Derivation of spectral cutoff lower limits in PeVatron searches with CTA

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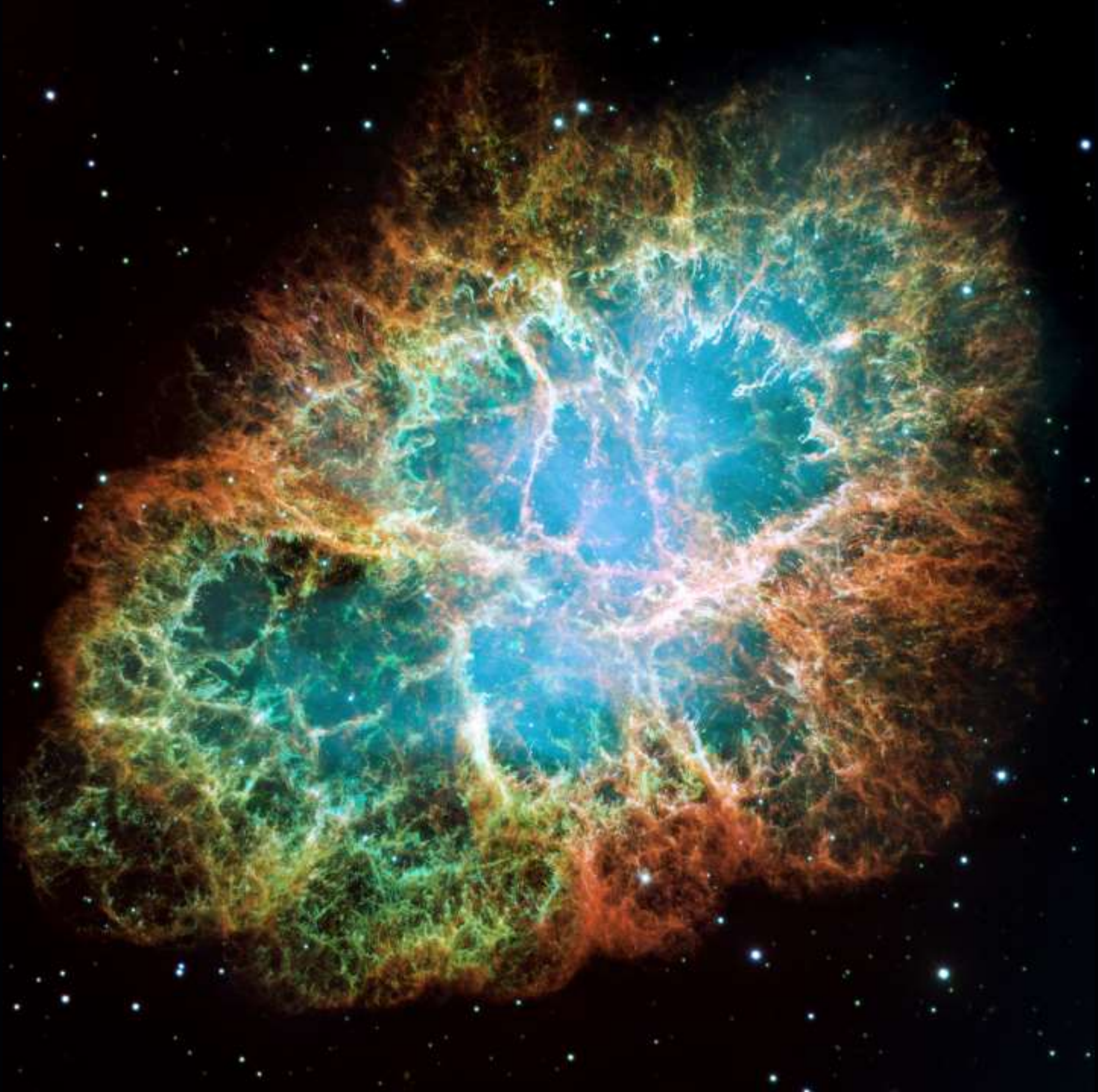
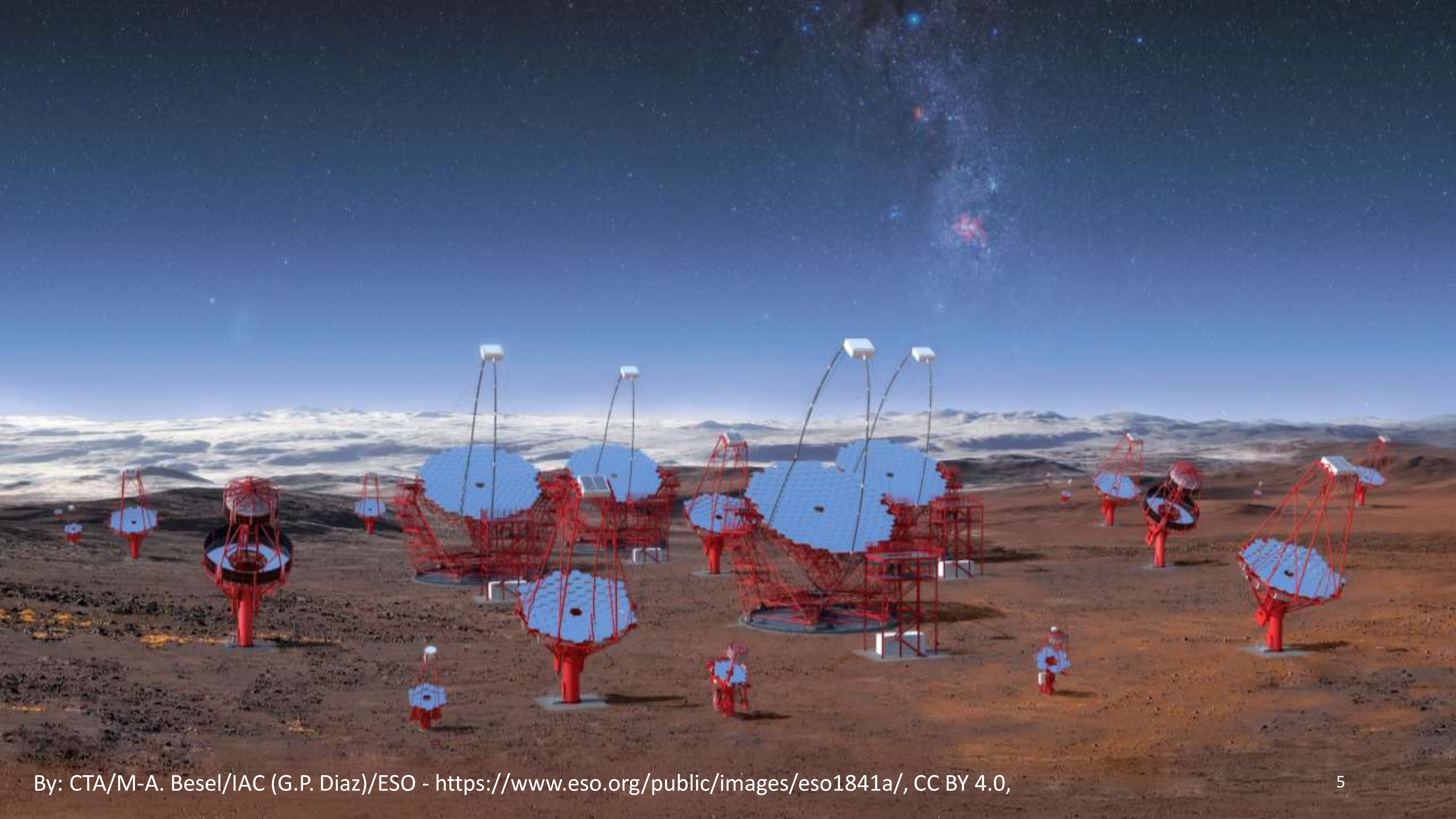
1. Introduction

