Statistics for Neutrino Oscillations Physics

The T2K Experiment

Statistics Seminar
Ph.D. cycle XXXVII

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Introduction

1. Neutrino Physics

2. T2K Experiment

3. Statistical Methods

4. Results

Introduction to Neutrinos

Neutrinos produced in weak processes ν_α are linear combination of mass eigenstates ν_i

$$|\nu(t)\rangle = \sum_i U^*_{\alpha i}\, e^{-iE_i t}\, |\nu_i\rangle = \sum_i U^*_{\alpha i}\, e^{-iE_i t} \sum_\beta U_{\beta i} |\nu_\beta\rangle$$

Neutrinos are observed to not preserve their flavor during propagation!

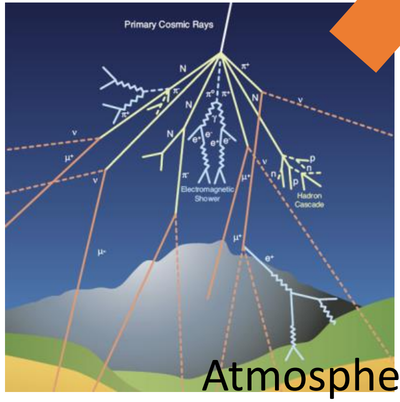
Nobel Prize 2015

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | \nu(t) \rangle|^2 = \left| \sum_i U_{\beta i} U^*_{\alpha i}\, e^{-iE_i t} \right|^2$$

Probability of oscillation is given by PMNS matrix

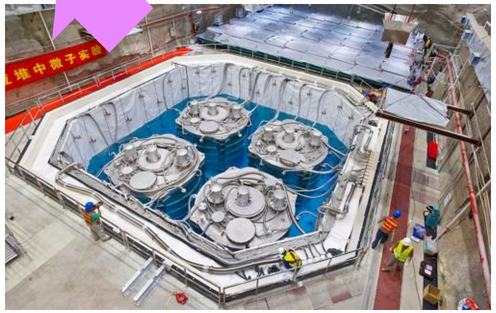
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

3 Mixing Angles
1 Phase

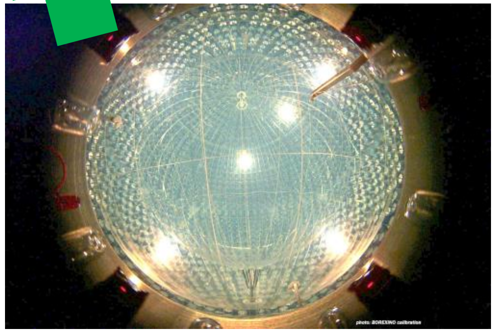


Atmospheric

Accelerators



Reactors



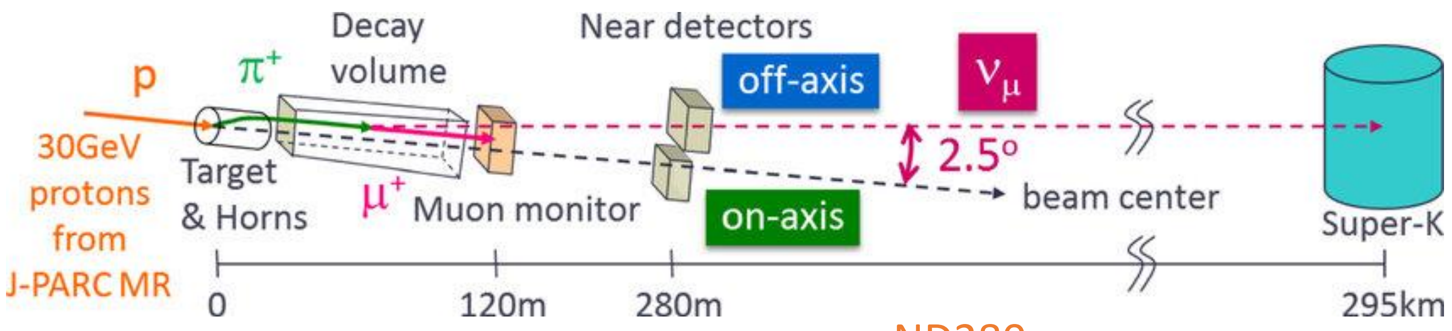
Solar

The T2K Experiment

Intense high purity muon (anti)neutrino beam from J-PARC to Super-Kamiokande

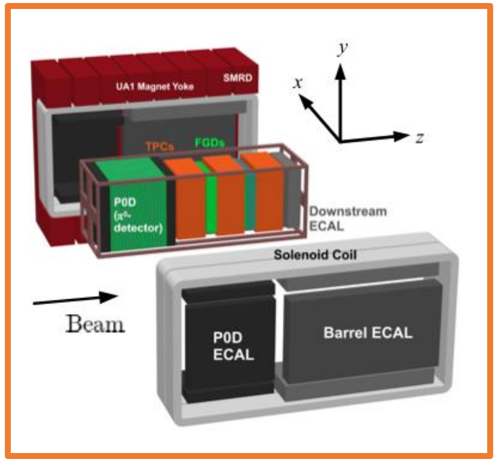
T2K compares the rate of neutrinos at Near and Far Detectors:

$$N_{ND} \sim \Phi_{ND} \cdot \sigma_{ND} \cdot \epsilon_{ND}$$
$$N_{FD} \sim \Phi_{FD} \cdot \sigma_{FD} \cdot \epsilon_{FD} \cdot \boldsymbol{P_{osc}}$$

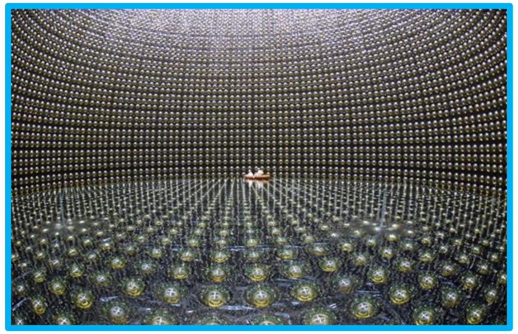


J-PARC

ND280



Super-K



Main goals and results

- Muon (anti)-neutrino disappearance $\nu_\mu \nrightarrow \nu_\mu (\overline{\nu_\mu} \nrightarrow \overline{\nu_\mu})$

  $\boldsymbol{\theta_{23}, \Delta m^2_{23}}$

- Electron (anti)-neutrino appearance $\nu_\mu \rightarrow \nu_e (\overline{\nu_\mu} \rightarrow \overline{\nu_e})$

  $\boldsymbol{\theta_{13}, \delta_{CP}}$

$$p + \text{graphite} \rightarrow \pi + K \rightarrow \mu + \nu_\mu$$

Muon neutrino beam (~ 600 MeV) is detected at near and far detectors

Neutrino Oscillations at T2K

ν_e appearance channel

Leading term for θ_{13}

Leading term for θ_{23}
Upper or Lower Octant?

$\overline{\nu_\mu} \to \overline{\nu_e}$

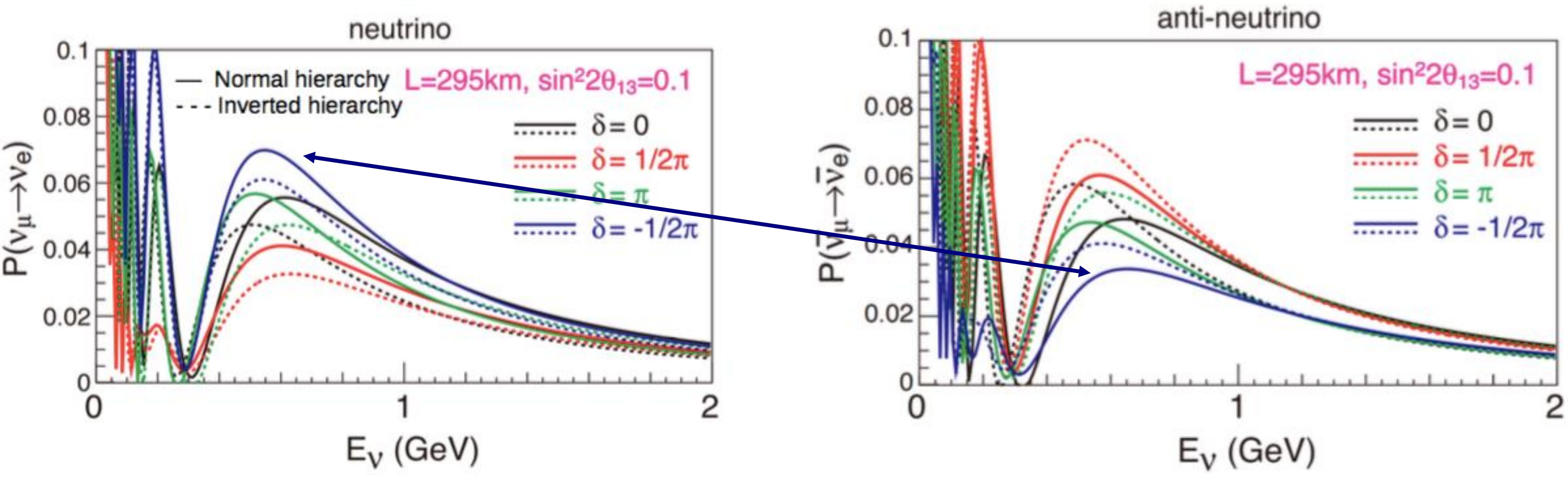
$$
\begin{aligned}
P(\nu_\mu \to \nu_e) = \; & 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \frac{\Delta m_{13}^2 L}{4E_\nu} \times \left[1 \pm \frac{2a}{\Delta m_{13}^2}(1 - s_{13}^2)\right] \\
& + 8c_{13}^2 s_{12} s_{13} s_{23} (c_{12} c_{23} \cos\delta_{CP} - s_{12} s_{13} s_{23}) \cos\frac{\Delta m_{23}^2 L}{4E_\nu} \sin\frac{\Delta m_{13}^2 L}{4E_\nu} \sin\frac{\Delta m_{12}^2 L}{4E\nu} \\
& \mp 8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin\delta_{CP} \sin\frac{\Delta m_{23}^2 L}{4E_\nu} \sin\frac{\Delta m_{13}^2 L}{4E_\nu} \sin\frac{\Delta m_{12}^2 L}{4E_\nu} \\
& + 4s_{12}^2 c_{13}^2 (c_{13}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - 2c_{12} c_{23} s_{12} s_{23} s_{13} \cos\delta_{CP}) \sin\frac{\Delta m_{12}^2 L}{4E_\nu} \\
& \mp 8c_{12}^2 s_{13}^2 s_{23}^2 \cos\frac{\Delta m_{23}^2 L}{4E_\nu} \sin\frac{\Delta m_{13}^2 L}{4E_\nu} \frac{aL}{4E_\nu}(1 - 2s_{13}^2)
\end{aligned}
$$

CP odd phase
$P(\nu_\mu \to \nu_e) \neq P(\overline{\nu_\mu} \to \overline{\nu_e})$
if $\sin\delta_{CP} \neq 0$

Matter Effect
$N.O. \to \nu_e$ appearance enhanced
$I.O. \to \overline{\nu_e}$ appearance enhanced

