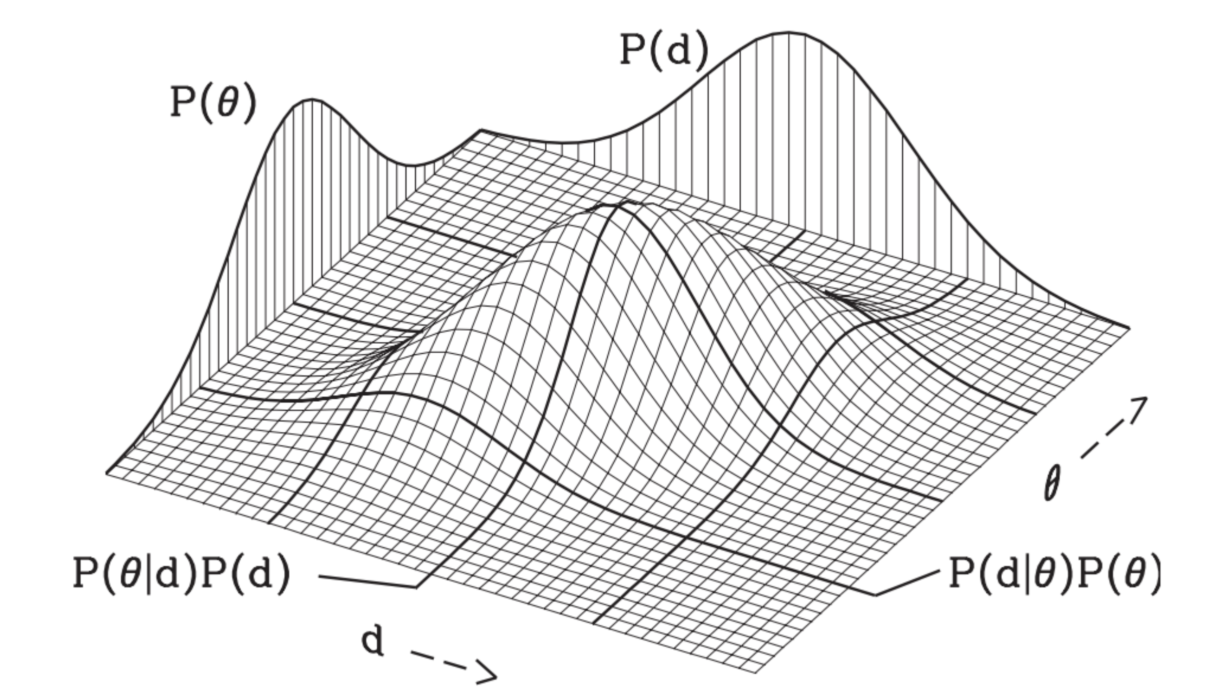


BAYESIAN INFERENCE OF PLASMA PROPERTIES FROM FAST-DIVING TRIPLE LANGMUIR PROBE MEASUREMENTS

Matteo Da Valle

Hall Thusters

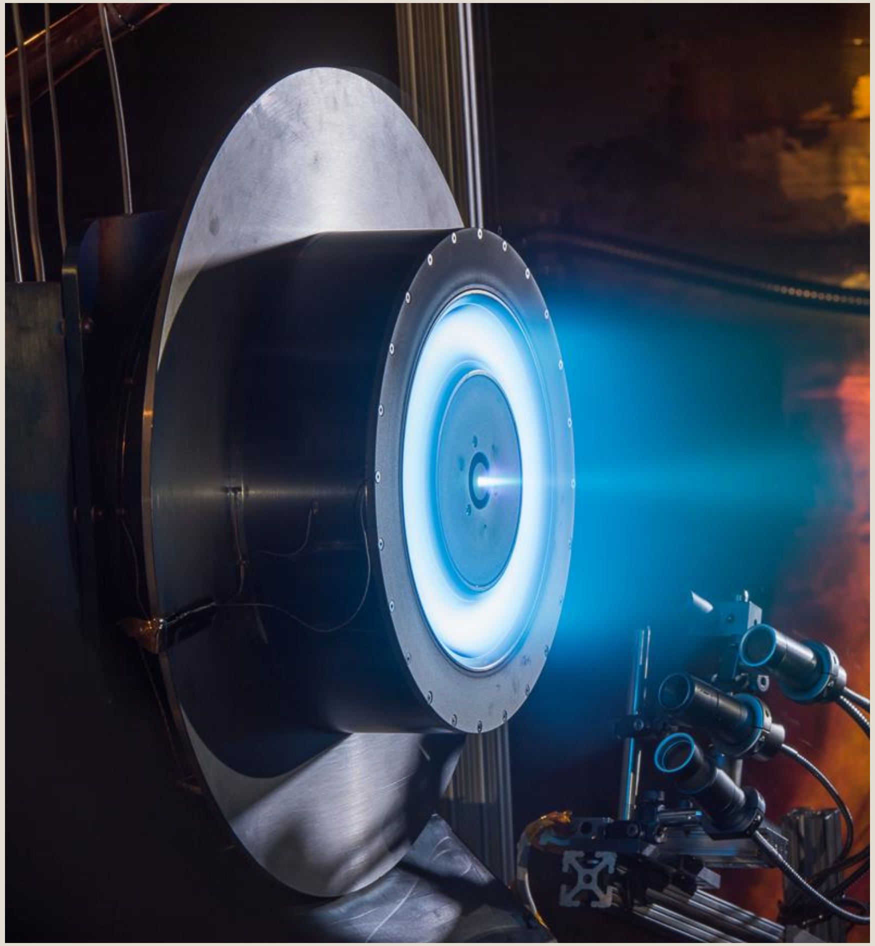
PROS

1. Compact (high Thrust to Area ratio)
2. Low propellant consumption (high I_{sp})
3. High Efficiency
4. Versatile

CONS

1. Strong (and expensive) experimental campaign

(Double) Langmuir Probe

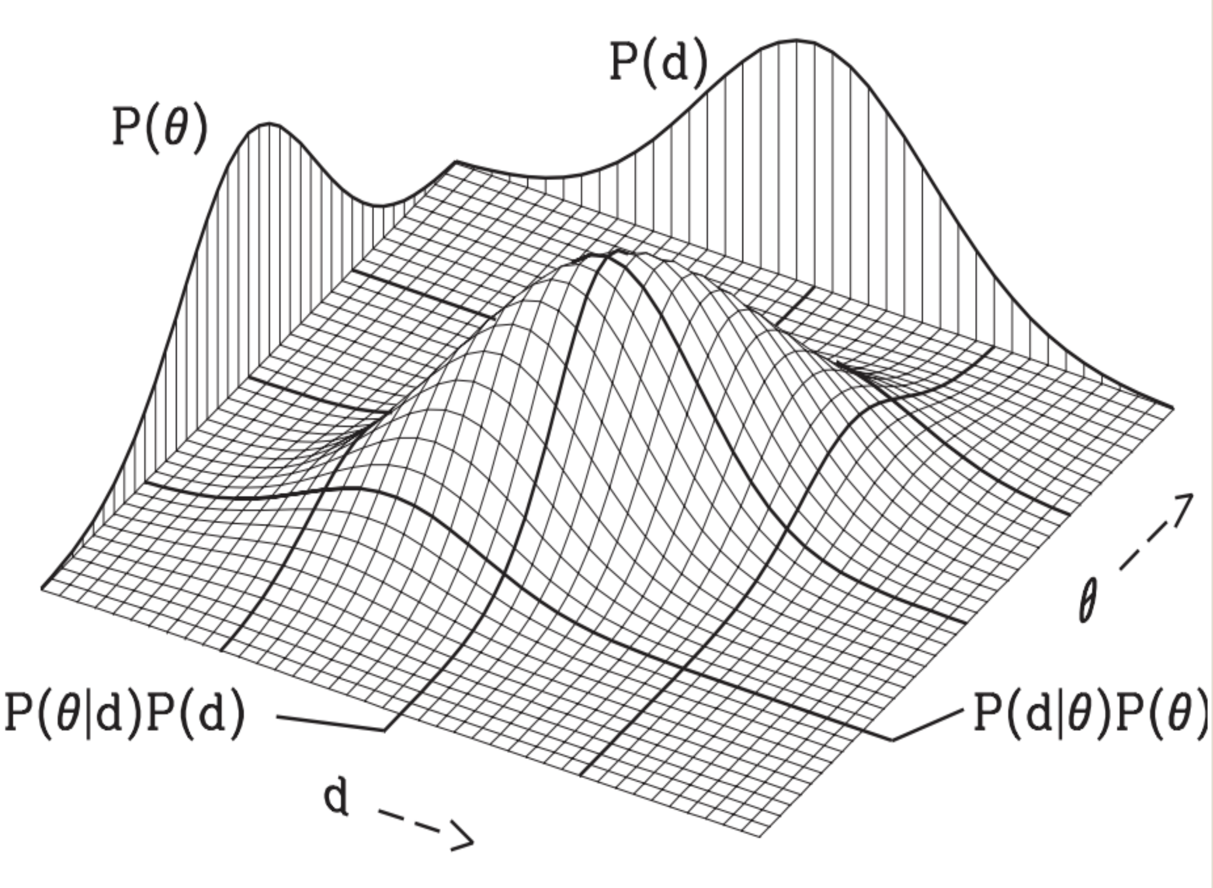


Bayesian Statistics

Likelihood

+

Expert Knowledge



Bayesian Statistics

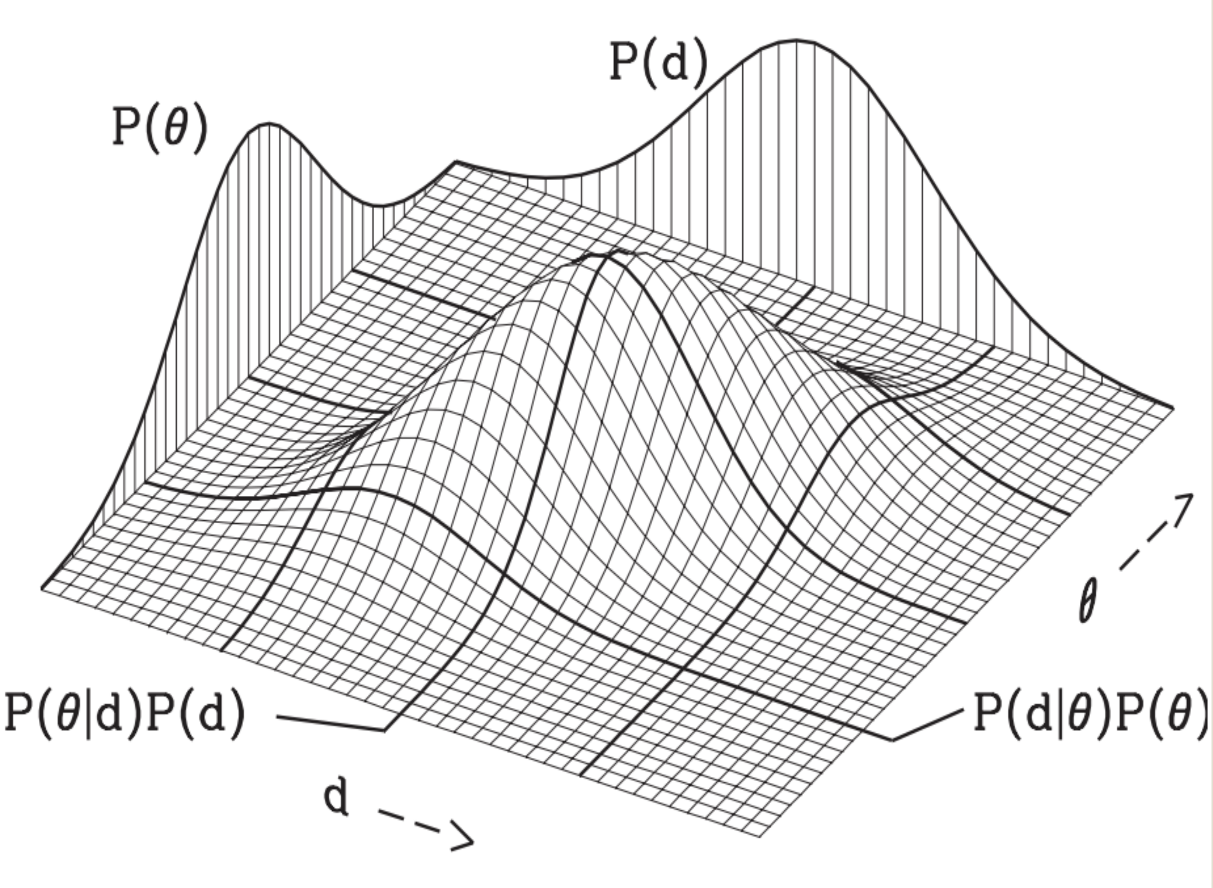
Product Rule

Prior x Likelihood = Evidence x Posterior

$$p(D,\theta|I) = p(\theta|I)p(D|\theta,I) = p(D|I)p(\theta|D,I)$$

Marginalization Rule

$$p(\theta_1|D,I) = \int_{-\infty}^{+\infty} p(\theta_1,\theta_2|D,I)d\theta_2$$



Bayesian Statistics - Example

