Coupled atoms in a cavity: some propositions for solid-state quantum battery

A non-exhaustive review

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Quantum battery: concept and advantage

Q. technologies call for high-speed energy transfers, while increasing miniaturization of devices call for quantum rather than classical models.

QM systems to store and release energy on demand, in a well-controlled way.

Search for quantum advantage: speedup and increased work extraction.

QB: ergotropy and optimal work extraction

[Alicki and Fannes, 2013]

Battery described by an internal non-degenerate Hamiltonian:

$$H = \sum_{j=1}^{d} \epsilon_j |j\rangle\langle j|, \text{ with } \epsilon_{j+1} > \epsilon_j$$

An generic (initial) state writes $\rho = \sum_j r_j |j\rangle\langle j|$

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Work extraction associated to a given unitary evolution:

$$W = Tr(\rho H) - Tr(U\rho U^\dagger H)$$

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Ergotropy :

$$W_{max} = \max_U W$$

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A state σ is *passive* if no work can be extracted, i.e. :

$$\forall U,\ Tr(\sigma H) \leq Tr(U\sigma U^\dagger H)$$

$$\longrightarrow \sigma = \sum_{j=1}^{d} p_j |j\rangle\langle j|, \text{ with } p_{j+1} \leq p_j$$

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This optimal passive state is obtained by rearranging the eigenvalues of ρ.

QB: ergotropy and optimal work extraction

[Alicki and Fannes, 2013]

$$\forall \rho, \exists! \ \beta \text{ such that } S(\rho) = S(\omega_\beta), \text{ with } \omega_\beta = \frac{e^{-\beta H}}{Z}$$

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Yet by definition, canonical Gibbs states minimize the free energy, hence:

$$Tr(\rho H) - \beta^{-1} S(\rho) \geq Tr(\sigma_\rho H) - \beta^{-1} S(\sigma_\rho) \geq Tr(\omega_\beta H) - \beta^{-1} S(\omega_\beta)$$

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Keeping in mind that the 3 states have same entropy, we end up with:

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Consider now the initial product state $\rho^{\otimes n}$, with Hamiltonian $H^{(n)} = \sum_{k=1}^{n} H_j$

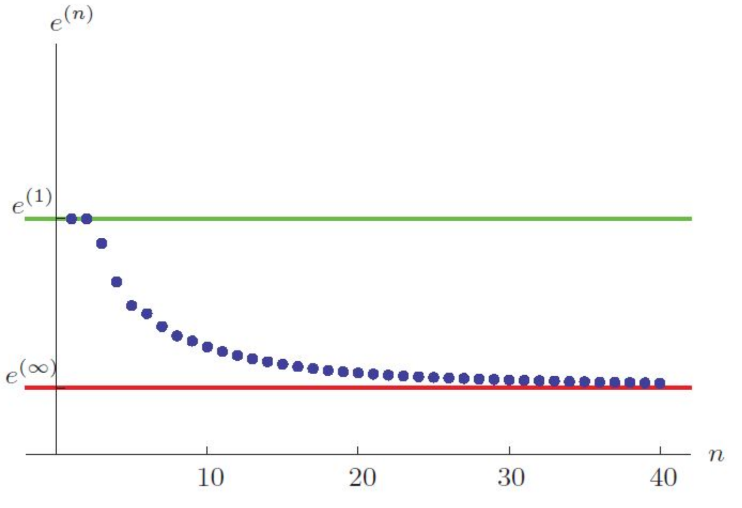

FIG. 1. (Color online) Energy per copy of passive state $\sigma_{\otimes^n \rho}$ associated with $\otimes^n \rho$.