Neutrino Oscillations at T2K

ν_e appearance channel



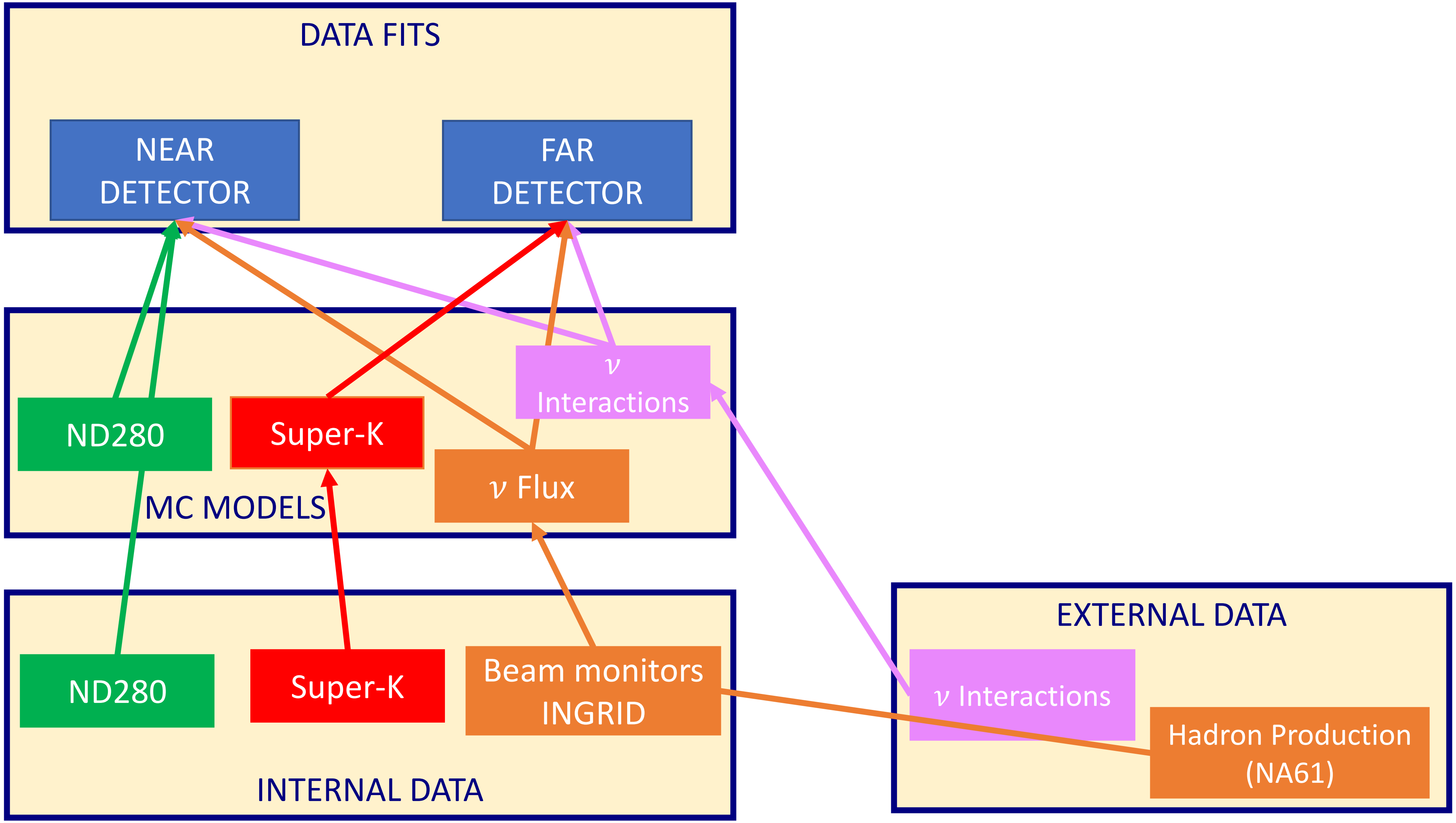
CP odd phase
$P(\nu_\mu \to \nu_e) \neq P(\overline{\nu_\mu} \to \overline{\nu_e})$
if $\sin\delta_{CP} \neq 0$

Up to $\pm 30\%$ effect

Matter Effect
$N.O. \to \nu_e$ appearance enhanced
$I.O. \to \overline{\nu_e}$ appearance enhanced

Up to $\pm 10\%$ effect

T2K Analysis Strategy



Parameter Estimation

Inference from data aims mainly to **parameter estimation** and **interval estimation**

A "best guess" and a "range" supported by data

- **Frequentist Approach**

It requires a full specification of the probability model $P(\boldsymbol{y}|H)$

- Parameters and hypotheses may be unknown, but they are fixed

Inferential procedure are evaluated under the (*hypothetical*) repeated sampling of the data

↓

An **approximate result** can be found by applying the Likelihood to only obtained data

Parameter Estimation

- **Bayesian Approach**

Unknown quantities contained in a probability model for the observed data are treated as random variables

Bayes' Theorem:

Likelihood function

Prior probability

Posterior ∝ Likelihood × Prior

$$P(\boldsymbol{\theta}|\boldsymbol{y}, I) = \frac{P(\boldsymbol{y}|\boldsymbol{\theta}, I)\pi(\boldsymbol{\theta}|I)}{p(\boldsymbol{y}|I)}$$

Posterior probability

Marginalization:

$$p(\theta_i|\boldsymbol{y}) = \int_{\theta-i} p(\boldsymbol{\theta}|\boldsymbol{y})d\boldsymbol{\theta}_{-i}$$

Normalizing Factor

$$p(\boldsymbol{y}|I) = \int_{\theta} p(\boldsymbol{y}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})d\boldsymbol{\theta}$$

Crucial points:

1. Prior Specification
2. Integration over large number of dimensions

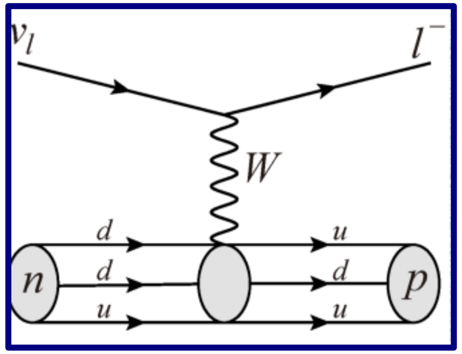
Near Detector Physics

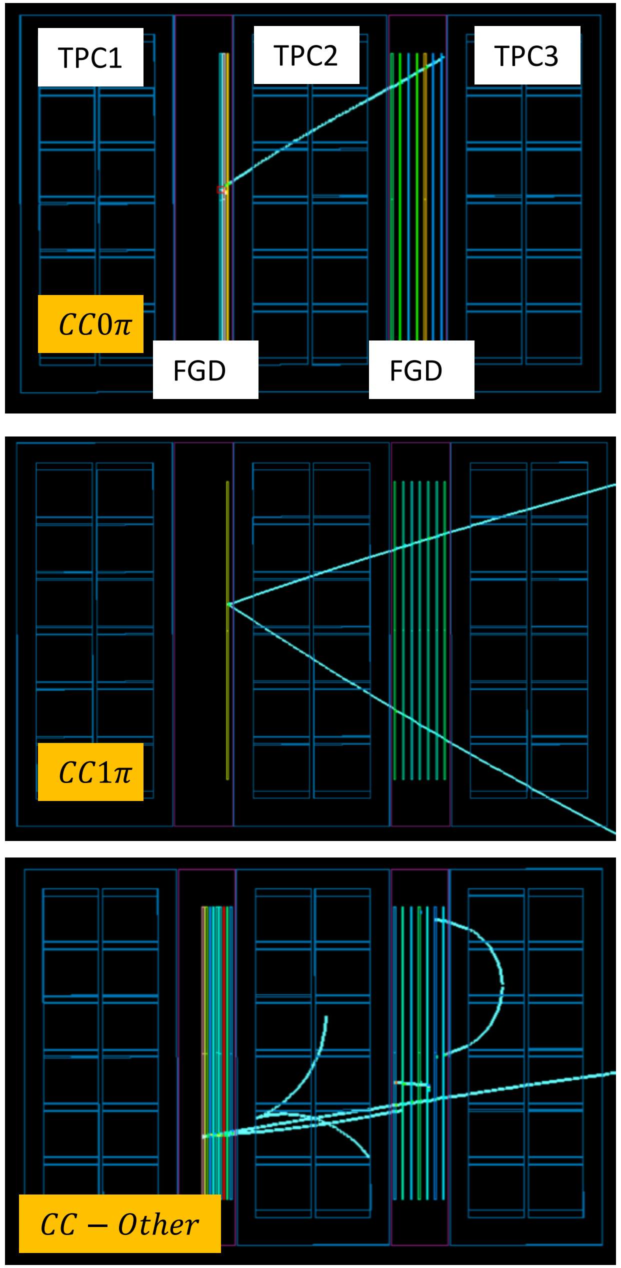
Neutrino interacts with Carbon and Water mainly through

Charged Current Quasi Elastic Scattering (CCQE)

$$\nu_\mu + n \rightarrow \mu^- + p$$
$$\overline{\nu_\mu} + p \rightarrow \mu^+ + n$$

Magnetic Field





ND addresses **systematic uncertainties**

Flux
- Data-driven simulation (NA61)
- High Statistic monitoring (INGRID)

Cross Section

Reactions that mimic CCQE
- Scattering into a correlated pair of nucleons
- Resonant production of pion(s)

Building a Likelihood Function – *Near Detector*

Data collected by Near Detector is binned as a function of two variables of detected muon:

- Momentum p_μ
- Angle w.r.t. beam direction $\cos\theta_\mu$

$$L_{total} = L_{stat} \times L_{stat}^{MC} \times L_{syst}$$

Statistical likelihood

$$-2\ln L_{stat} - 2\ln L_{stat}^{MC} = 2\sum_{i}^{samples}\sum_{j}^{bins}\left[\left(N_{MC} - N_{Data} + N_{Data}\ln\frac{N_{Data}}{N_{MC}}\right) + \frac{\left(\beta_j - 1\right)^2}{2\sigma_{\beta_j}^2}\right]$$

Poissonian term

Barlow-Beeston term
Including the uncertainty of finite MC simulation

Building a Likelihood Function – *Near Detector*

Data collected by Near Detector is binned as a function two variables of detected muon:

- Momentum p_μ
- Angle w.r.t. beam direction $\cos\theta_\mu$

$$L_{total} = L_{stat} \times L_{stat}^{MC} \times L_{syst}$$

Systematic uncertainties

$$-2\ln L_{syst} = (\vec{x} - \vec{\mu})^T V^{-1} (\vec{x} - \vec{\mu})$$

Systematic uncertainties:
- ND response
- Neutrino interactions
- Neutrino flux

Gaussian penalty term that includes the covariance matrix **V**
- $\vec{\mu}$ systematic uncertainty parameters **before** fit
- $\vec{x}$ systematic uncertainty parameters **during** fit

Covariance Matrix V from ND fit

Constraints on the flux at FD!