Data distribution $$p(D|\theta,\sigma,I) = \left(\frac{1}{2\pi\sigma^2}\right)^{N/2} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(d_i-\theta)^2\right)$$

Assumed Prior $$p(\theta|I) = \left(\frac{1}{2\pi S^2}\right)^{1/2} \exp\left(-\frac{1}{2S^2}(\theta-\theta_0)^2\right)$$

Posterior $$p(\theta|D,I) = \frac{p(\theta|I)p(D|\theta,I)}{p(D|I)} \rightarrow \begin{cases} \langle\theta\rangle = \dfrac{N\bar{d}/\sigma^2 + \theta_0/S^2}{N/\sigma^2 + \theta_0/S^2} \\ \langle\Delta\theta^2\rangle = \left(\dfrac{1}{1+\left(\frac{\sigma^2}{NS^2}\right)}\right)\dfrac{\sigma^2}{N} \end{cases}$$

Bayesian Statistics

Multi-dimensional numerical integration

$$\langle \psi(\boldsymbol{\theta}) \rangle = \frac{1}{Z} \int \psi(\boldsymbol{\theta}) p(\boldsymbol{\theta}|I) p(\boldsymbol{d}|\boldsymbol{x}, \boldsymbol{\theta}, I) d\boldsymbol{\theta}$$