Figure from [Alicki and Fannes, 2013]

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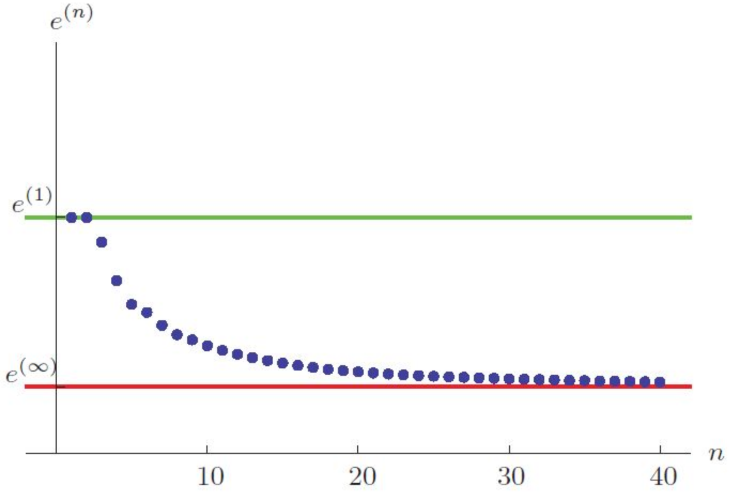

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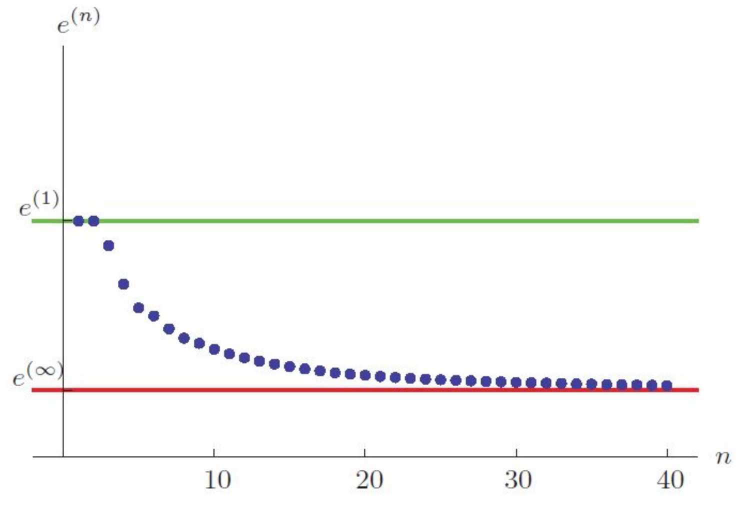

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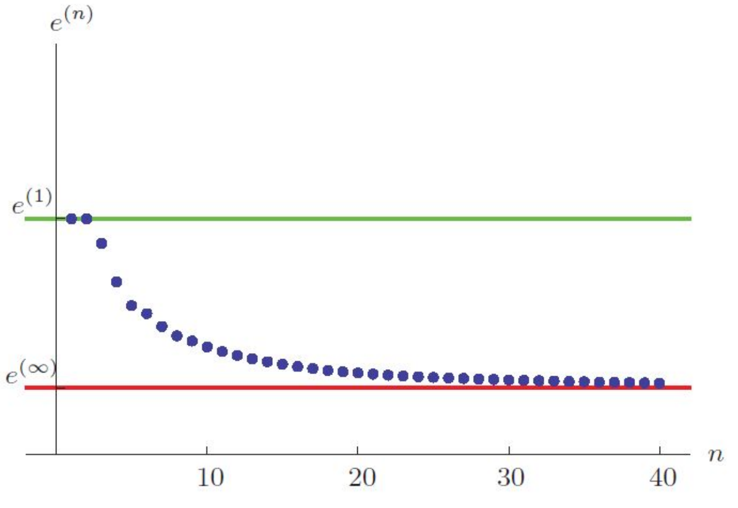

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But in general the product of *n* passive states is not itself passive.

$$\text{If } \sigma_{\rho^{\otimes n}} \neq \sigma_\rho^{\otimes n}, \text{ then } U_{\rho^{\otimes n}} \neq U_\rho^{\otimes n}$$