$$\max \boldsymbol{L} \leftrightarrow \min(-2 \times \ln \boldsymbol{L})$$

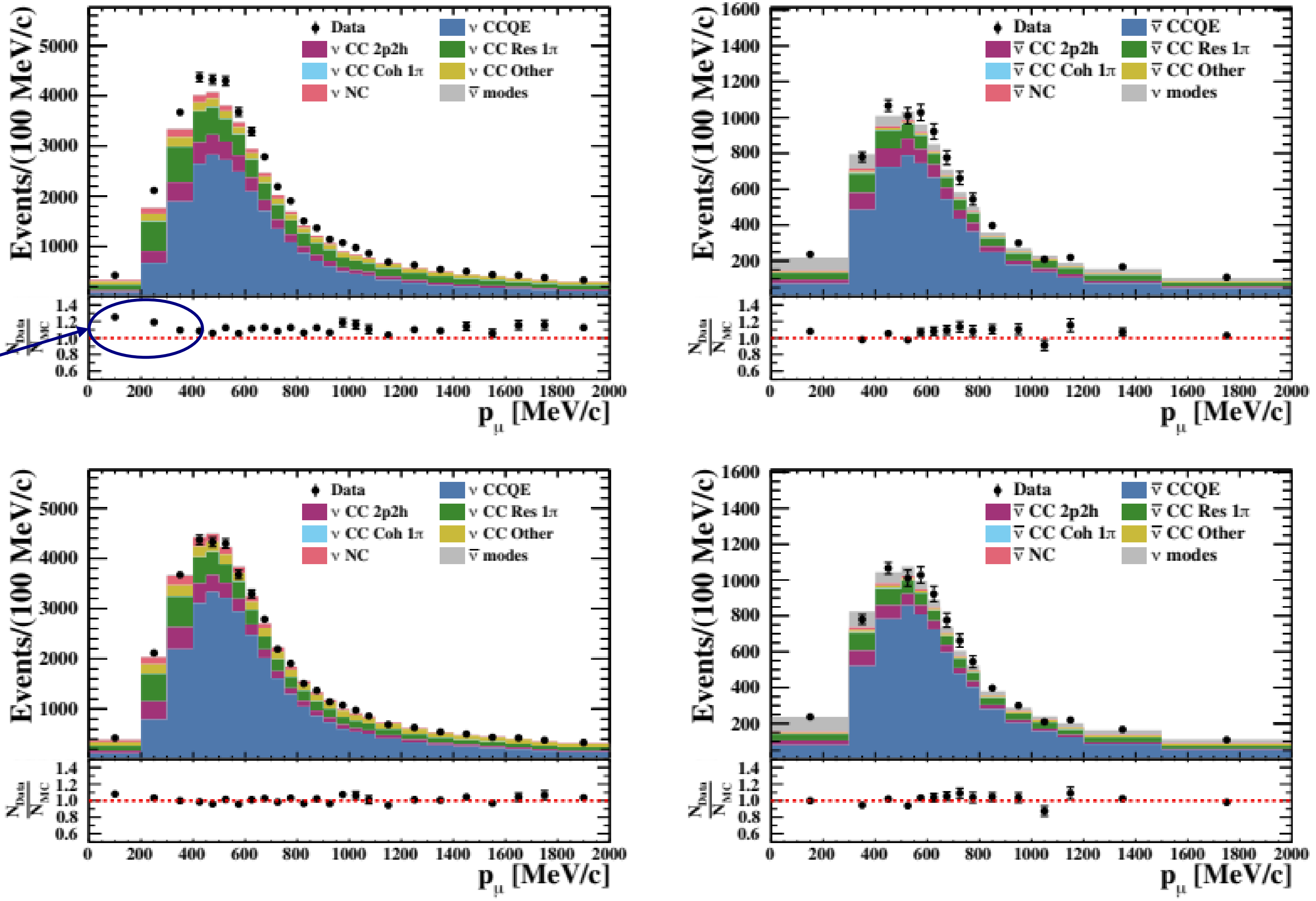
$$\frac{\partial \ln L}{\partial \theta_i} = 0, \qquad i = 1, \dots, N$$

Maximum Likelihood estimators for $\boldsymbol{\theta}$

Near Detector Results

Pre-fit

Underprediction!



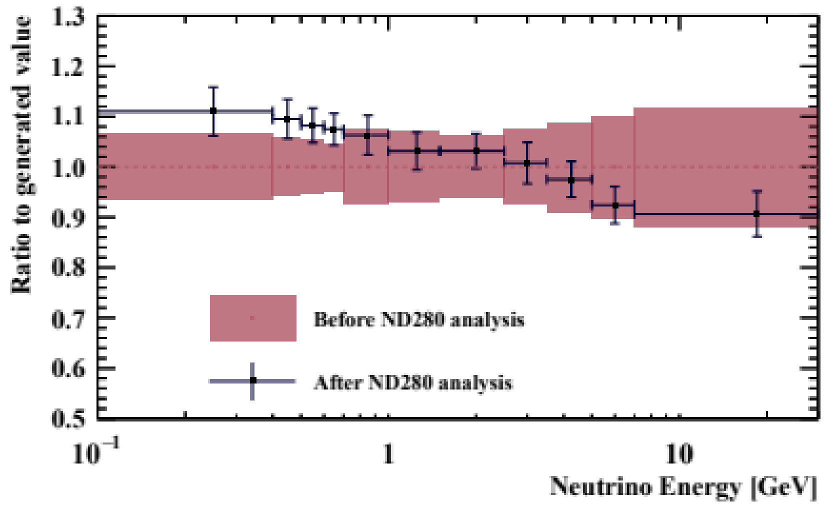
Post-fit

(a) FGD1 ν-mode ν_μ CC0π

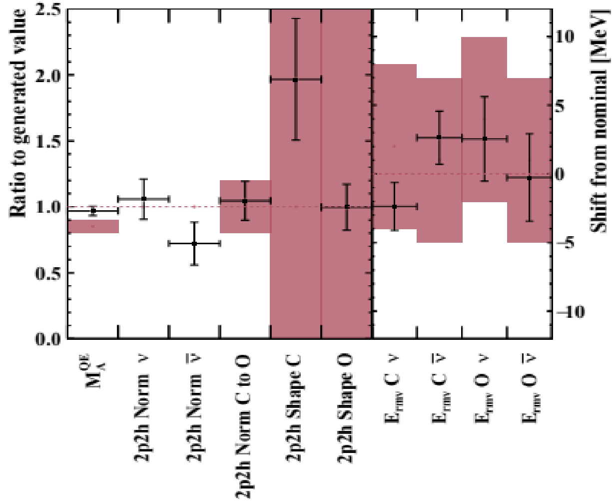
(b) FGD2 $\overline{\nu}$-mode $\overline{\nu}_\mu$ CC0π

Near Detector Results

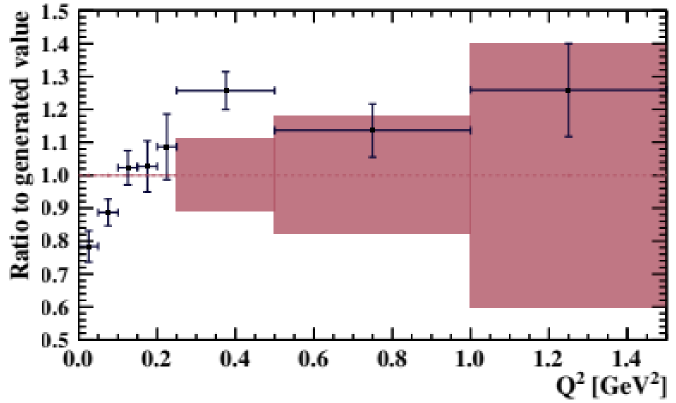
Flux



Cross Section Parameters



Q^2 Parameter



Uncertainty at Far Detector

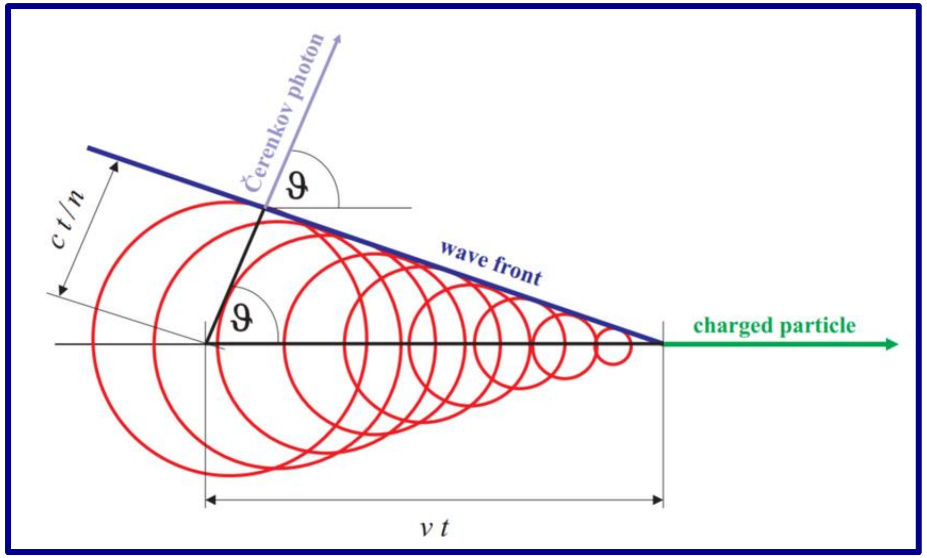
After ND Fit

Sample		Uncertainty source (%) Flux	Interaction	FD + SI + PN	Flux⊗Interaction (%)	Total (%)
1Rμ	ν	2.9 (5.0)	3.1 (11.7)	2.1 (2.7)	2.2 (12.7)	3.0 (13.0)
	$\bar{\nu}$	2.8 (4.7)	3.0 (10.8)	1.9 (2.3)	3.4 (11.8)	4.0 (12.0)
1Re	ν	2.8 (4.8)	3.2 (12.6)	3.1 (3.2)	3.6 (13.5)	4.7 (13.8)
	$\bar{\nu}$	2.9 (4.7)	3.1 (11.1)	3.9 (4.2)	4.3 (12.1)	5.9 (12.7)
1Re1de	ν	2.8 (4.9)	4.2 (12.1)	13.4 (13.4)	5.0 (13.1)	14.3 (18.7)

Before ND Fit

Building a Likelihood Function – *Far Detector*

Event reconstruction in Super-K uses both **charge** and **timing** information from **hits** in the PMTs ⟶ **Cherenkov radiation**



- **Event topology hypothesis $\boldsymbol{\Gamma}$ (e.g., single-ring e-like)**

- **Kinematic parameters $\boldsymbol{\theta}$**
    1. Vertex position
    2. Particle creation times
    3. Angles of direction
    4. Particles momenta

Recorded data (each PMT):

- **Charge**
- **Hit time**

$$L(\Gamma,\theta) = \prod_{j}^{unhit} P_j(unhit|\Gamma,\theta) \times$$

$$\prod_{i}^{hit} [1 - P_i(unhit|\Gamma,\theta)] \times f_q(q_i|\Gamma,\theta) f_t(t_i|\Gamma,\theta)$$

Observing charge q_i

Observing charge t_i

It can be rewritten in term of expected **photo-electrons** $\mu_i(\Gamma,\theta)$

Particle Identification (PID)

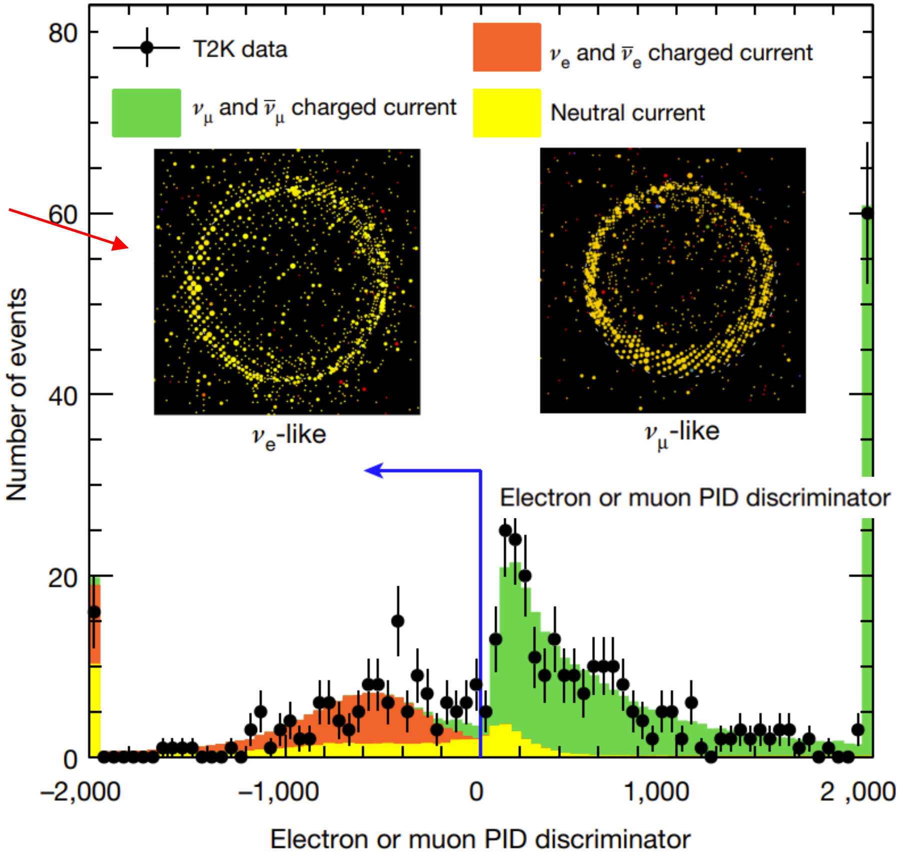
An event may involve **one** or **more** rings that has to be reconstructed