Evidence

Bayesian Statistics

Multi-dimensional numerical integration

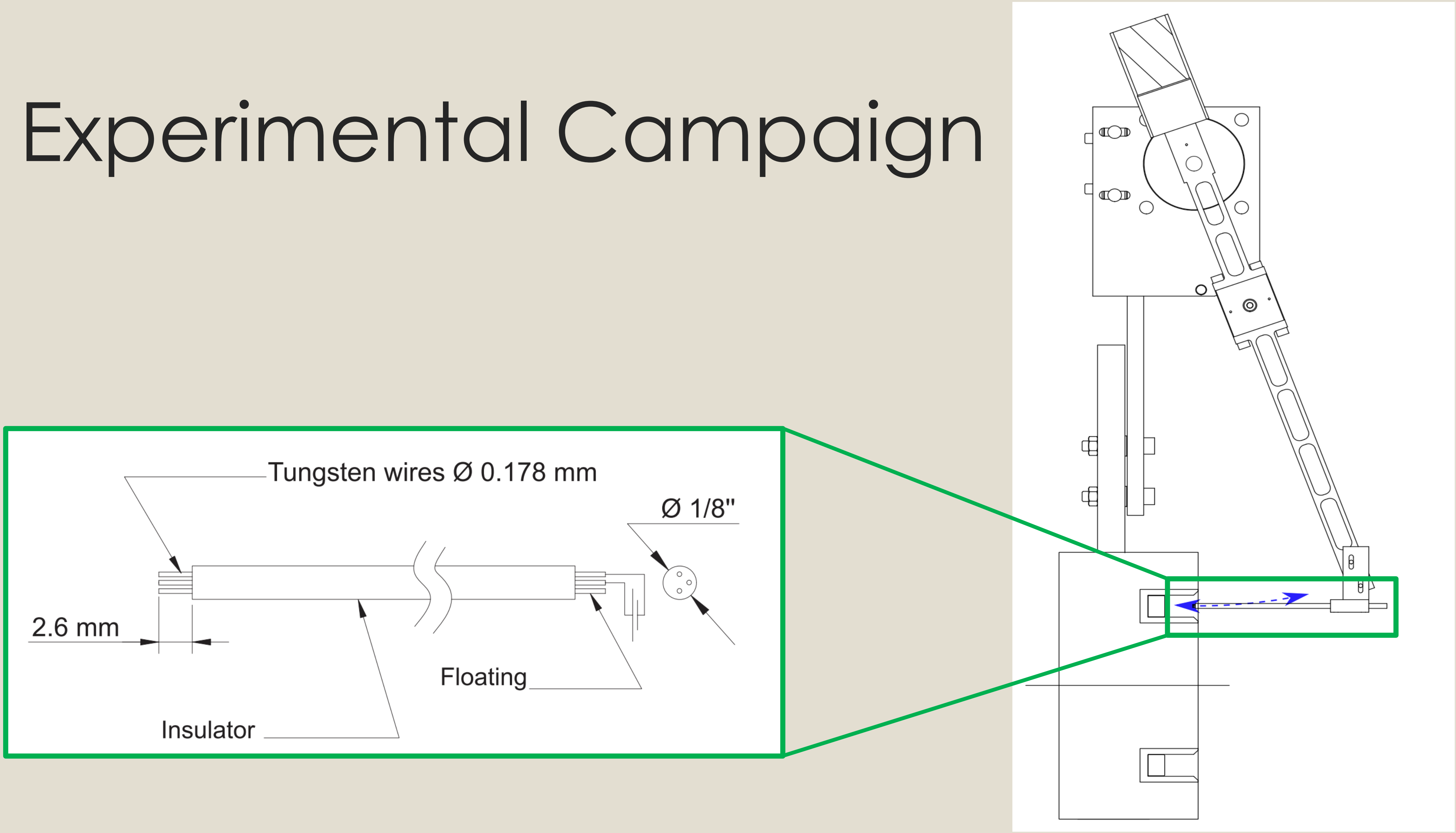
$$\langle \psi(\boldsymbol{\theta}) \rangle = \frac{1}{Z} \int \psi(\boldsymbol{\theta}) p(\boldsymbol{\theta}|I) p(\boldsymbol{d}|\boldsymbol{x}, \boldsymbol{\theta}, I) d\boldsymbol{\theta}$$

Evidence

Markov Chain Monte Carlo

(MCMC) techniques

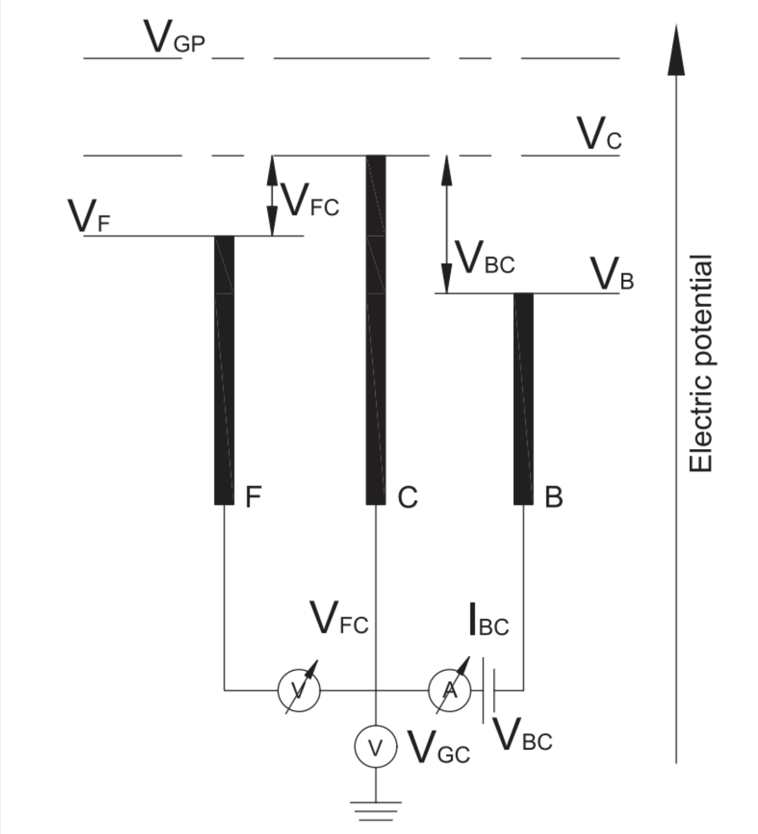
Experimental Campaign



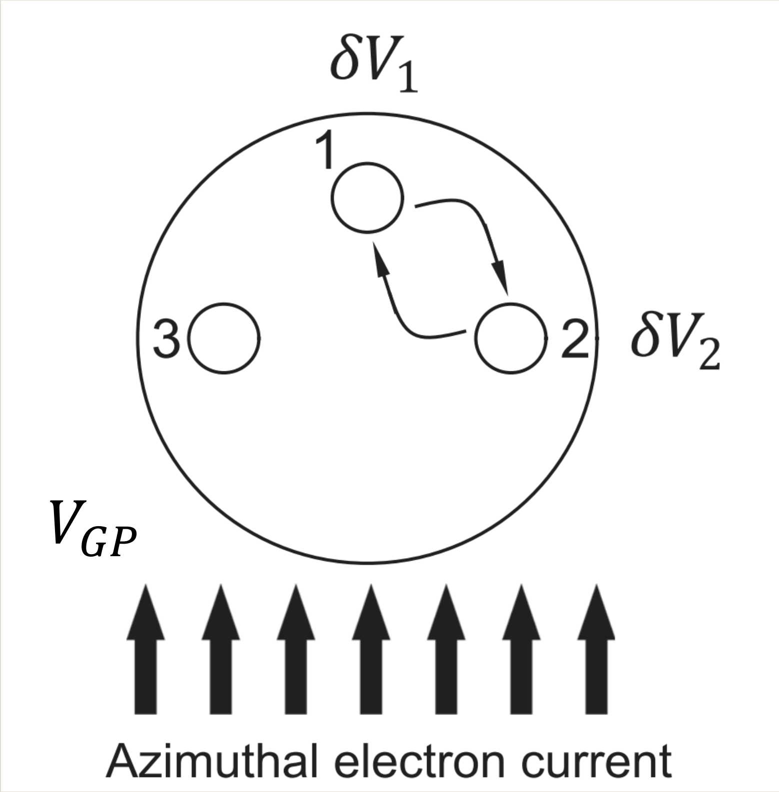
Experimental Campaign

Three Bias Voltages:

- V = 0 V
- V = 15 V
- V = 30 V



Experimental Campaign



Two probe configurations:

Probe 1 = F
Probe 2 = C
Probe 3 = B

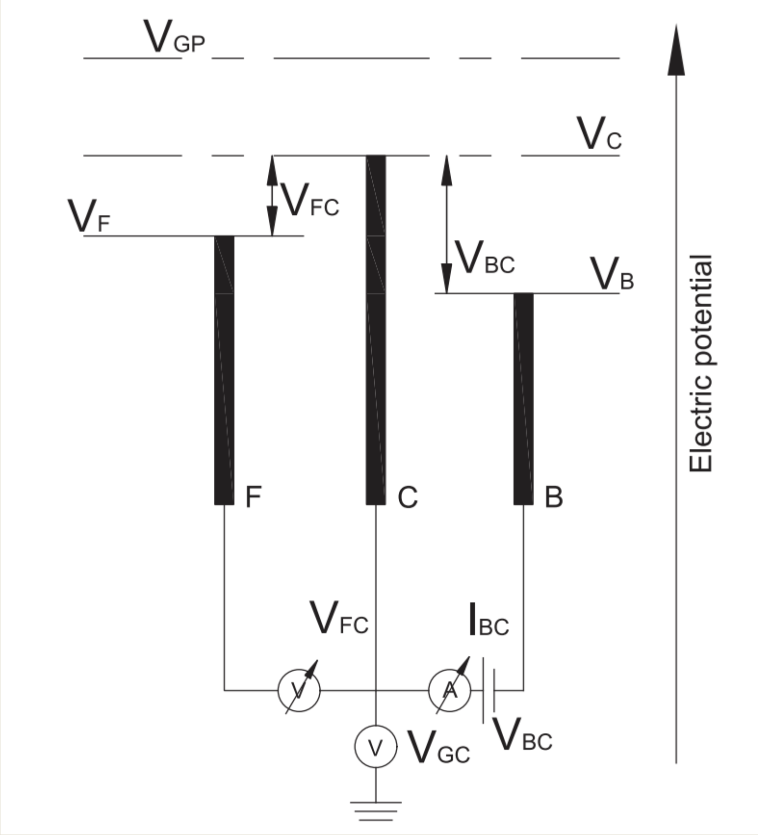
Probe 1 = C
Probe 2 = F
Probe 3 = B

Three Bias Voltages:

V = 0 V

V = 15 V

V = 30 V



Langmuir Probe Model

Laframboise solution $I(V_{PR}, T_e, n, V_{GP}) = Aen\sqrt{\frac{T_e}{2\pi m_e}}\left[\exp\left(e\frac{V_{PR}-V_{GP}}{T_e}\right) - a\sqrt{\frac{m_e}{m_i}}\left(b - e\frac{V_{PR}-V_{GP}}{T_e}\right)^c\right]$

Langmuir Probe Model

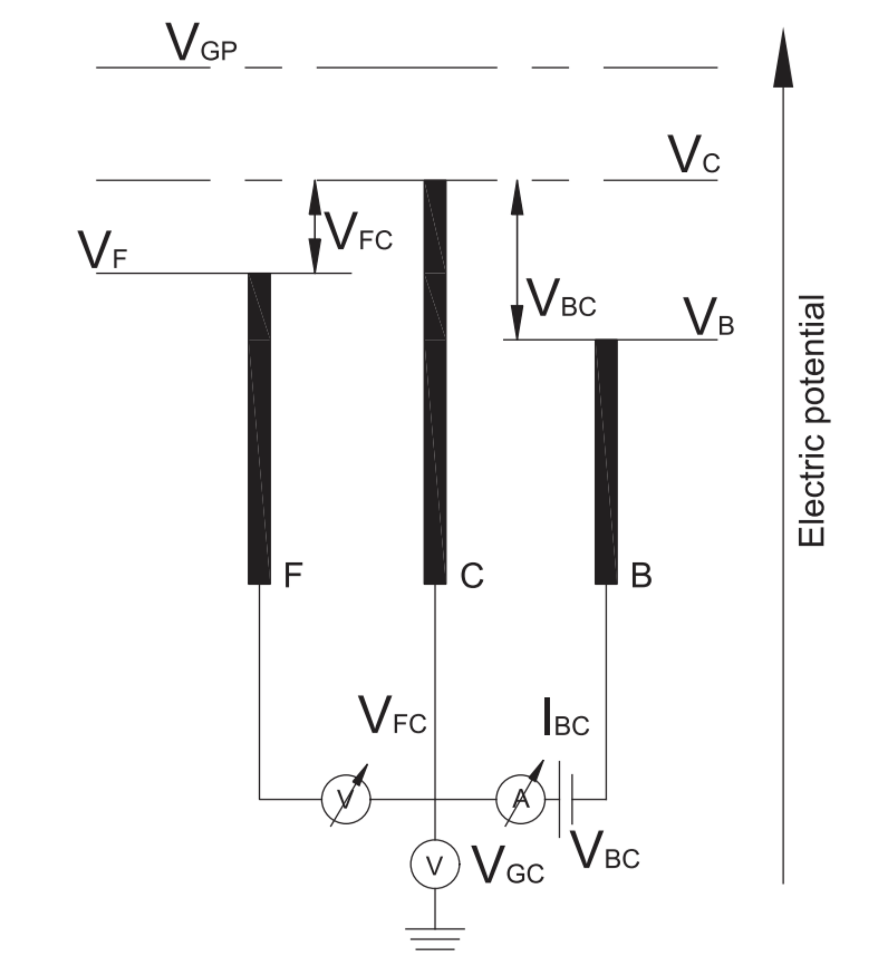
Laframboise solution $I(V_{PR}, T_e, n, V_{GP})$