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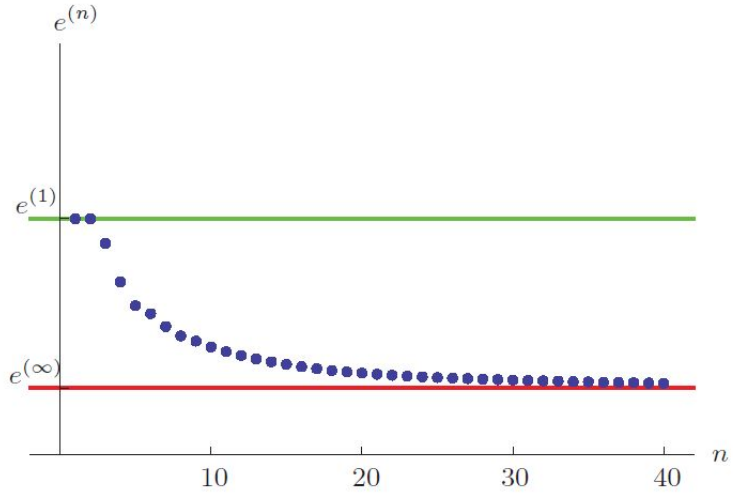

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Optimal work extraction needs *entangling* (i.e. interacting) unitary evolution operators.

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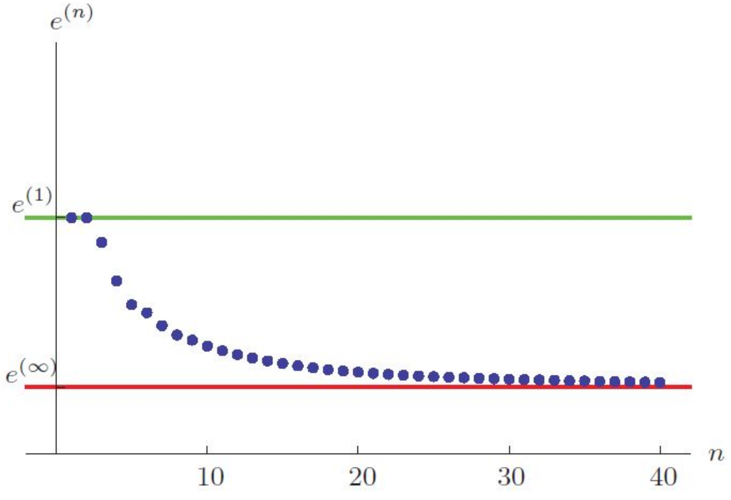

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The bound on the ergotropy is asymptotically reachable:

$$\lim_{n\to\infty} \frac{1}{n}[Tr(\rho^{\otimes n} H^{(n)}) - Tr(\sigma_{\rho^{\otimes n}} H^{(n)})]$$

$$= Tr(\rho H) - Tr(\omega_\beta H)$$

QB: a simplified proof of the quantum advantage

[Binder et al, 2015]

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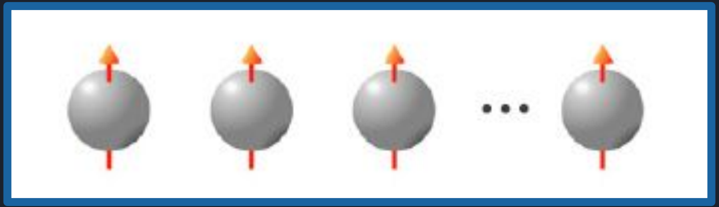

Array of N 2-level-systems (qubits):

$$H_q = -\epsilon\sigma_z$$
$$= \epsilon(|1\rangle\langle 1| - |0\rangle\langle 0|)$$

