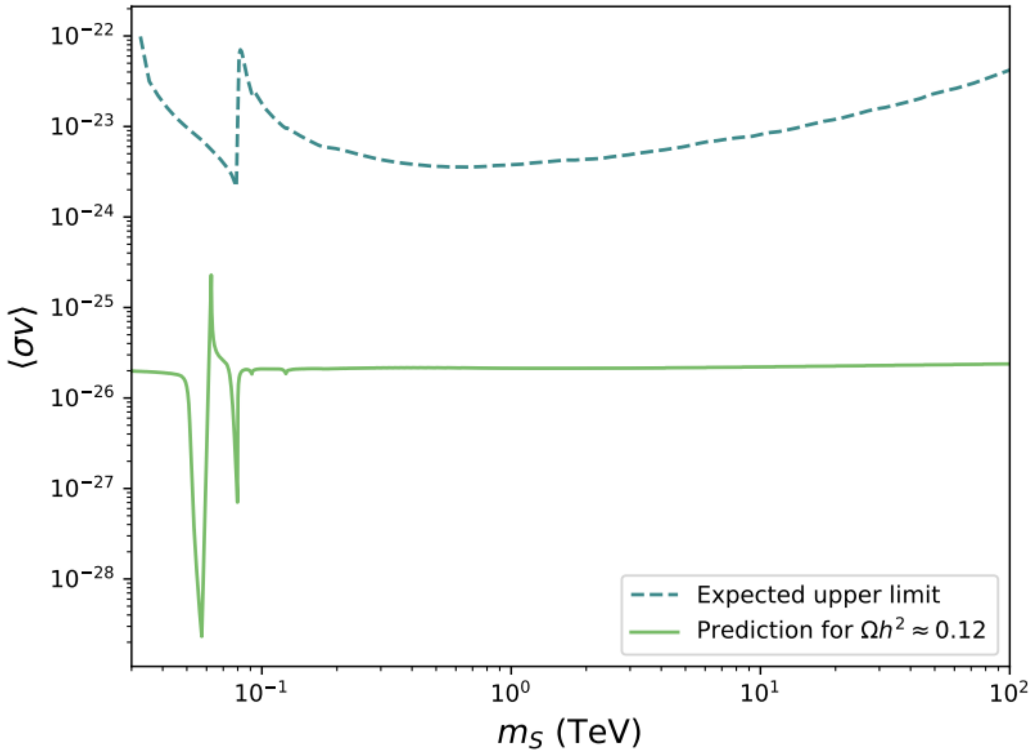
Examples:

$$TS = \begin{cases} 0 & for\ \widehat{\langle\sigma v\rangle} > \langle\sigma v\rangle \\ -2\ln\dfrac{\mathcal{L}\left(\langle\sigma v\rangle, \hat{\hat{N}}_B\left(\widehat{\langle\sigma v\rangle}\right), \hat{\hat{J}}\left(\widehat{\langle\sigma v\rangle}\right)\right)}{\mathcal{L}\left(\widehat{\langle\sigma v\rangle}, \hat{N}_B, \hat{J}\right)} & for\ 0 \le \widehat{\langle\sigma v\rangle} \le \langle\sigma v\rangle \\ -2\ln\dfrac{\mathcal{L}\left(\langle\sigma v\rangle, \hat{\hat{N}}_B\left(\widehat{\langle\sigma v\rangle}\right), \hat{\hat{J}}\left(\widehat{\langle\sigma v\rangle}\right)\right)}{\mathcal{L}\left(0, \hat{\hat{N}}_B(0), \hat{\hat{J}}(0)\right)} & for\ \widehat{\langle\sigma v\rangle} < 0 \end{cases}$$

https://iopscience.iop.org/article/10.1088/1475-7516/2022/11/055/pdf

Examples:





- $m_S = m_h/2 \approx 62.5GeV$ corresponds to Higgs resonance.
- $m_S \approx m_W \approx 80GeV$, DM particles get heavier and produce more energetic SM particles