↓

$$I_B = I(V_{GC} - V_{BC}, T_e, n, V_{GP}) = -I_{BC}$$

$$I_C = I\left(V_{GC}, T_e, n, V_{GP} + \delta V_j\right) = I_{BC}$$

$$I_F = I(V_{GC} - V_{FC}, T_e, n, V_{GP} + \delta V_k) = 0$$



Langmuir Probe Model

Laframboise solution $I(V_{PR}, T_e, n, V_{GP})$

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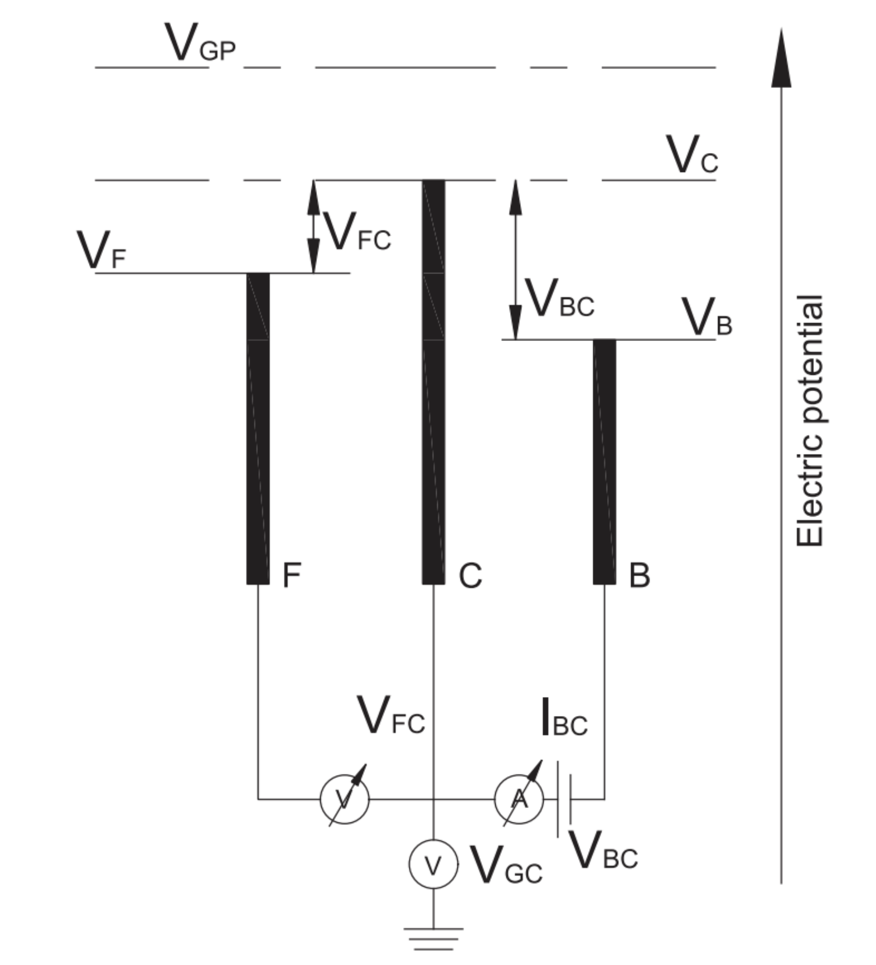
$$I_B = I(V_{GC} - V_{BC}, T_e, n, V_{GP}) = -I_{BC}$$

$$I_C = I\left(V_{GC}, T_e, n, V_{GP} + \delta V_j\right) = I_{BC}$$

$$I_F = I(V_{GC} - V_{FC}, T_e, n, V_{GP} + \delta V_k) = 0$$

$3\ data$

$5\ unknowns$



Data Analysis – Priors

Uniform probabilities

$$0.01\ eV < T_e < 100\ eV$$

$$10^{15}\ m^{-3} < n_e < 10^{20}\ m^{-3}$$

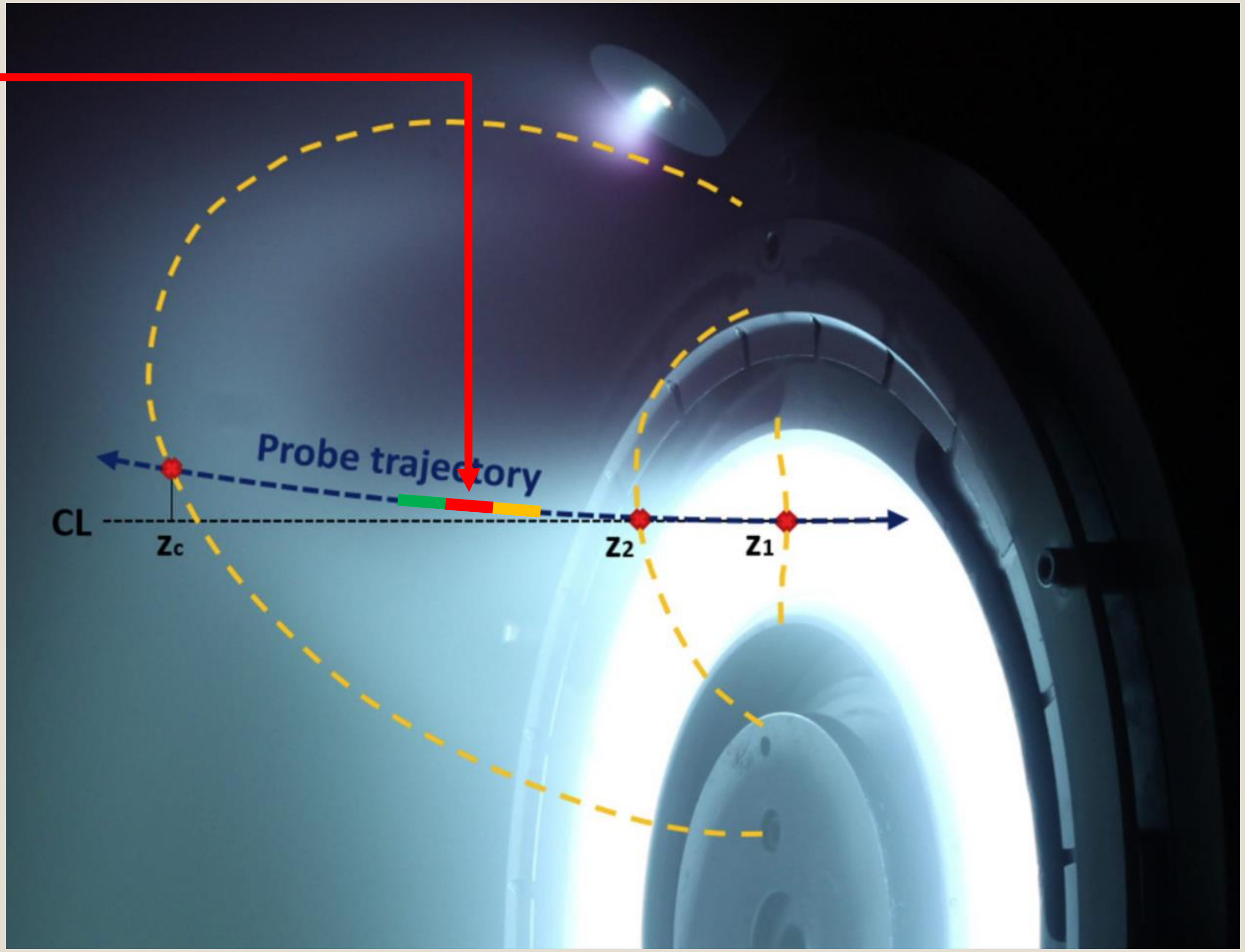
$$\mathbf{-25\ V < V_{GP} < 330\ V}$$

LIF measurements

Previous Probes measurements

Data Analysis – Data Management

Spatial bins of 2.6 mm



Multivariate Gaussian Likelihood

$$p(\boldsymbol{D_i}|\boldsymbol{\theta}, I) = \frac{\exp\left(-\frac{1}{2}(\boldsymbol{D_i} - \boldsymbol{Y})^T \boldsymbol{\Sigma_i}^{-1}(\boldsymbol{D_i} - \boldsymbol{Y})\right)}{\sqrt{(2\pi)^3 |\boldsymbol{\Sigma_i}|}}$$

$$\boldsymbol{Y} = \begin{bmatrix} V_{GC} \\ V_{FC} \\ I_{BC} \end{bmatrix} = \boldsymbol{f}\left(\begin{bmatrix} T_e \\ n \\ V_{GP} \\ \delta V_1 \\ \delta V_2 \end{bmatrix}\right) = \boldsymbol{f}(\boldsymbol{\theta})$$

Data Analysis - Procedure

Dataset V = 0 V

Computation of

$p(\boldsymbol{\theta}|\{\boldsymbol{D}\}_{V0}, I)$

$p(\delta V_1|\{\boldsymbol{D}\}_{V0}, I)$

$p(\delta V_2|\{\boldsymbol{D}\}_{V0}, I)$

Data Analysis - Procedure

Dataset V = 0 V

Computation of

$p(\boldsymbol{\theta}|\{\boldsymbol{D}\}_{V0}, I)$

$p(\delta V_1|\{\boldsymbol{D}\}_{V0}, I)$

$p(\delta V_2|\{\boldsymbol{D}\}_{V0}, I)$

New priors:

$\mathrm{p}(\delta V_1|I) = p(\delta V_1|\{\boldsymbol{D}\}_{V0}, I)$

$\mathrm{p}(\delta V_2|I) = p(\delta V_2|\{\boldsymbol{D}\}_{V0}, I)$

Dataset V = 15 V / 30 V

Computation of

$p\left(\boldsymbol{\theta}\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$

$p\left(T_e\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$

$p\left(n\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$

$p\left(V_{GP}\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$

Data Analysis – Smoothing

Smoothing prior:

$$\log\left(\mathrm{p}(\theta_z|I)\right) = -\frac{1}{2\sigma^2_{\frac{d^2\theta}{dz^2}}}\sum_{i=2}^{N_z-1}\left[\frac{\frac{\theta_{i+1}-\theta_i}{z_{i+1}-z_i}-\frac{\theta_i-\theta_{i-1}}{z_i-z_{i-1}}}{z_{i+1}-z_{i-1}}\right]^2$$

$$p\left(T_e\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$$

$$p\left(n\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$$

$$p\left(V_{GP}\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$$

Dataset V = 15 V / 30 V

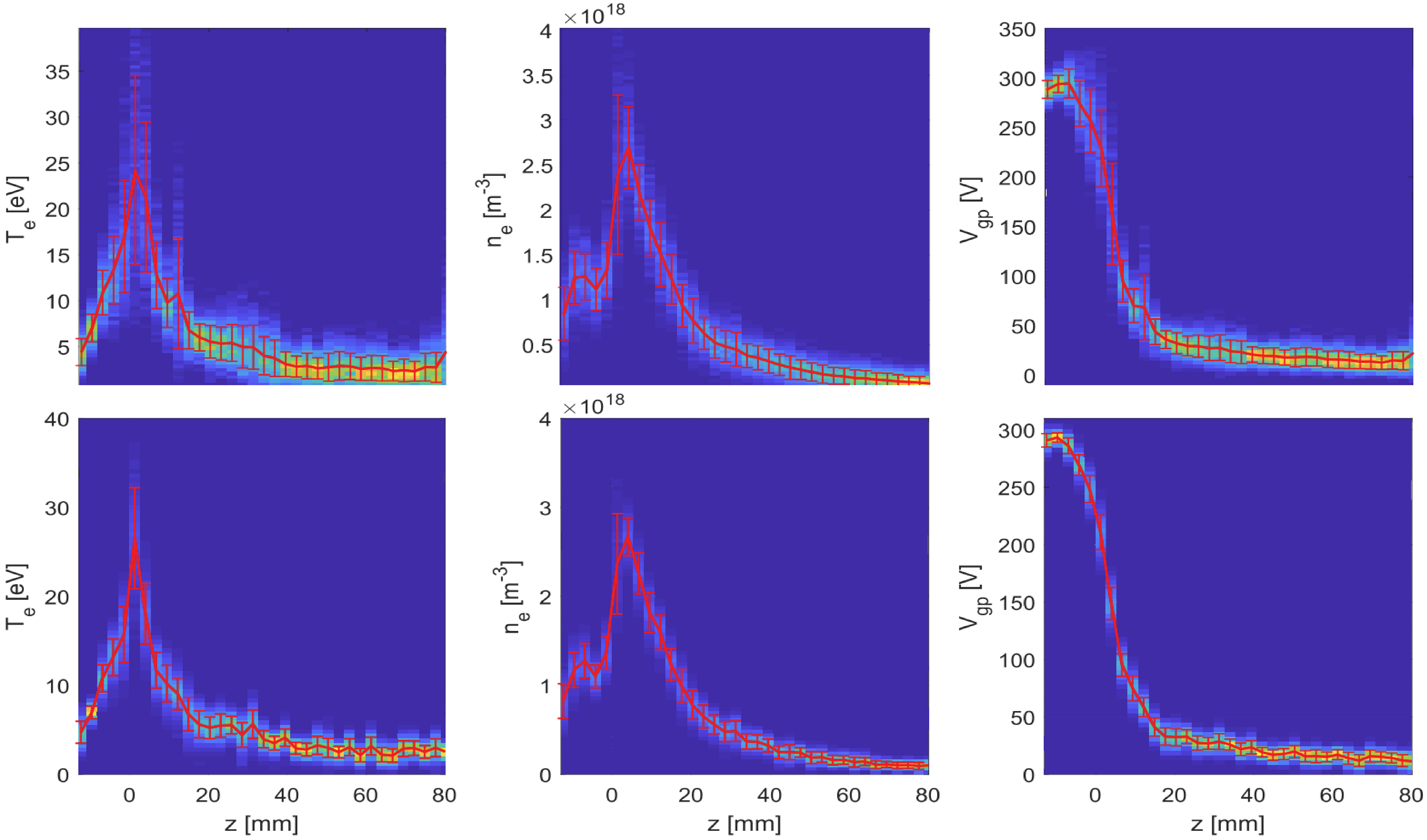
Computation of

$$p\left(\boldsymbol{\theta}\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$$

$$p\left(T_e\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$$

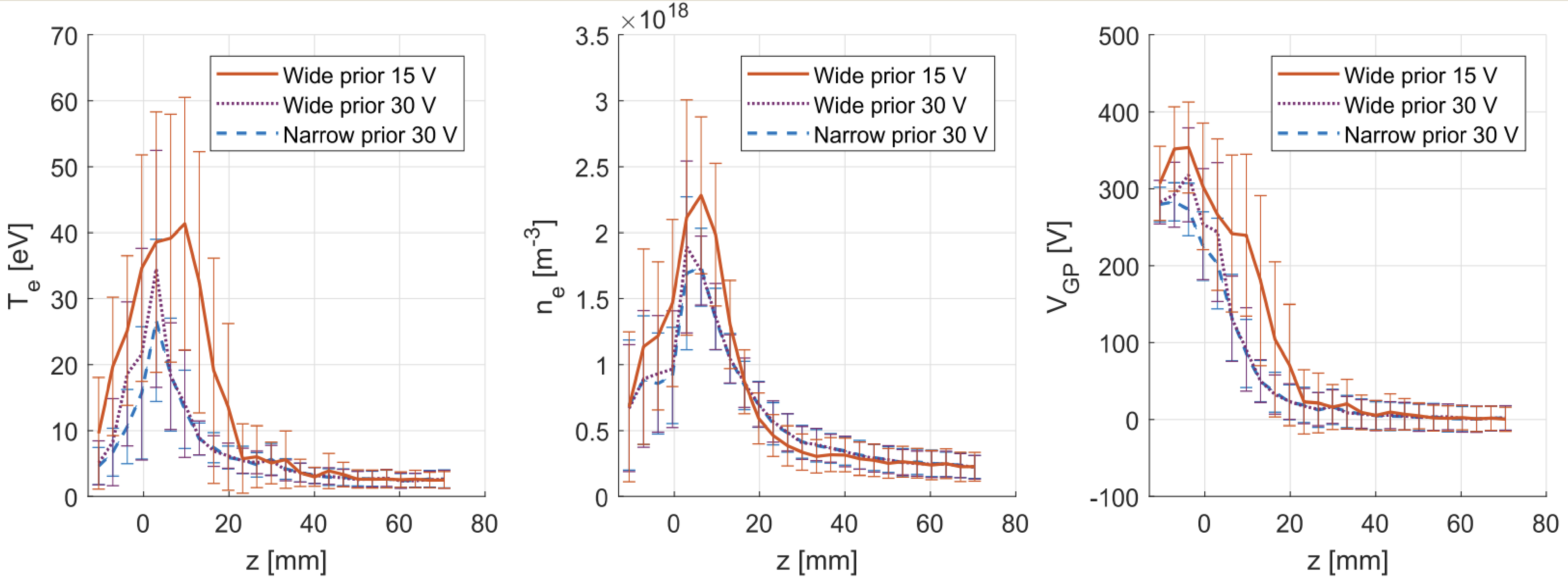
$$p\left(n\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$$