- PeVatron is a term used to describe astrophysical sources that are able to accelerate particles up to $10^{15} eV$ (1 PeV)
- Several source classes have been proposed as potential PeVatrons, but Supernova Remnants (SNRs) have been the preferred candidates
- Galactic PeVatron have been detected, but none of them are proven to be related to SNRs
- Crab nebula is an example of a leptonic PeVatron (in this work hadronic PeVatrons are being searched for)



By NASA, ESA, J. Hester and A. Loll (Arizona State University)

- CTA is the next generation Imaging Atmosferic Cherenkov Telescope (IACT) system.
- It will be located at Paranal Observatory (Chile) and Roque de los Mucachos Observatory (Spain)→whole sky observations
- Energy range from 20 GeV to 200 TeV
- Improved sensitivity is expected to lead to discovery of many more astrophysical sources.



By: CTA/M-A. Besel/IAC (G.P. Diaz)/ESO - https://www.eso.org/public/images/eso1841a/, CC BY 4.0,

2. Simulation and analysis of CTA data

- The simulation and analysis of CTA dana are based on the instrument response functions (IRFs)
- The morphology of extended γ-ray sources is modelled using 2D symmetric Gaussians, and source extensions are given as the width (σ) of the Gaussian.
- Simulated CTA event data are drawn from Poisson distributed random variables around their bin-wise expectation.
- A binned 3D-likelihood analysis is performed in the framework of gammapy.
- The population of Galactic SNRs is simulated with a Monte Carlo approach, in which the distribution of SNe in time and space is randomly drawn in multiple samples.

3. Derivation of spectral cutoff lower limits

- PeVatron searches with CTA rely on the derivation of statistical statements on the inverse energy cutoff parameter λ.
- When a significant cutoff detection is impossible, frequentist upper limits λ^{UL} on the inverse cutoff parameter at a given confidence level CL are of high relevance.
- Limits on the inverse spectral cutoff λ are investigated within γ-ray emission models.

The Poisson likelihood:

$$L(\lambda, \theta|\vec{c}) \coloneqq \prod_{i=1}^{N} \exp(-n_i)\,\frac{{n_i}^{c_i}}{c_i!}$$

- $\boldsymbol{\theta}$ – nuisance parameters
- λ – the inverse energy cutoff
- $\vec{c} = (c_1, \dots, c_N)$ – simulated event counts
- $\vec{n} = \vec{n}(\lambda, \boldsymbol{\theta})$ – predicted counts

Cash statistic:

$$C(\lambda, \theta) = 2\sum_{i}(n_i - c_i \ln n_i)$$

3.1. Profile likelihood

- This method is an example for the inversion of a frequentist hypothesis test

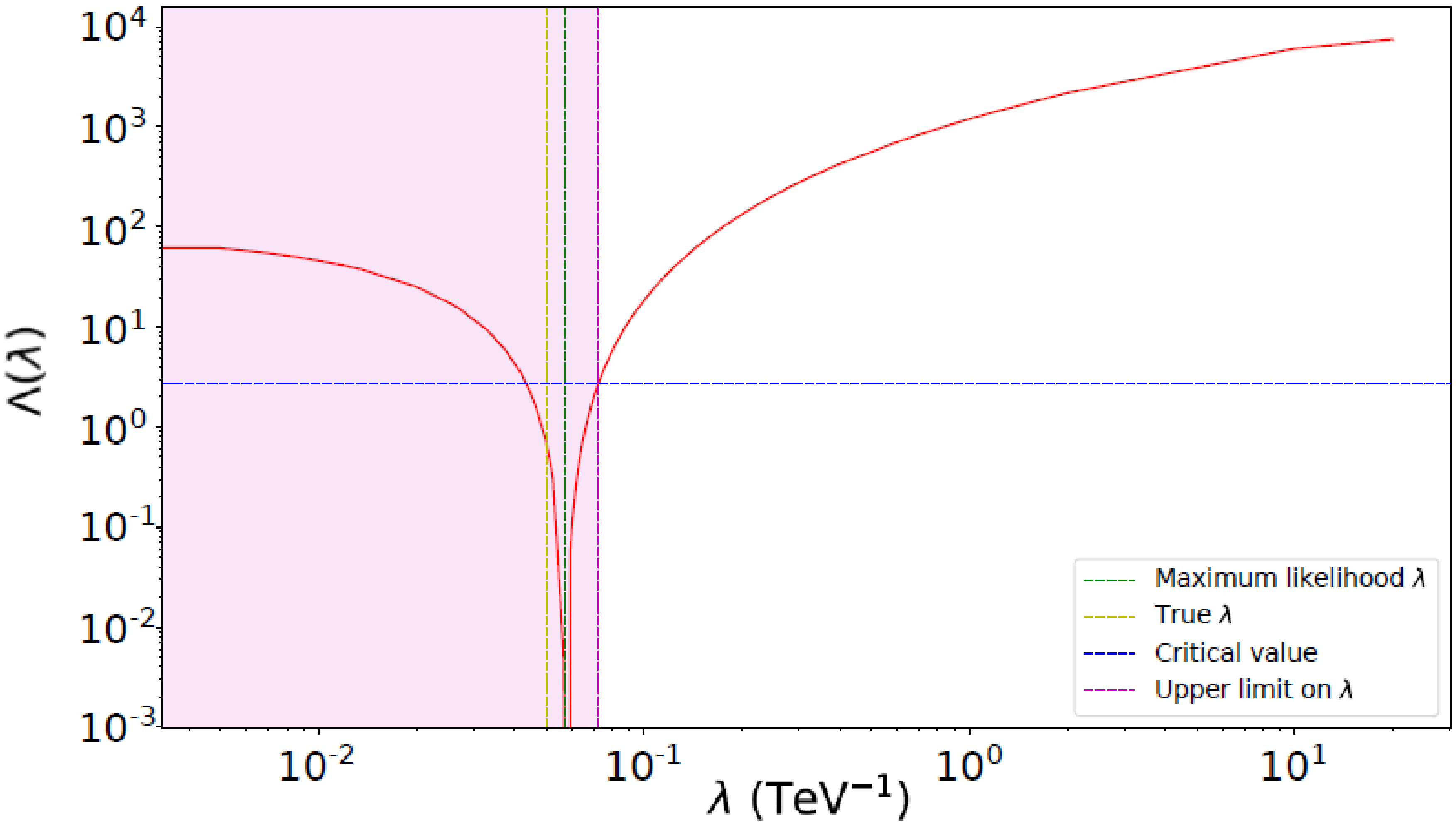
- Let $L(\lambda) = max_\theta L(\lambda, \theta|\vec{c})$ be profile likelihood, and $\mathrm{C}(\lambda) = max_\theta \mathrm{C}(\lambda, \theta)$ corresponding Cash statistic