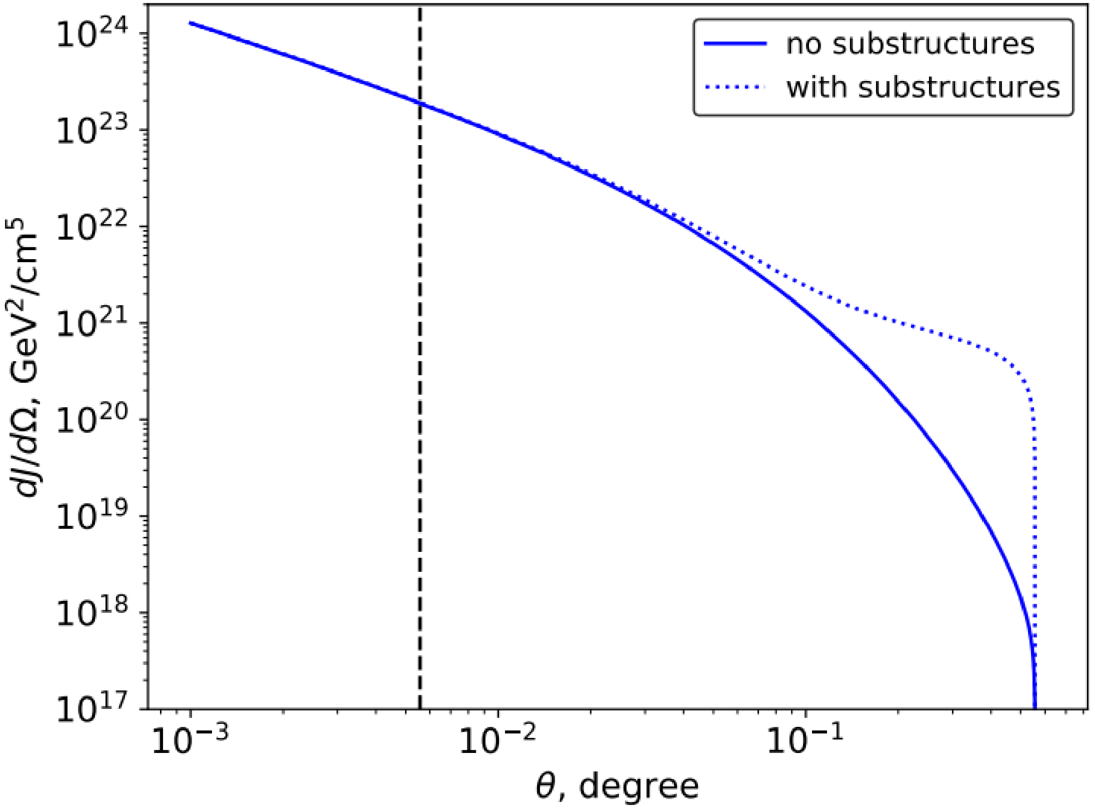
https://iopscience.iop.org/article/10.1088/1475-7516/2022/11/055/pdf

Examples: [Thorpe-Morgan et al.]

- 'Annihilating Dark Matter Search with 12 Years of Fermi-LAT Data in Nearby Galaxy Clusters'
- AIM: Studying observed data of fermi-LAT of galaxy clusters to derive constraints on the DM annihilation cross-section to discuss the potential of a boost to the signal due to the presence of substructures in the DM halos
- Assumptions:
- Cases with assuming the presence of substructures that can significantly boost the expected signal from the outer parts of halos.
- The spatial and mass distributions are taken from Sánchez-Conde & Prada (2014)

https://arxiv.org/pdf/2010.11006.pdf

Examples:



$$dJ/d\Omega = \int_{l.o.s} \rho^2(l)dl$$

https://arxiv.org/pdf/2010.11006.pdf

Examples:

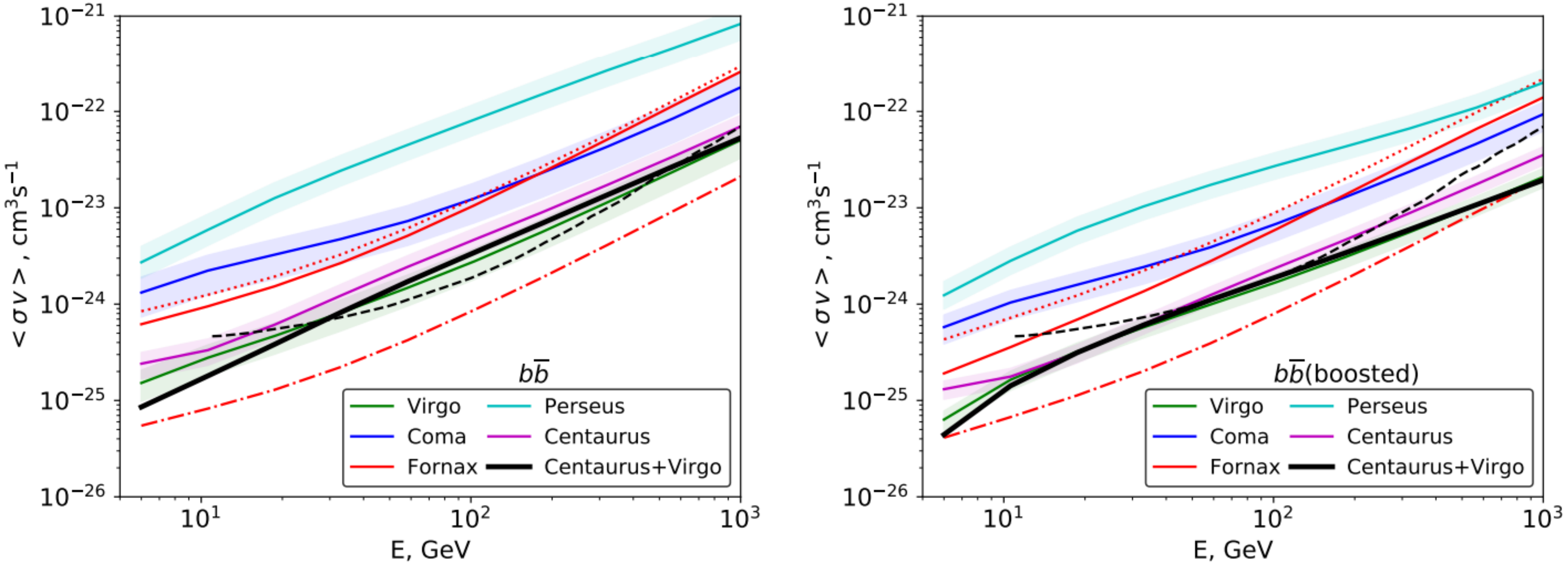


Fig: The left panel shows the results for smooth DM halos, while the right one shows the results when the presence of substructures is included

https://arxiv.org/pdf/2010.11006.pdf

Examples:

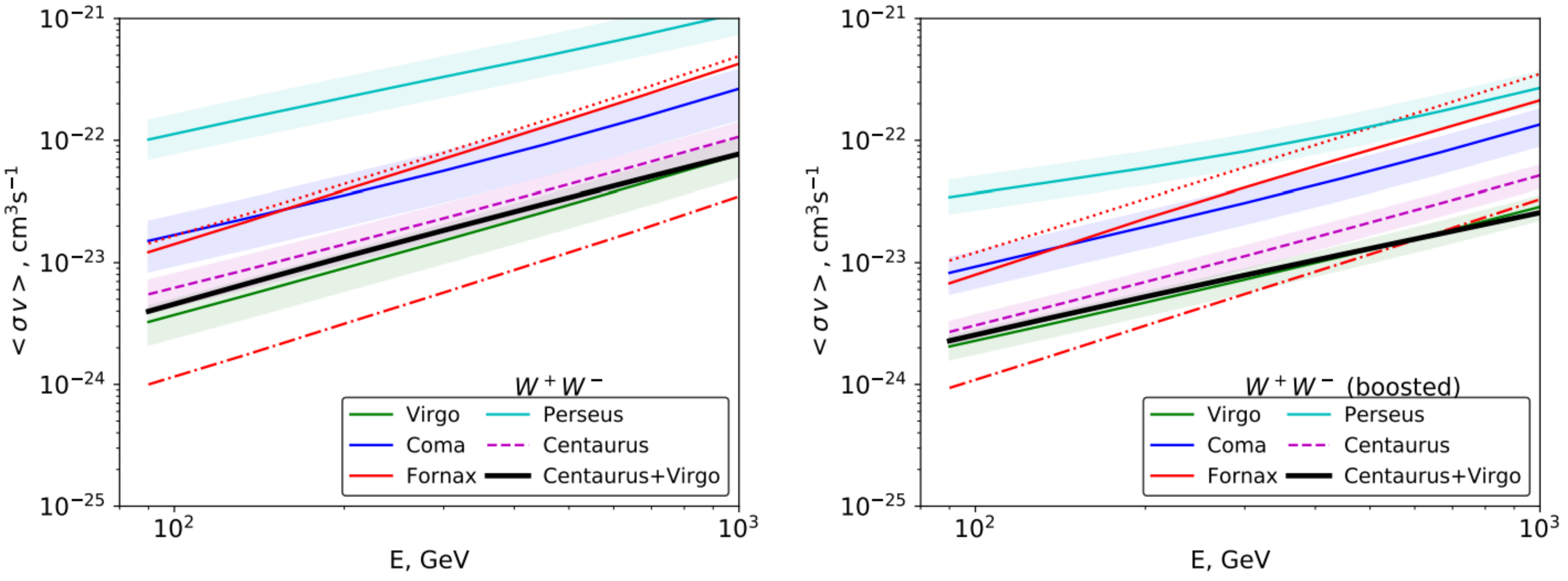


Fig: The left panel shows the results for smooth DM halos, while the right one shows the results when the presence of substructures is included

https://arxiv.org/pdf/2010.11006.pdf

References:

[1] Asymptotic formulae for likelihood-based tests of new physics [url]

[2] Dark matter indirect detection limits from complete annihilation patterns [url]

[3] Annihilating Dark Matter Search with 12 Years of Fermi LAT Data in Nearby Galaxy Clusters [url]

[4] Fundamental Physics Searches with IACTs [url]

[5] Limits and confidence intervals in the presence of nuisance parameters [W.A. Rolke et al. (2005)]

Thank you

Extra Slides

Applications

- In this publication of 'Dark matter indirect detection limits from complete annihilation patterns' :

$$\mathrm{TS} = \begin{cases} 0 & \text{for } \widehat{\langle \sigma v \rangle} > \langle \sigma v \rangle\,, \\ -2\ln \dfrac{\mathcal{L}(\langle \sigma v \rangle, \hat{\hat{\boldsymbol{N}}}_{\mathrm{B}}(\langle \sigma v \rangle), \hat{\hat{J}}(\langle \sigma v \rangle))}{\mathcal{L}(\widehat{\langle \sigma v \rangle}, \hat{\boldsymbol{N}}_{\mathrm{B}}, \hat{J})} & \text{for } 0 \le \widehat{\langle \sigma v \rangle} \le \langle \sigma v \rangle\,, \\ -2\ln \dfrac{\mathcal{L}(\langle \sigma v \rangle, \hat{\hat{\boldsymbol{N}}}_{\mathrm{B}}(\langle \sigma v \rangle), \hat{\hat{J}}(\langle \sigma v \rangle))}{\mathcal{L}(0, \hat{\hat{\boldsymbol{N}}}_{\mathrm{B}}(0), \hat{\hat{J}}(0))} & \text{for } \widehat{\langle \sigma v \rangle} < 0\,, \end{cases}$$

https://iopscience.iop.org/article/10.1088/1475-7516/2022/11/055/pdf

Formalism

$$\lambda(\mu) = \frac{L(\mu, \hat{\hat{\boldsymbol{\theta}}})}{L(\hat{\mu}, \hat{\boldsymbol{\theta}})}$$