$$p\left(V_{GP}\middle|\{\boldsymbol{D}\}_{V15/V30}, I\right)$$

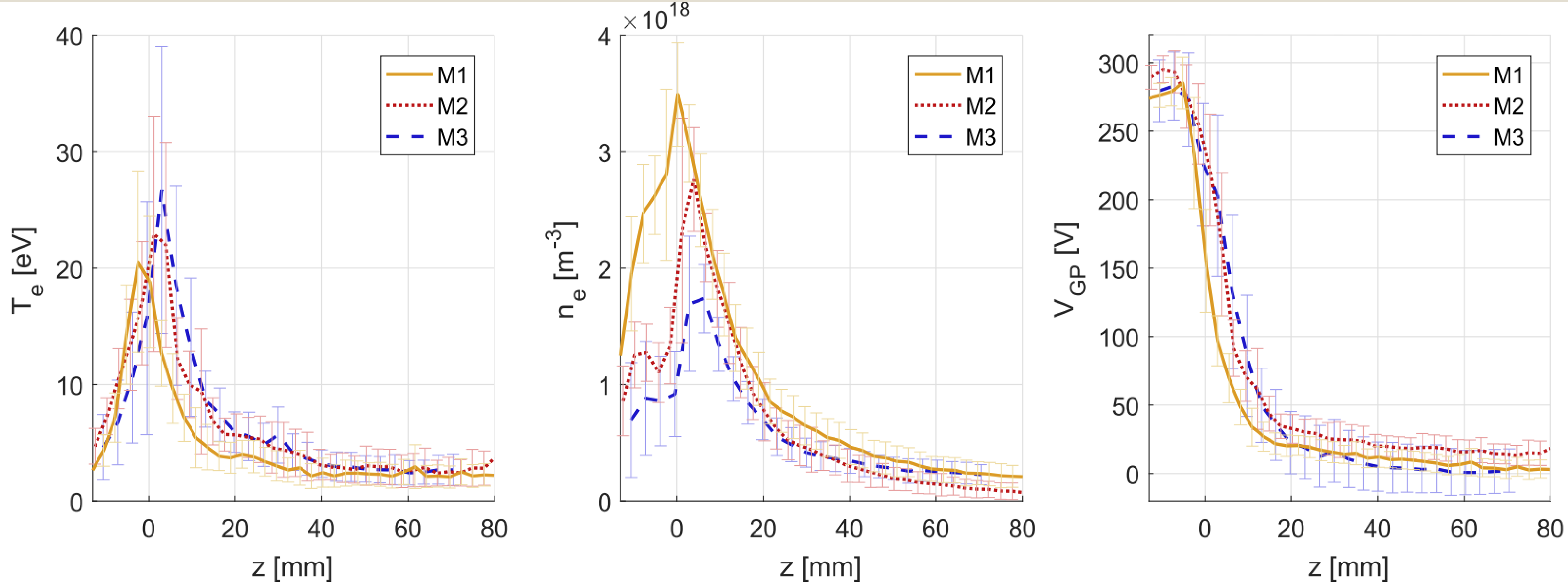


T_e [eV]
$\times10^{18}$
n_e [m^{-3}]
V_{gp} [V]
T_e [eV]
$\times10^{18}$
n_e [m^{-3}]
V_{gp} [V]
z [mm]
z [mm]
z [mm]

Main Results – Different Priors



Main Results – Different B Config



Conclusions

- Robust inference method for plasma parameters in most of the thruster channel
- A clever identification of the priors is more important in the high T_e area, and this dependance may be reduced with higher V_{BC}
- Possibility of investigation of the effect of magnetic shielding on plasma

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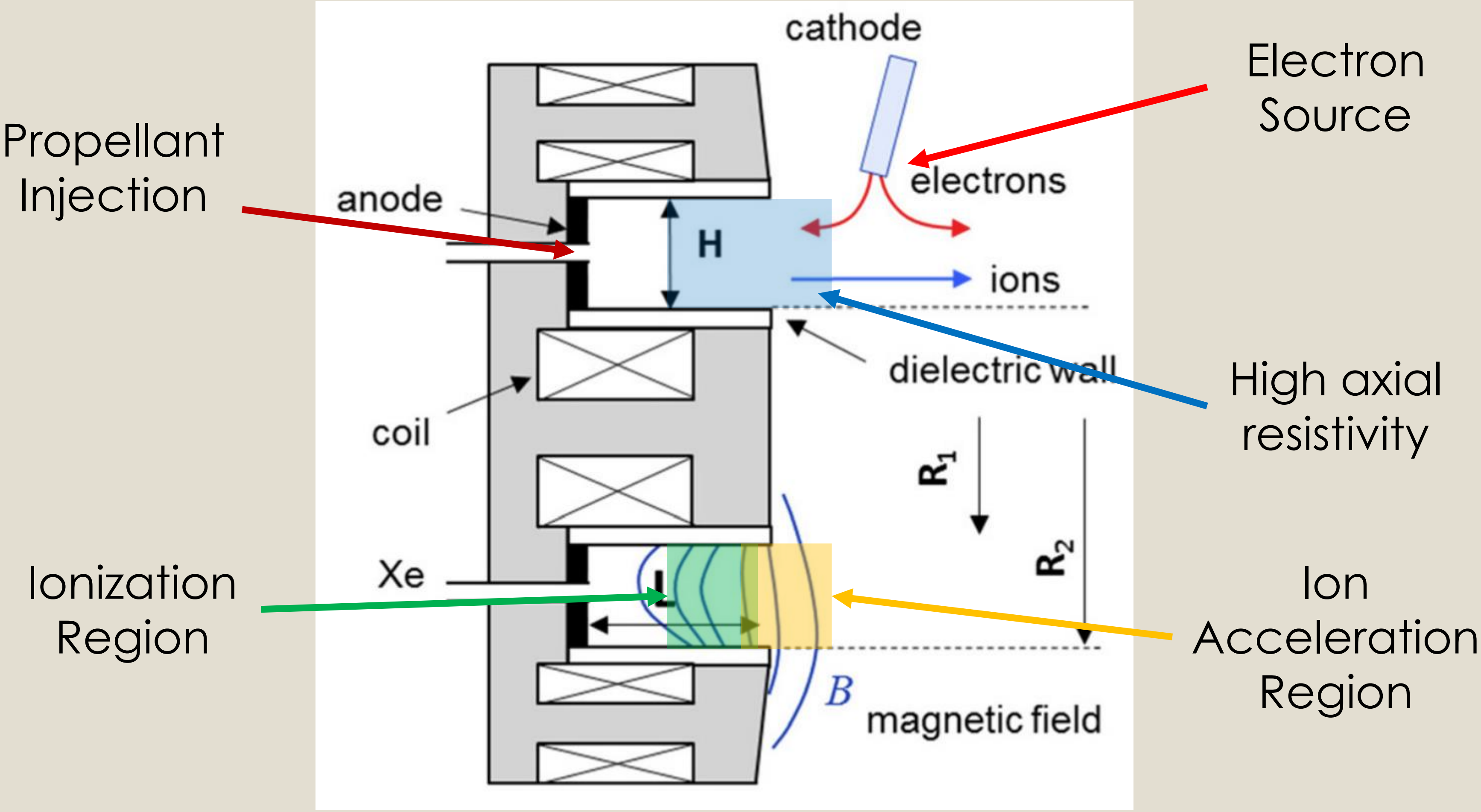
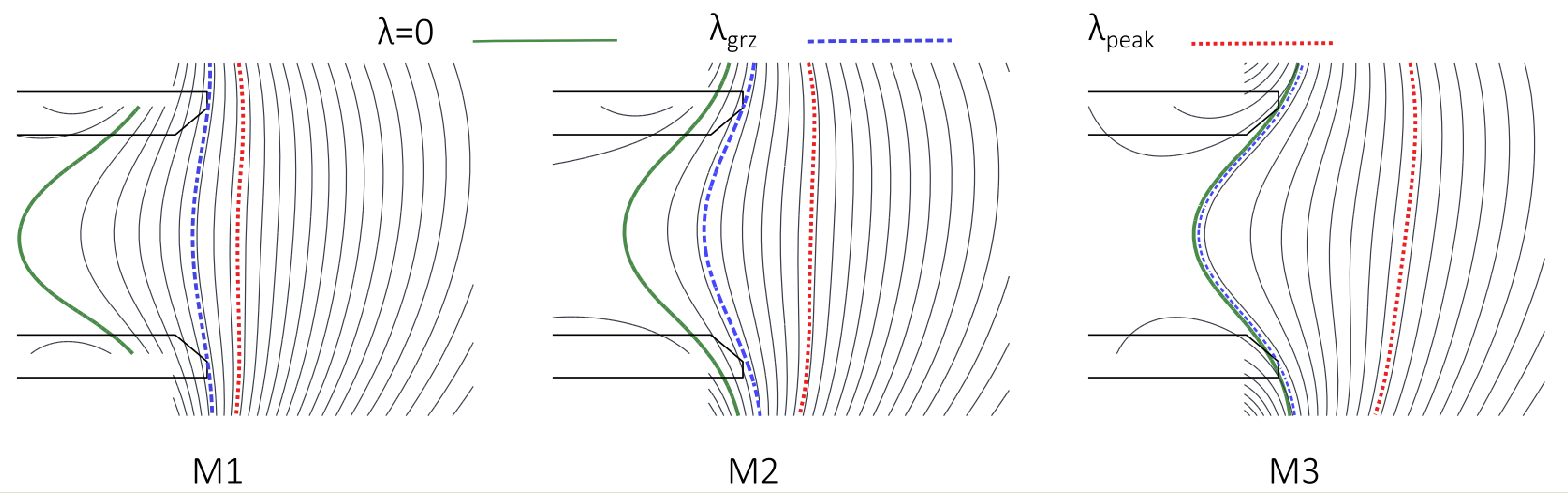
Thanks for your attention