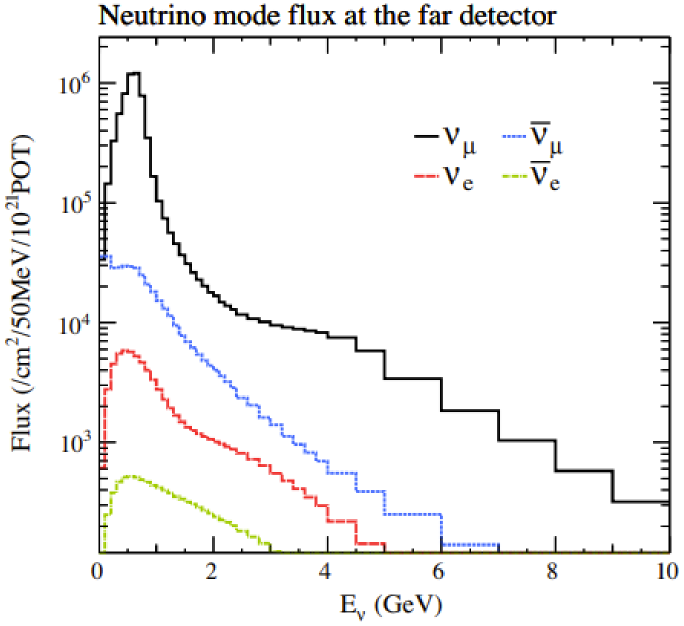
- **Single-ring**

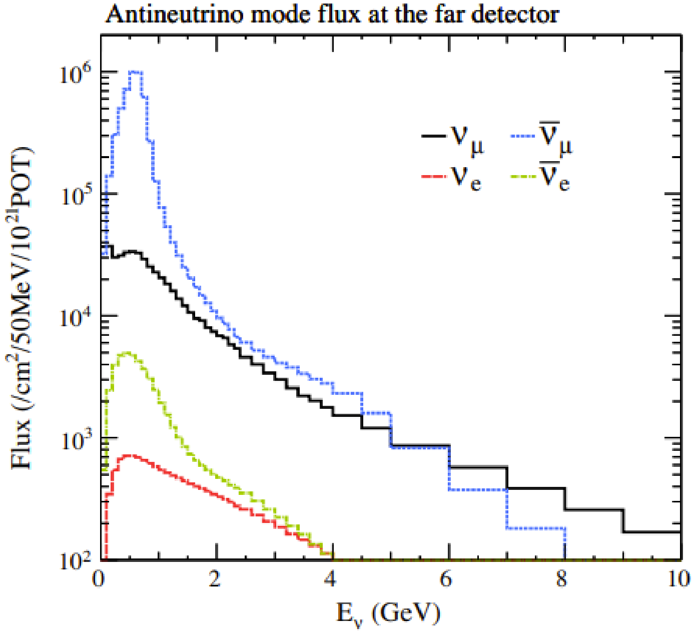
Three hypotheses Γ

$e, \mu, \pi^{\pm}$ → separated by Log-likelihood Ratio $\ln \frac{L_\mu}{L_e}$

"Fuzzier"







Hypothesis Testing

- **Frequentist Approach**

A **p-value** is calculated

A statistic t is defined → Quantification of agreement between data and the hypothesis

Probability that a model with a test statistic equal to or larger than observed data is found

$$p = \int_{t_{obs}}^{\infty} f(t|H_0)dt$$

p-value is not the probability for the Hypothesis!

How many toy datasets give as a result…

$$(-2\ln L)_{min}^{Toy} \geq (-2\ln L)_{min}^{Data}$$

- An a priori criteria of $p > 0.05$ is needed

Toys datasets are created by varying uncertainties

Likelihood contributor			p-value
ν_μ in ν-mode	0π	FGD1	0.93
		FGD2	0.93
$\overline{\nu}_\mu$ in $\overline{\nu}$-mode	0π	FGD1	0.20
		FGD2	0.15
ν_μ in $\overline{\nu}$-mode	0π	FGD1	0.54
		FGD2	0.45
All samples			0.82
Neutrino flux			0.46
ND detector			0.06
Cross section			0.01
All samples, all syst.			0.74

Hypothesis Testing

- **Bayesian Approach**

Bayes Factor

Consider data $\boldsymbol{y}$ arisen under two Hypotheses H_1, H_2 with probability density $p(\boldsymbol{y}|H_1)$ and $p(\boldsymbol{y}|H_2)$

$$B_{12} = \frac{p(\boldsymbol{y}|H_1)}{p(\boldsymbol{y}|H_2)} = \frac{\int p(\boldsymbol{\theta_1}|H_1)p(\boldsymbol{y}|\boldsymbol{\theta_1}, H_1)d\boldsymbol{\theta_1}}{\int p(\boldsymbol{\theta_2}|H_2)p(\boldsymbol{y}|\boldsymbol{\theta_2}, H_2)d\boldsymbol{\theta_2}} = \frac{p(H_1|\boldsymbol{y})}{p(H_2|\boldsymbol{y})} \times \frac{\pi(H_2)}{\pi(H_1)}$$

Marginalized Likelihoods

$$\text{Bayes Factor} \propto \frac{\text{Posterior Odds}}{\text{Prior odds}}$$

Interpretation

Summary of evidence provided by data

In case of test against null hypothesis H_0:

$2\log_e(B_{10})$	(B_{10})	Evidence against H_0
0 to 2	1 to 3	Not worth more than a bare mention
2 to 6	3 to 20	Positive
6 to 10	20 to 150	Strong
>10	>150	Very strong

Kass-Raftery classification

Oscillation Analysis

From ND and FD analysis to... → **Three-flavour oscillation analysis**

No experiment is sensitive to all mixing parameters

T2K relies on external constraints for:

Solar
$\sin^2\theta_{12}, \Delta m^2_{21}$

Reactor
$\sin^2\theta_{13}$

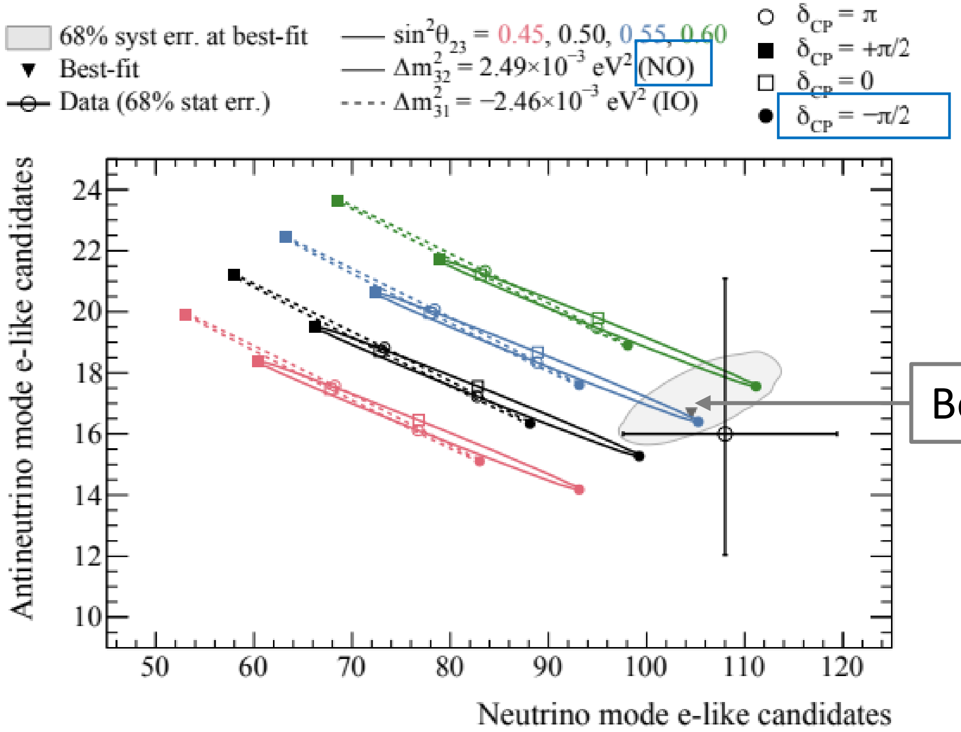
Constraints from PDG 2019

Crucial effect on δ_{CP}

Predictions

Sample		True δ_{CP} (rad.) $-\pi/2$	0	$\pi/2$	π	Data
1Rμ	ν-mode	346.61	345.90	346.57	347.38	318
	$\bar{\nu}$-mode	135.80	135.45	135.81	136.19	137
1Re	ν-mode	96.55	81.59	66.89	81.85	94
	$\bar{\nu}$-mode	16.56	18.81	20.75	18.49	16
1Re1de	ν-mode	9.30	8.10	6.59	7.79	14

Fixed values of parameters + varying δ_{CP}



Best-fit predicted # of events

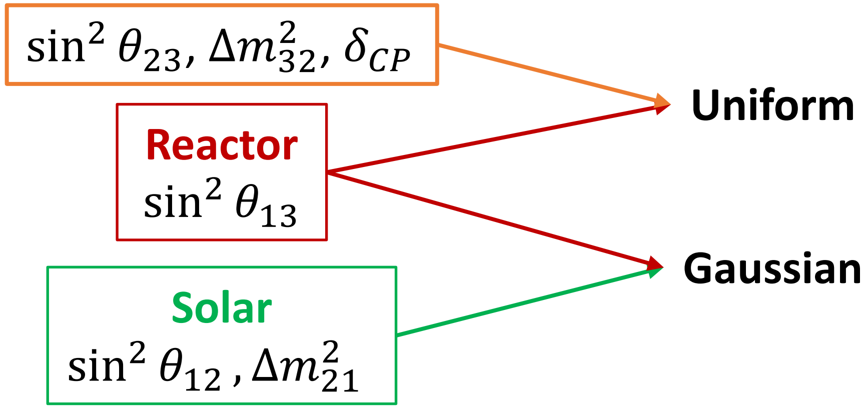
Bayesian Results

Sampling of posterior distribution through MCMC → Simultaneous ND and FD selections

External constraints on systematic uncertainties parameters → Multivariate gaussian

1. Priors distribution:

Different based on how parameters are obtained



2. Credible regions:

Extracted from lower dimensional **marginalised** posterior distributions

Reminder:

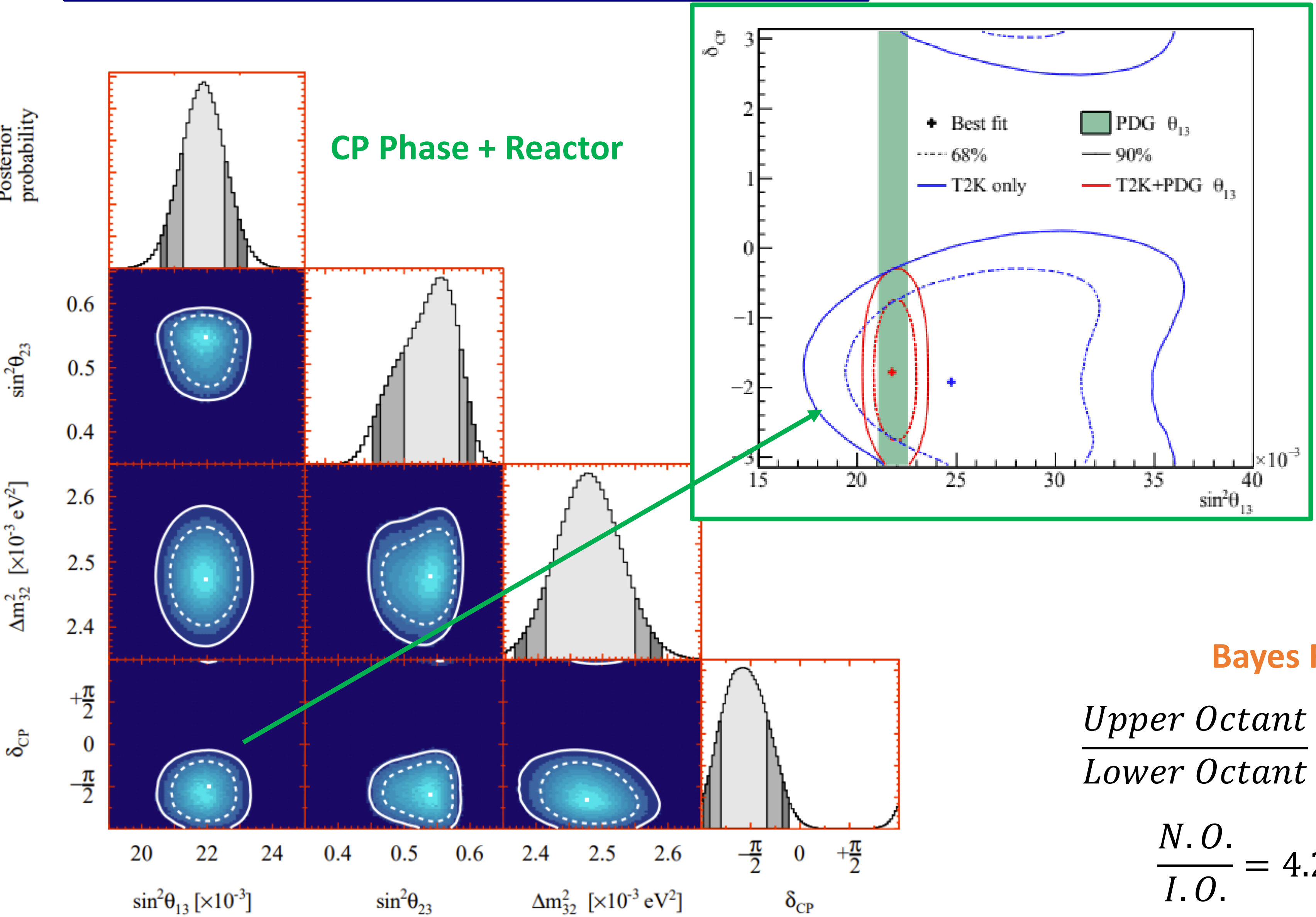
$$p(\boldsymbol{\theta}|\boldsymbol{y}, I) = \frac{p(\boldsymbol{y}|\boldsymbol{\theta}, I)\pi(\boldsymbol{\theta}|I)}{p(\boldsymbol{y}|I)}$$

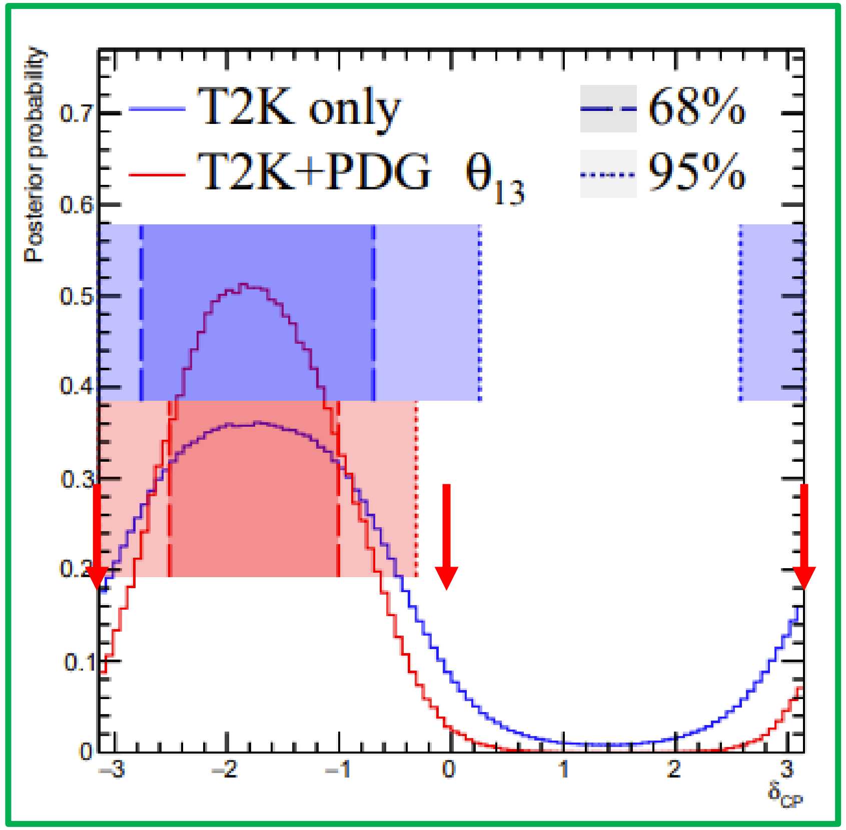
Marginalization:

$$p(\theta_i|\boldsymbol{y}) = \int_{\theta-i} p(\boldsymbol{\theta}|\boldsymbol{y})d\boldsymbol{\theta}_{-i}$$

Bayesian Results

Posterior probability obtained with marginalization





Bayes Factor

$$\frac{Upper\ Octant}{Lower\ Octant} = 3.35\ (1.43)$$

$$\frac{N.O.}{I.O.} = 4.21\ (1.83)$$

Kass-Raftery

- Positive evidence

Frequentist Results

Results are obtained by using the marginal likelihood

$$L_{marg}(\theta) = \int d\eta\, p(\eta) L(\theta, \eta)$$

→ Binned Poisson likelihood for
- Parameters of interest θ
- Nuisance parameters η

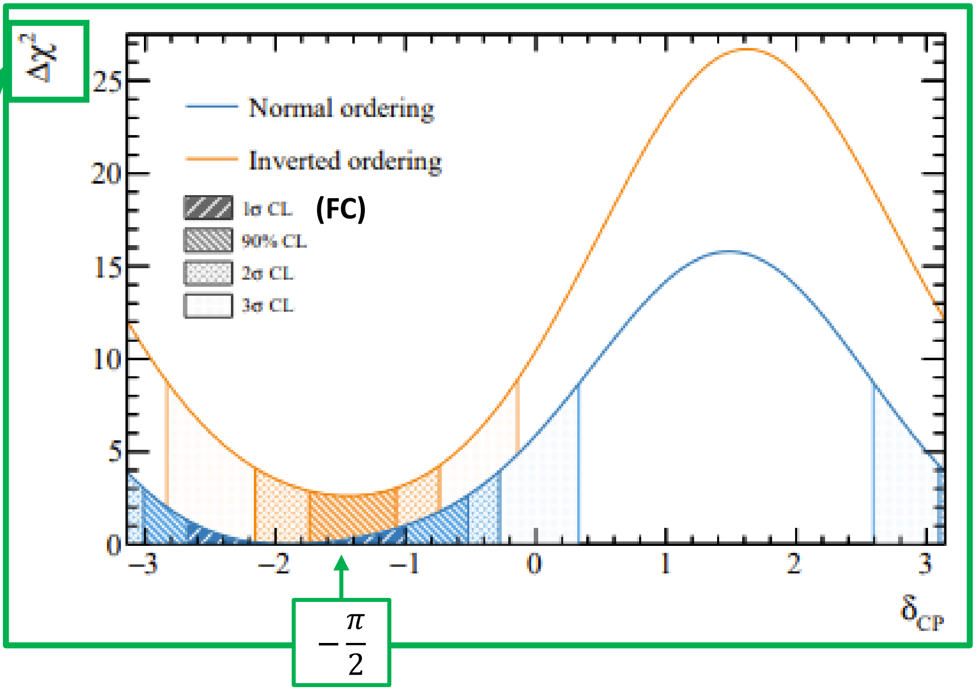
- **Confidence Interval**

Feldman-Cousins (FC) method

$$R = \frac{L(\boldsymbol{y}|\boldsymbol{\theta})}{L(\boldsymbol{y}|\boldsymbol{\theta}_{best})}$$

$$\Delta\chi^2(\theta) = \chi^2(\theta) - \min_{\theta'} \chi^2(\theta')$$

CP violating phase



Conclusions

T2K has measured:

- Oscillation parameters $\Delta m^2_{32}, \sin^2\theta_{13}, \sin^2\theta_{23}, \delta_{CP}$
- Jarlskog invariant
- Mass Ordering

Next Steps

1. Results are limited by **statistics** → Currently in shutdown for **Upgrade**

The Upgrade of ND is designed to:

- Decrease **systematic uncertainties**
- Increase **flux**

Change at FD:

- **Gadolinium** doping of pure water

2. Joint Analysis with **NOvA**

Thanks for your attention!

Bibliography

- Abe, K., et al. "Measurements of neutrino oscillation parameters from the T2K experiment using $3.6\times10^{21} $ protons on target." *arXiv preprint arXiv:2303.03222* (2023).

- Abe, Ko, et al. "Improved constraints on neutrino mixing from the T2K experiment with 3.13× 10 21 protons on target." *Physical Review D* 103.11 (2021): 112008.

- Jiang, M., et al. "Atmospheric neutrino oscillation analysis with improved event reconstruction in Super-Kamiokande IV." *Progress of Theoretical and Experimental Physics* 2019.5 (2019): 053F01.