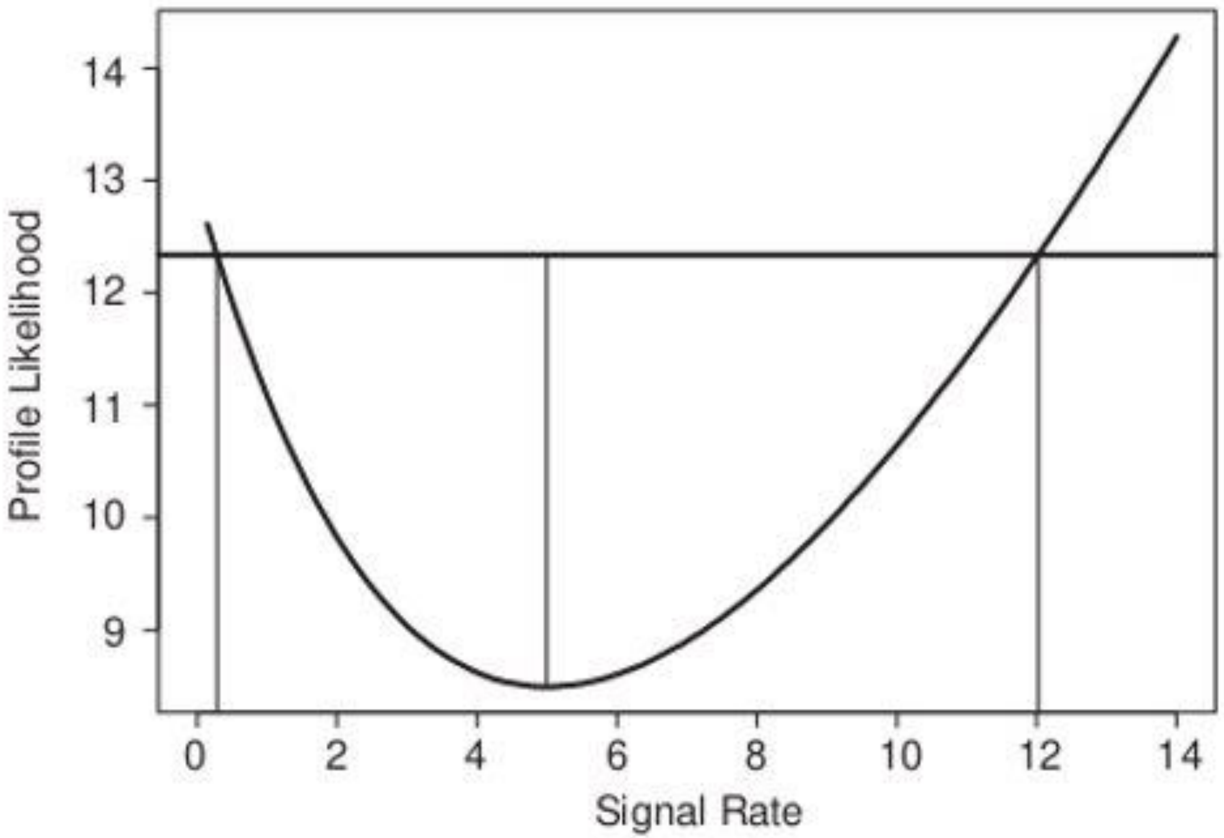
- Here, $\hat{\hat{\boldsymbol{\theta}}}$ denotes the value of θ that maximizes L for the specified μ. (conditional maximum-likelihood (ML) estimator)
- The denominator is the maximized (unconditional) likelihood function.
- We generally define the test statistic as:

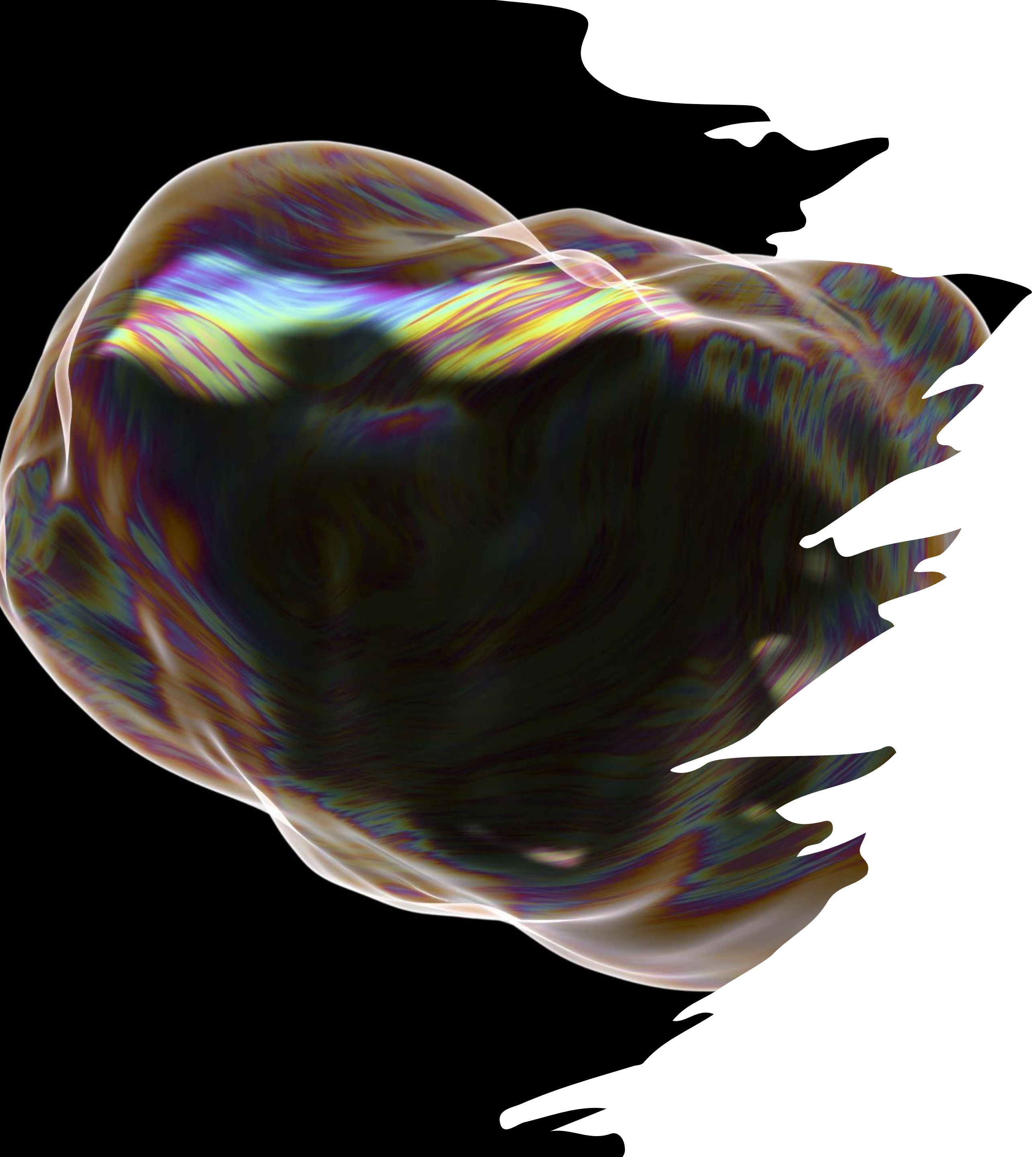
$$t_\mu = -2 \ln \lambda(\mu)$$

As when using q_μ, the upper limit on μ at confidence level $1-\alpha$ is found by setting $p_\mu = \alpha$ and solving for μ, which reduces to the same result as found when using q_μ, namely,

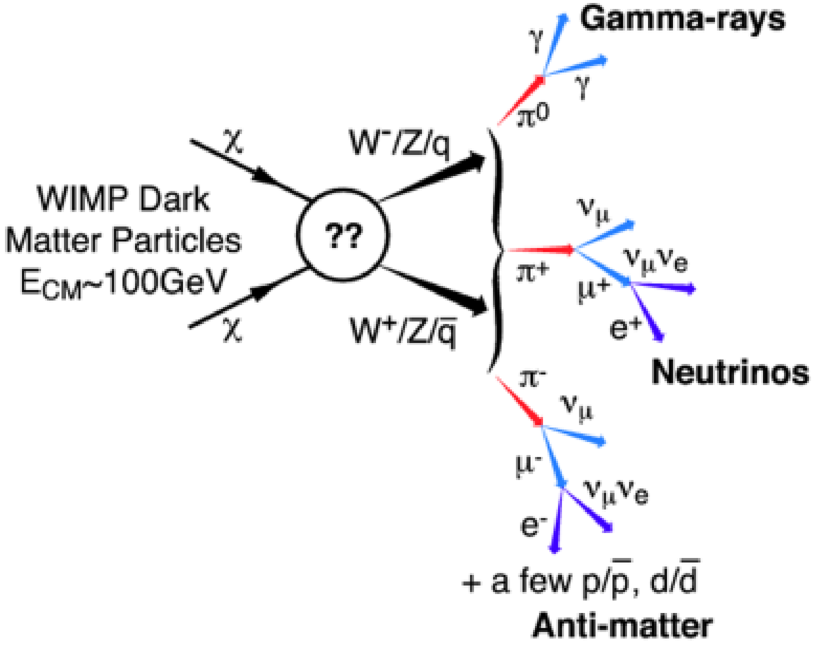
$$\mu_{\rm up} = \hat{\mu} + \sigma \Phi^{-1}(1-\alpha) \; . \qquad (69)$$

That is, to the extent that the Wald approximation holds, the two statistics q_μ and $\tilde{q}_\mu$ lead to identical upper limits.


Profile Likelihood
14
13
12
11
10
9
0
2
4
6
8
10
12
14
Signal Rate



Search for dark matter



Statistical analysis

- A widely used procedure to establish discovery is based on a frequentist significance test using a likelihood ratio as a test statistic.
- We use log-likelihood ratio statistical test to constrain the DM annihilation cross-section setting upper limits.