Hall Thusters



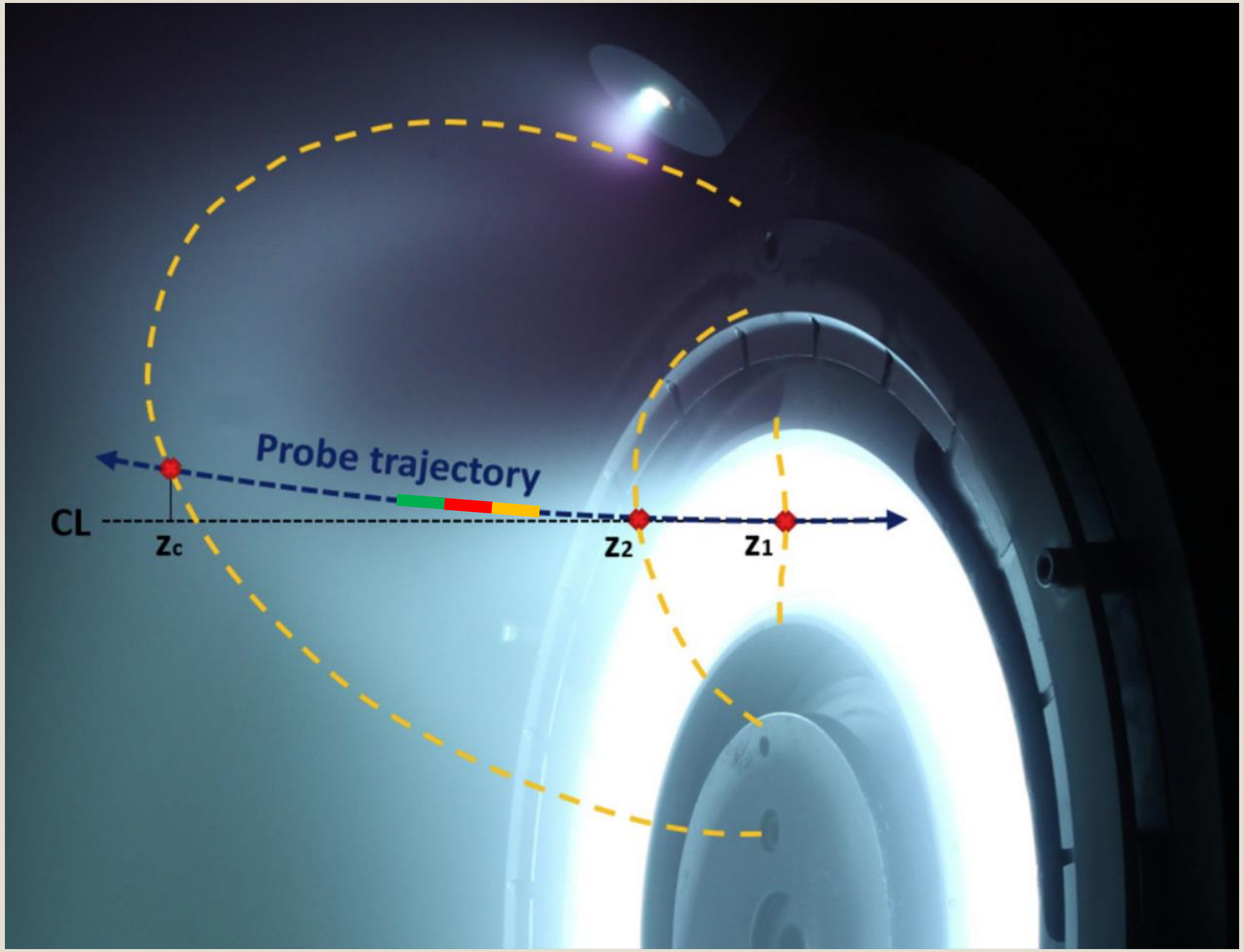
Magnetic Field Configurations



Data Analysis – Data Management

$$\boldsymbol{D_i} = \left\langle \begin{bmatrix} V_{GC} \\ V_{FC} \\ I_{BC} \end{bmatrix} \right\rangle_{t_{z_i}<t<t_{z_{i+1}}}$$

$$\boldsymbol{\Sigma_i} = \left\langle \left(\begin{bmatrix} V_{GC} \\ V_{FC} \\ I_{BC} \end{bmatrix} - \boldsymbol{D_i} \right) \left(\begin{bmatrix} V_{GC} \\ V_{FC} \\ I_{BC} \end{bmatrix} - \boldsymbol{D_i} \right)^T \right\rangle_{t_{z_i}<t<t_{z_{i+1}}}$$



Main Results – Breathing Mode