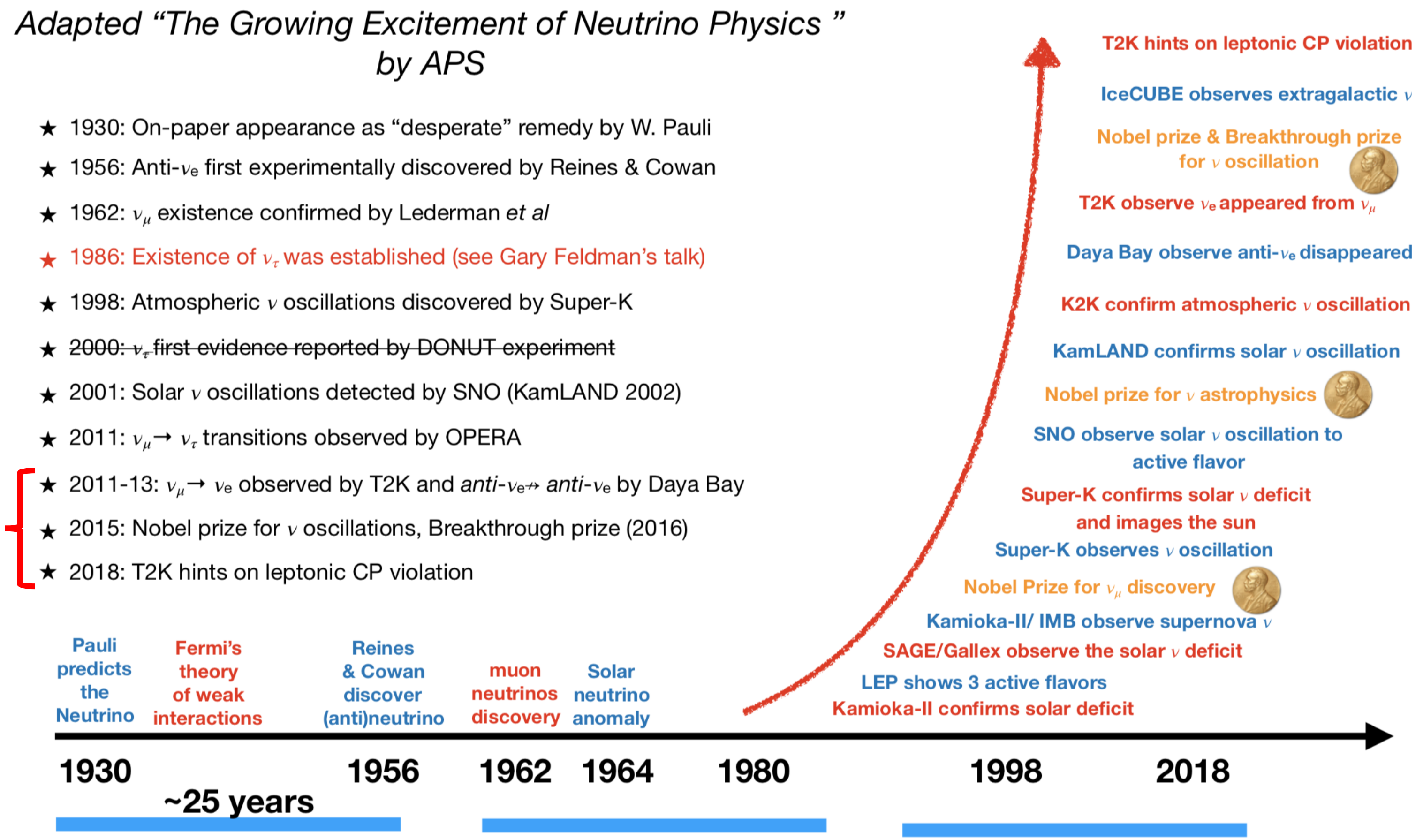
BACKUP

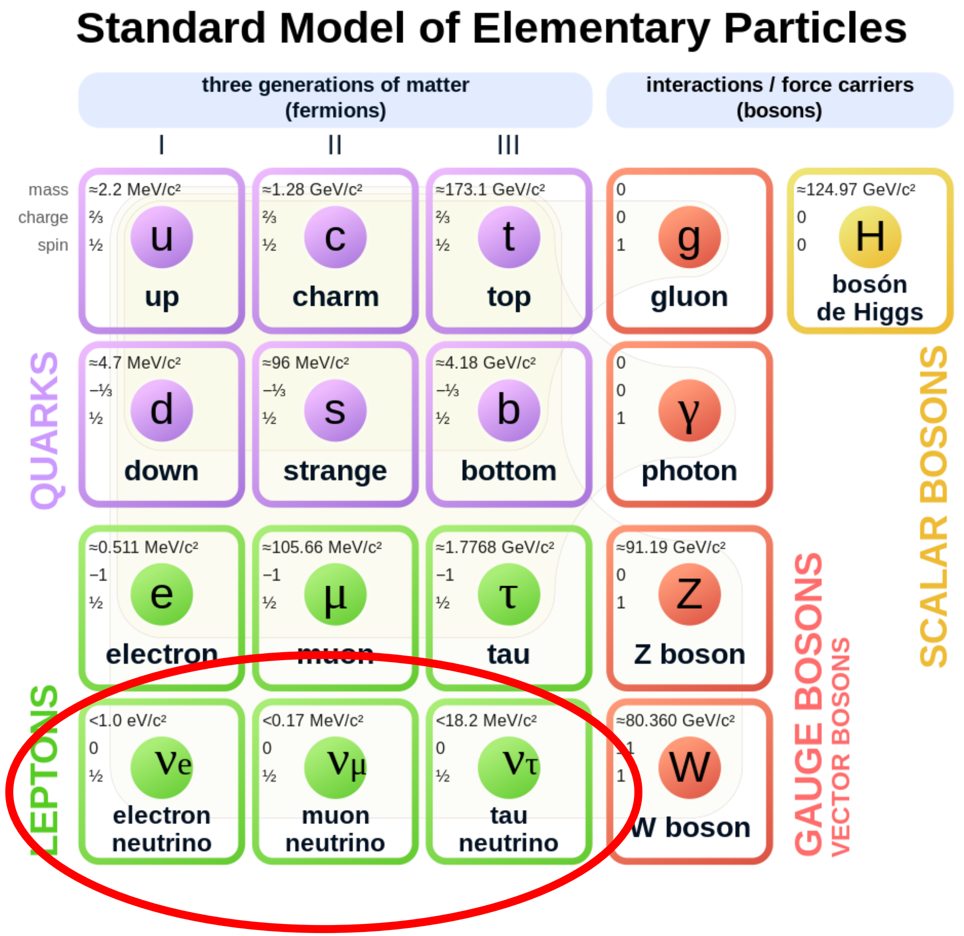
Introduction to Neutrinos

According to Standard Model, neutrinos are 3 of the fundamental particles

- **Neutral**
- **Massless**

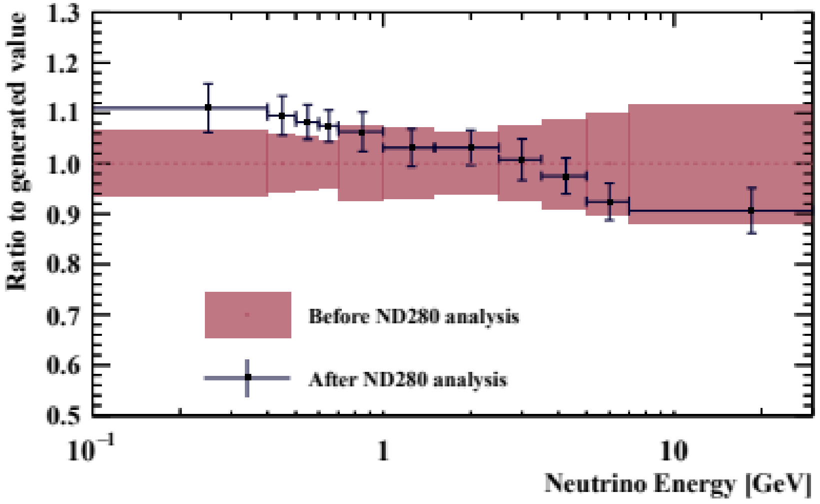
They can be observed through the products of their interactions