The likelihood ratio test statistic:

$$\Lambda(\lambda) \coloneqq -2\ln\frac{L(\lambda)}{L(\hat{\lambda})} = C(\lambda) - C(\hat{\lambda})$$

- $\hat{\lambda}$ - maximum likelihood estimator for the inverse energy cutoff over the constrained range $\lambda \geq 0$

Likelihood ratio statistic for the analysis of a typical γ-ray source:



- The likelihood ratio test statistic equation is the test statistic of the null hypothesis (H_0: $\lambda = \hat{\lambda}$) against the alternative hypothesis (H_1: $\lambda = \lambda$)
- The alternative hypothesis is accepted when the test statistic is smaller than or equal to the critical value at a given confidence level CL (the horizontal line in the previous slide)

3.2. Markov-Chain Monte Carlo (MCMC)

- Upper limit of the inverse cutoff parameter can be derived when the probability distribution of model parameters is expressed in the framework of Bayesian terminology.

Posterior probability density

$$p(\lambda, \theta|\vec{c}) = \frac{L(\lambda, \theta|\vec{c})\, p(\lambda, \theta)}{p(\vec{c})}$$

- $p(\lambda, \theta)$ – probability density for the model parameters
- $p(\vec{c}) \coloneqq \int d\lambda d\theta\, L(\lambda, \theta|\vec{c})\, p(\lambda, \theta)$

3.3. Bootstrap

- This method resamples binned γ-ray events ($\vec{c}$) as bootstrap samples ($\vec{c}_*$)
- The percentile method is used to get the smallest positive upper limit on the inverse cutoff parameter (λ^{UL}) which satisfies:

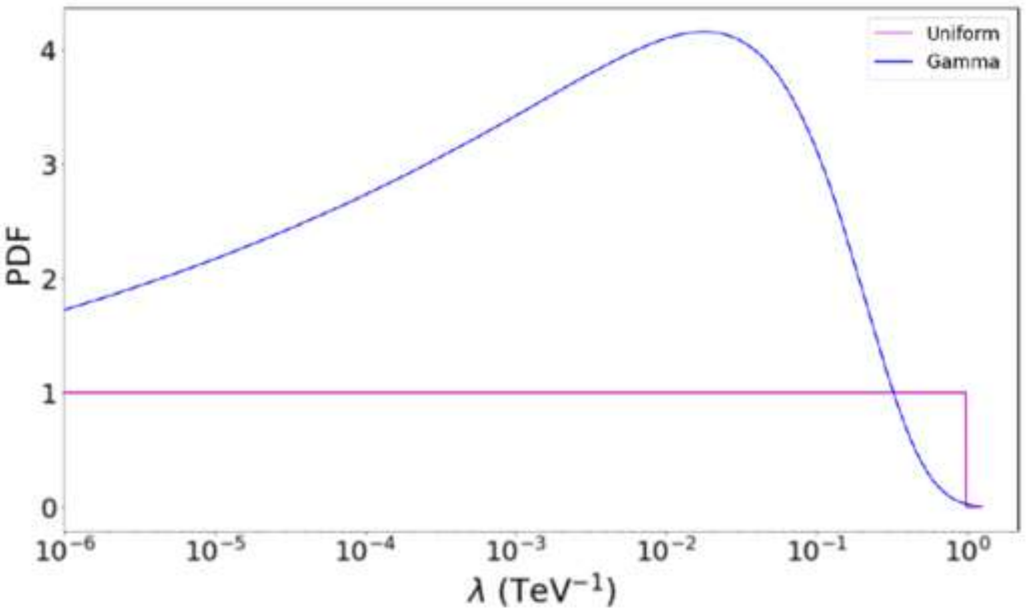
$$CL \le \int_{-\infty}^{\lambda^{UL}} d\lambda_* f(\lambda_*)$$

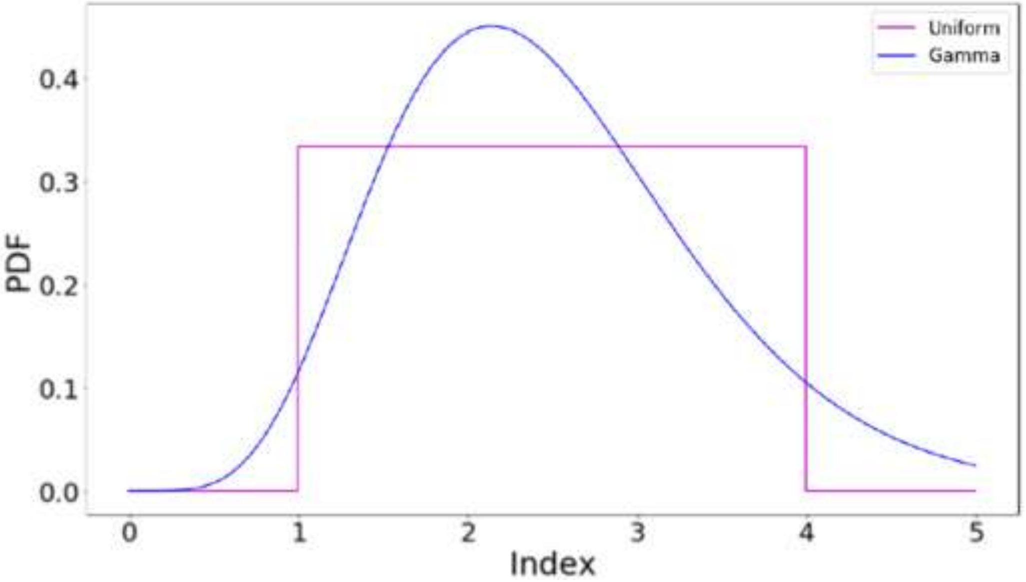
- Non-parametric bootstrap
- Parametric bootstraps:
  - Poisson bootstrap
  - Best fit bootstrap
- Difference between the parametric and non-parametric bootstrap is that the total number of events is a random variable

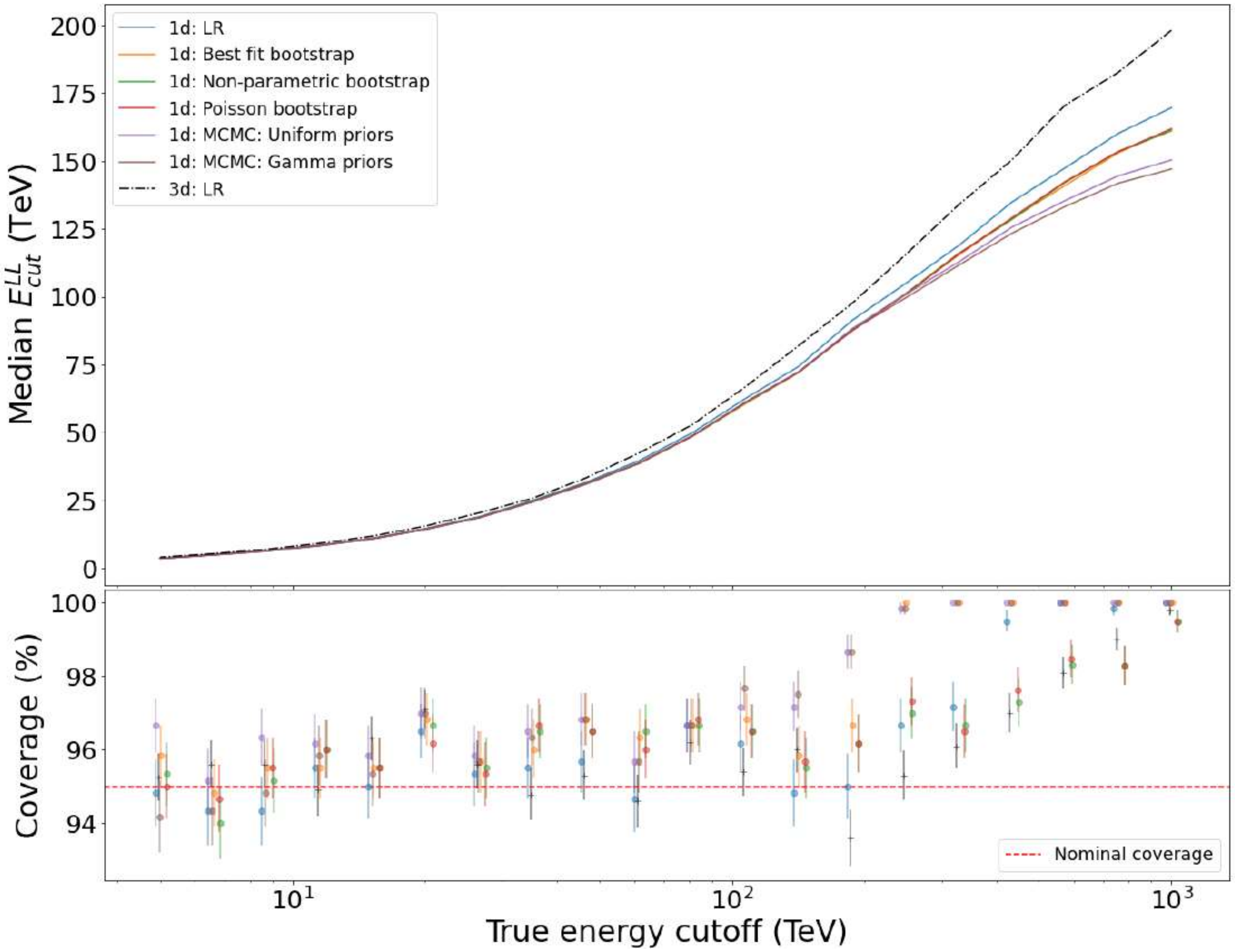
3.4. Performance comparison

Point-like source analysis

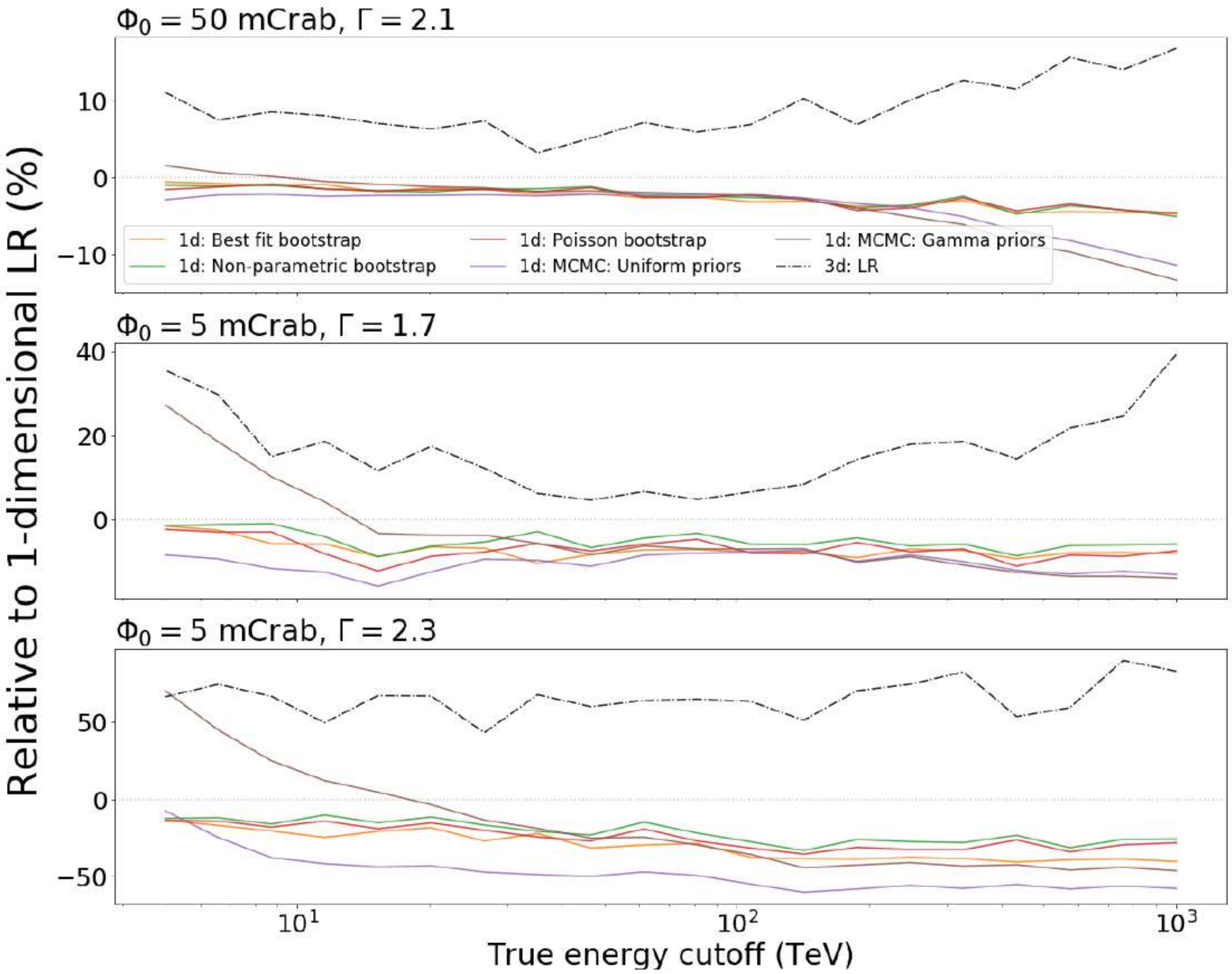
- The energy cutoff limits obtained with the bootstrap and MCMC methods are calculated with a precision better than 2%.
- Two different sets of prior density distributions for the model parameters are investigated for the MCMC method.
- The uniformity of prior density depends on the choice of the parameter.
- Results based on uniform prior densities are compared to results obtained with priors based on gamma distributed random variables.







A comparison of the coverage and sensitivity for the specific γ-ray point-like source



Point-like source energy cutoff sensitivity as a function of the true energy cutoff for different methods relative to the respective sensitivity achievable with a 1-dimensional profile likelihood analysis.

The different panels show the relative sensitivity for different point-like source parameters in terms of flux normalization ϕ_0 and index Γ.

Conclusions

- The profile likelihood method provides a computationally very efficient way to derive lower limits on the energy cutoff.
- Other methods are less sensitive or a possible sensitivity improvement in restricted parameter ranges results from the choice of the prior distributions(MCMC).
- The computational effort to derive reasonably precise limits is larger for bootstrap and MCMC implementations than for the profile likelihood method.
- Bootstrap and MCMC methods can provide an important alternative in cases where the profile likelihood method cannot be applied.

Bibliography

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Thank you for the attention