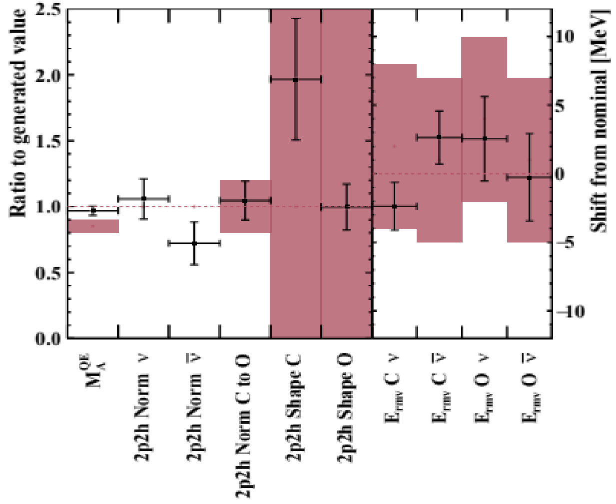


Near Detector Results

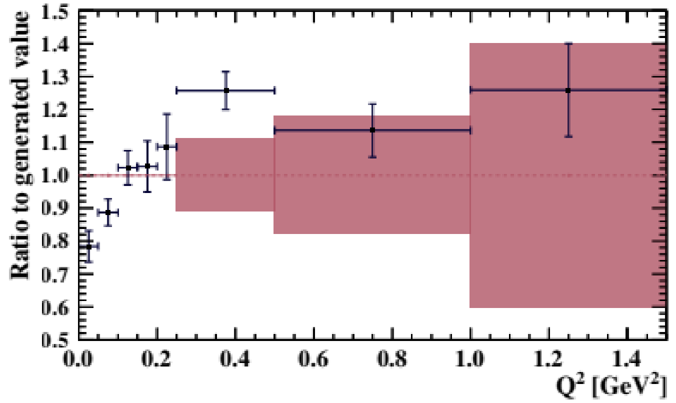
Flux



Cross Section Parameters

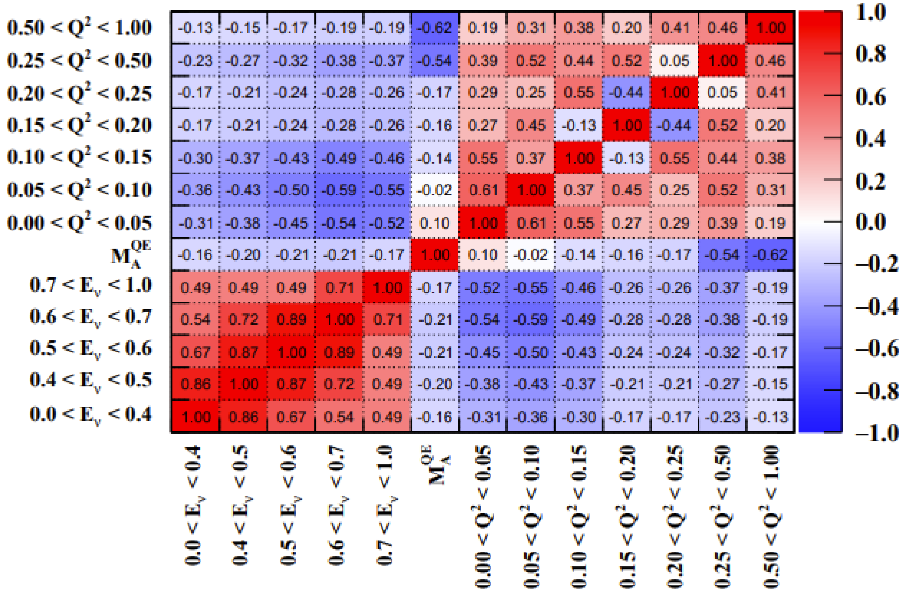


Q^2 Parameter

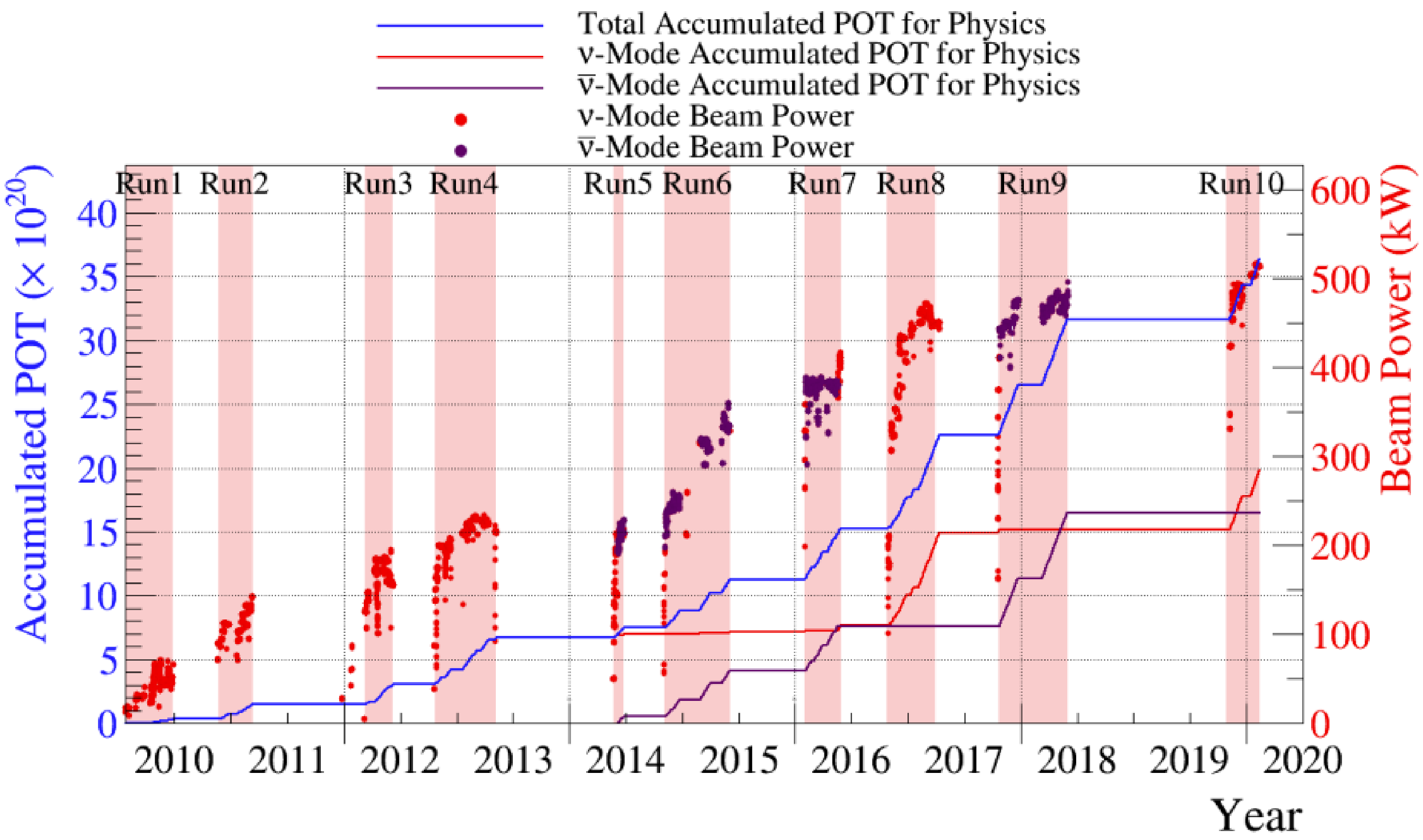


Covariance Matrix

- Q^2 parameters have a strong anti-correlation with flux parameters
- Strong anti correlation reduces uncertainty on the prediction at FD



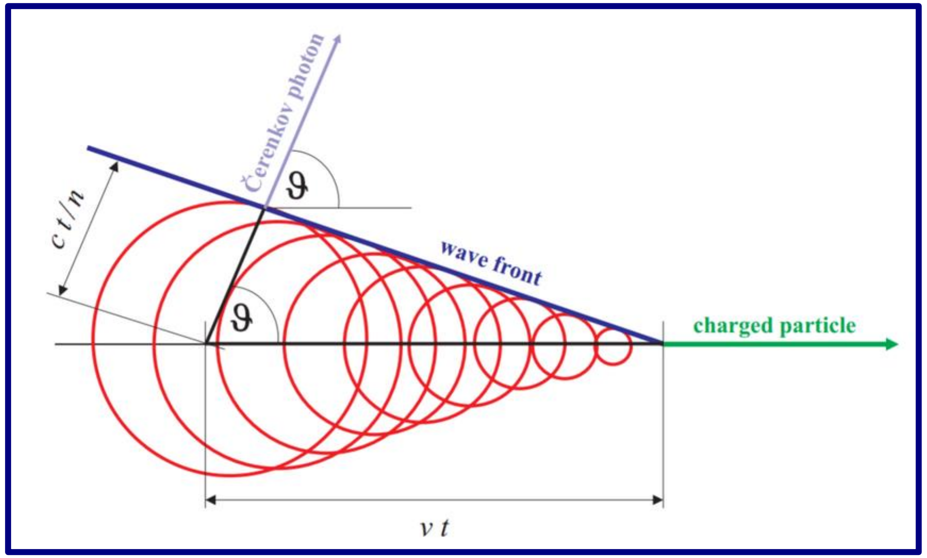
Data Taking



Total Accumulated POT for Physics
ν-Mode Accumulated POT for Physics
ν̄-Mode Accumulated POT for Physics
ν-Mode Beam Power
ν̄-Mode Beam Power
Run1 Run2 Run3 Run4 Run5 Run6 Run7 Run8 Run9 Run10
Accumulated POT (× 10^20)
Beam Power (kW)
0 5 10 15 20 25 30 35 40
0 100 200 300 400 500 600
2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020
Year

Building a Likelihood Function – *Far Detector*

Event reconstruction in Super-K uses both **charge** and **timing** information from **hits** in the PMTs → **Cherenkov radiation**



An event topology hypothesis Γ (e.g., single-ring e-like) is considered in a likelihood fit, along with its kinematic parameters θ:

1. Vertex position
2. Particle creation times
3. Angles of direction
4. Particles momenta

Recorded data (each PMT):

- **Charge**
- **Hit time**

$$L(\Gamma,\theta) = \prod_{j}^{unhit} P_j(unhit|\Gamma,\theta) \times$$

$$\prod_{i}^{hit} [1 - P_i(unhit|\Gamma,\theta)] \times f_q(q_i|\Gamma,\theta) f_t(t_i|\Gamma,\theta)$$

Observing charge q_i

Observing charge t_i

Depends on Γ:

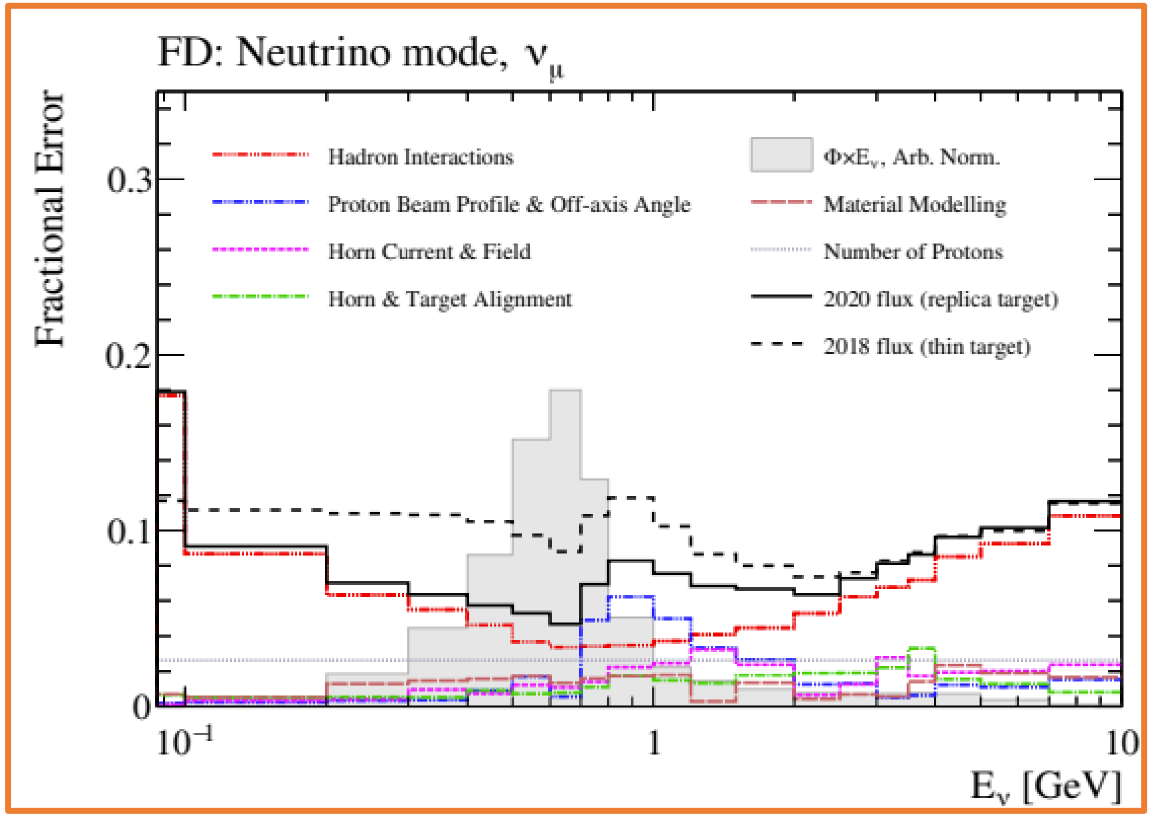
- Momentum p
- Expected hit time t_i^{exp}

$$f_t(t_i|t_i^{exp},\Gamma,p,\mu_i)$$

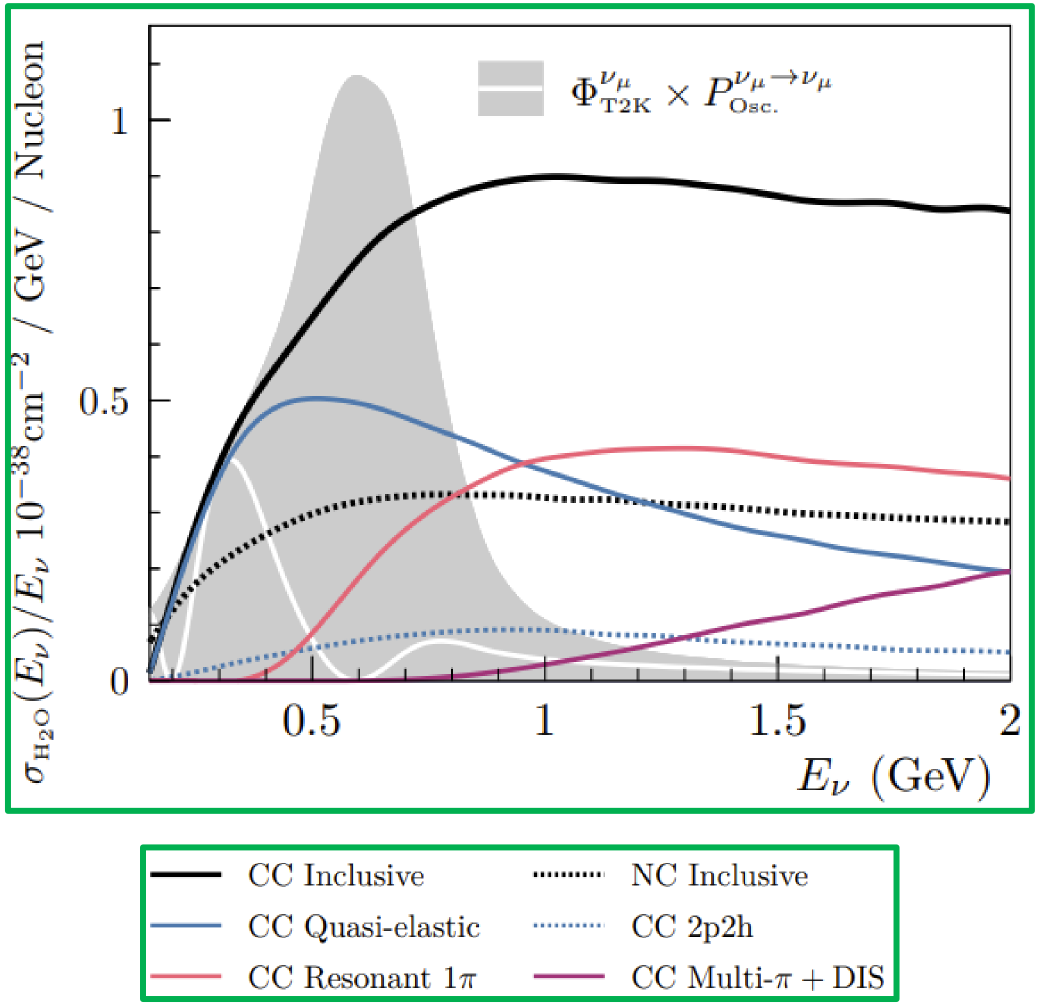
Direct hit starting mid-track

Near Detector Physics

Flux



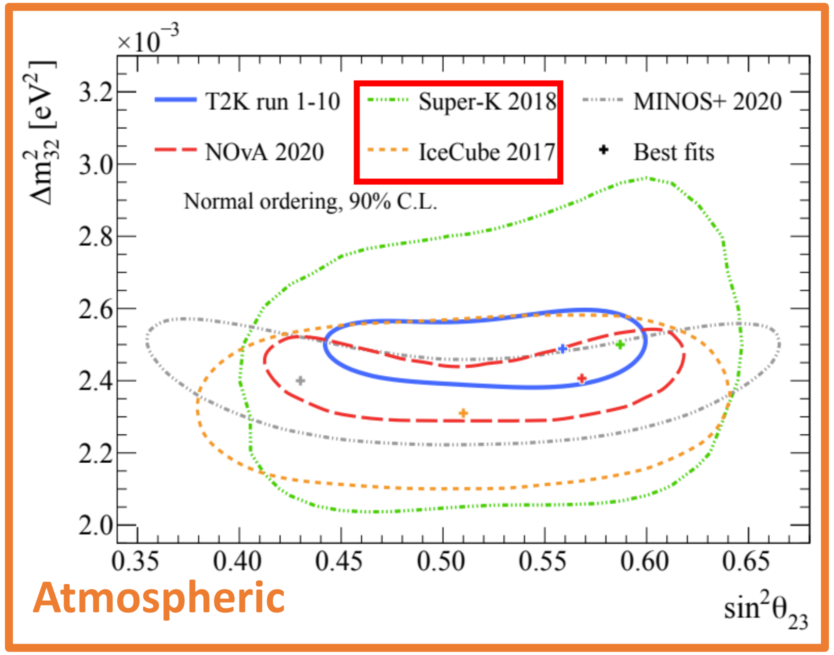
Cross Section



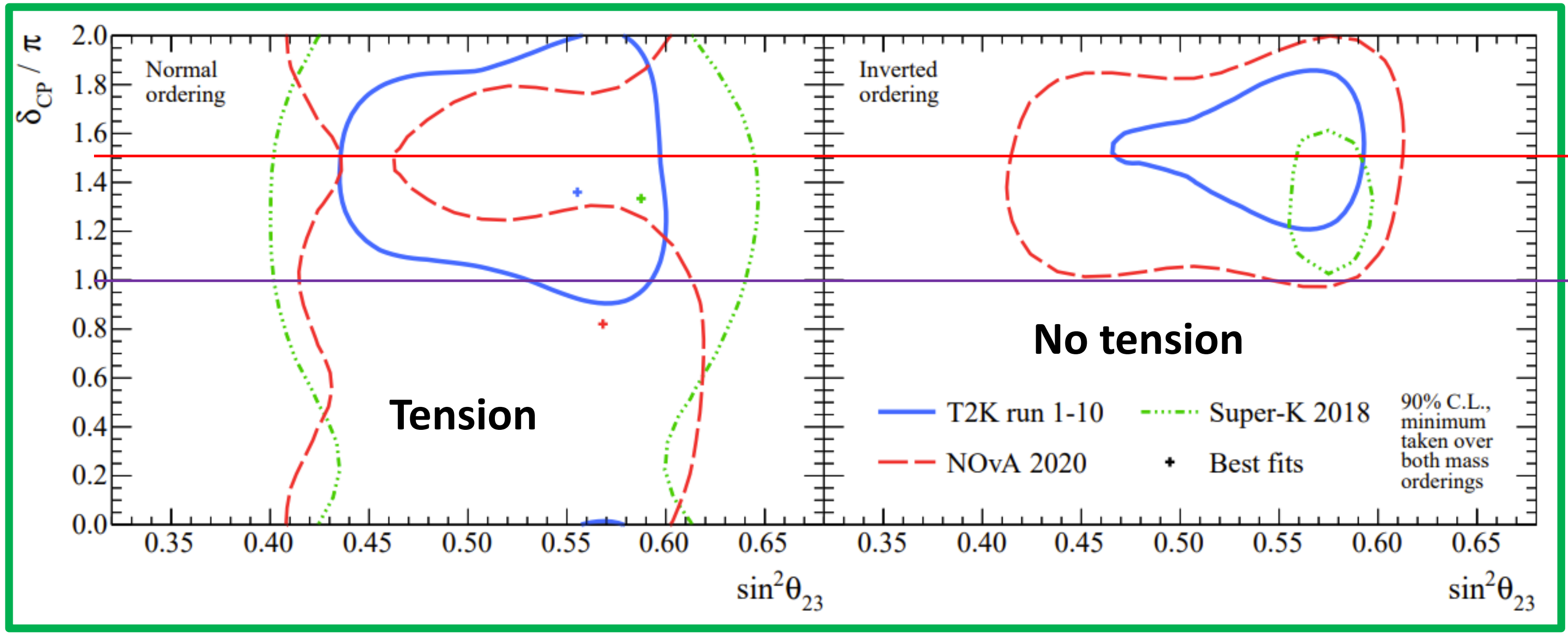
Results and Comparison with other experiments

Leading constraints on $\Delta m^2_{32}, \sin^2\theta_{23}, \delta_{CP}$

- **General agreement**



CP Phase



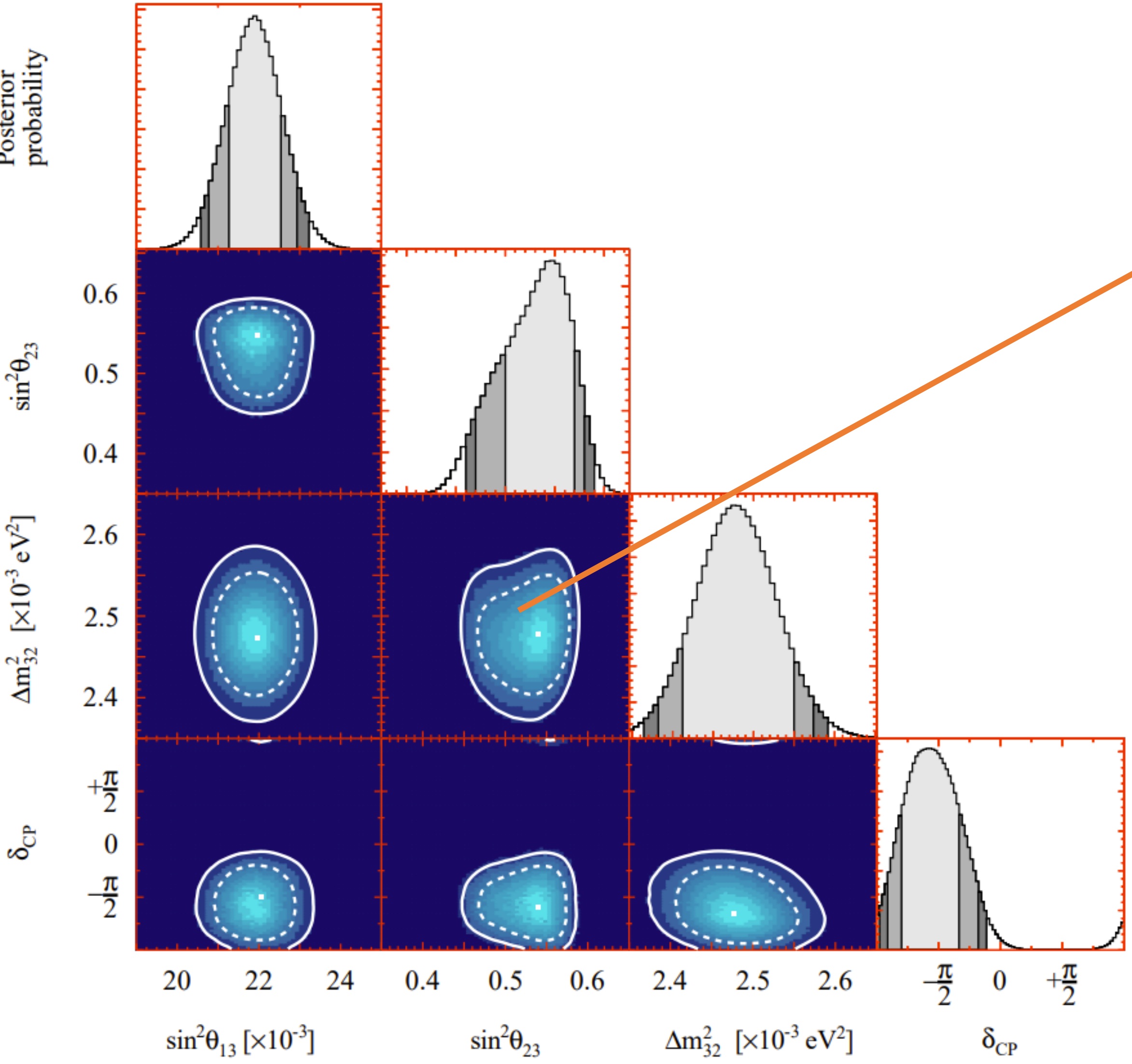
Maximal violation $\left(k + \frac{1}{2}\right) \cdot \pi$

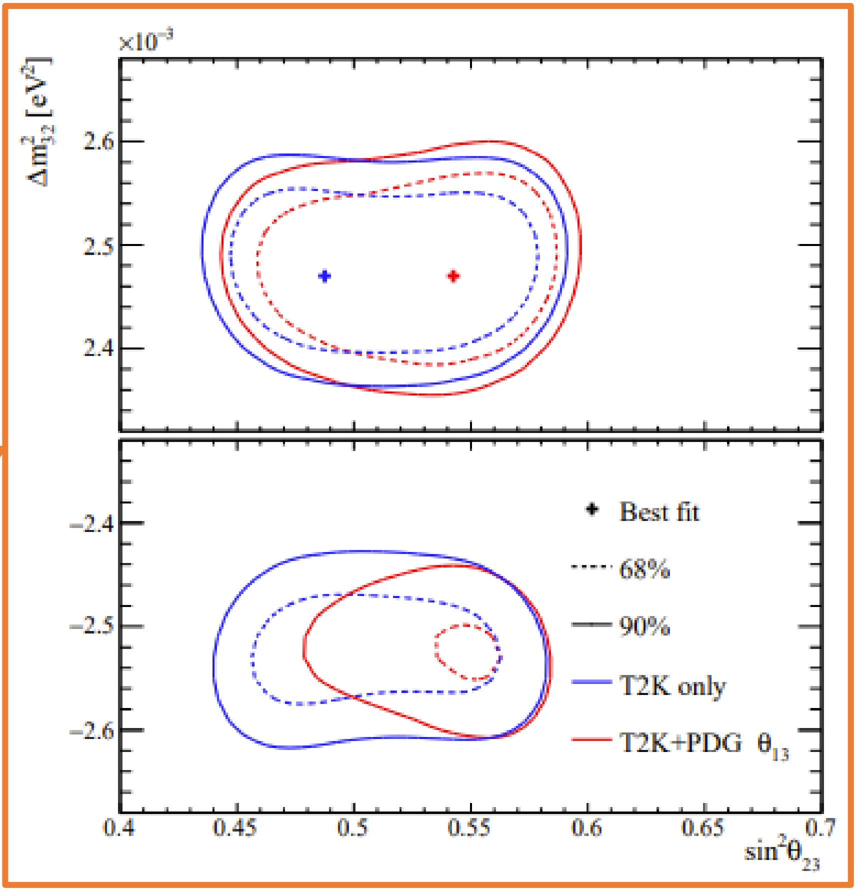
Conservation $k \cdot \pi$

- **Tension with NOvA results**

Bayesian Results

Posterior probability obtained with marginalization





Atmospheric

Frequentist Results

Results are obtained by using the marginal likelihood

$$L_{marg}(\theta) = \int d\eta \, p(\eta, \theta)$$

Binned Poisson likelihood for
- Parameters of interest θ
- Nuisance parameters η

- **Confidence Interval**

Feldman-Cousins (FC) method

$$R = \frac{L(\boldsymbol{y}|\boldsymbol{\theta})}{L(\boldsymbol{y}|\boldsymbol{\theta}_{best})}$$

Atmospheric