⇒

Low energy state $|0\rangle$

High energy state $|1\rangle$

Charging process: $|0\rangle^{\otimes N} \longrightarrow |1\rangle^{\otimes N}$

QB: a simplified proof of the quantum advantage

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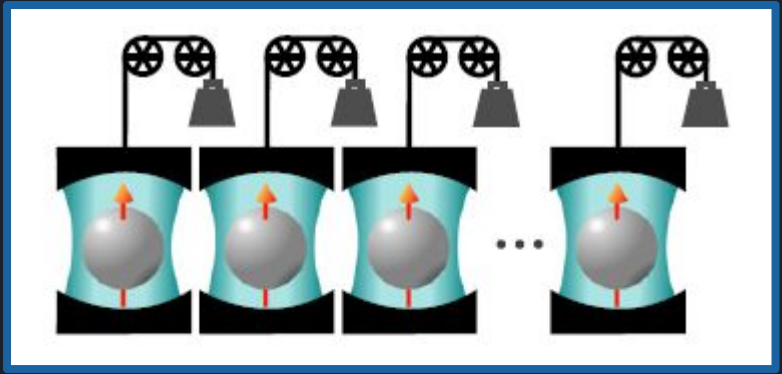
Parallel process: external local potential

$$V_{\parallel} = \sum_{\mu=1}^{N} V_q = v_0 \sum_{\mu=1}^{N} \sigma_x^{\mu}$$



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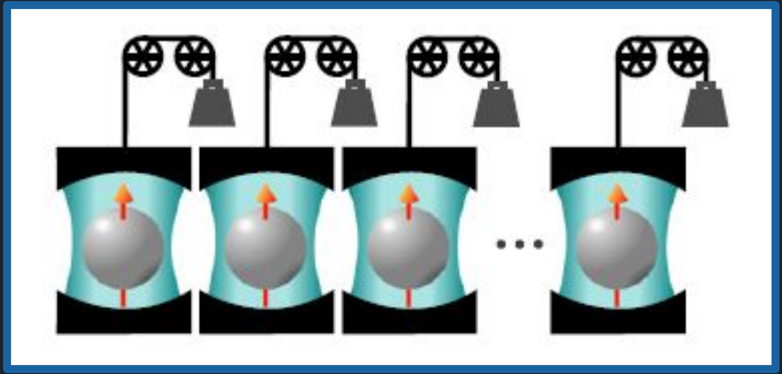

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Parallel process: external local potential

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$$U_{\parallel} \propto \prod_{\mu=1}^{N} \exp\{-itv_0\sigma_x^{\mu}\} \longrightarrow \tau_{\parallel} = \frac{\pi}{2v_0}$$

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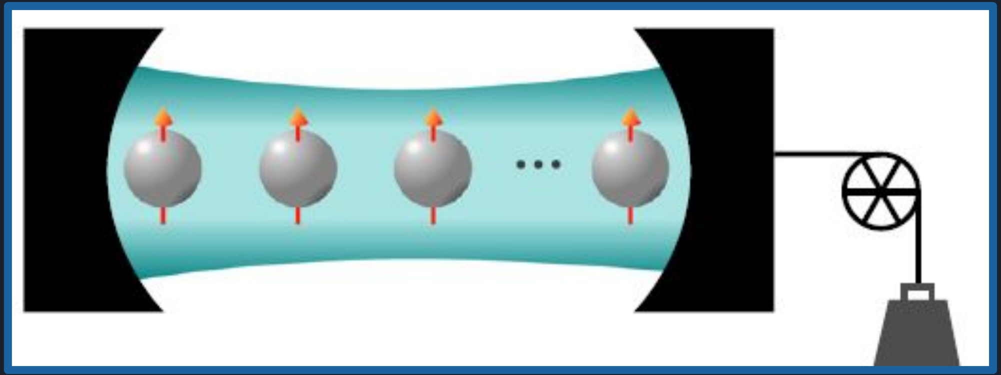
<u>Global process:</u> *entangling* non-local potential

$$V_{\#} = \alpha_{\#} V_q^{\otimes N} = N v_0 \bigotimes_{\mu=1}^{N} \sigma_x^{\mu}$$

with $\alpha_{\#}$ such that $||V_{\#}|| = ||V_{\parallel}||$



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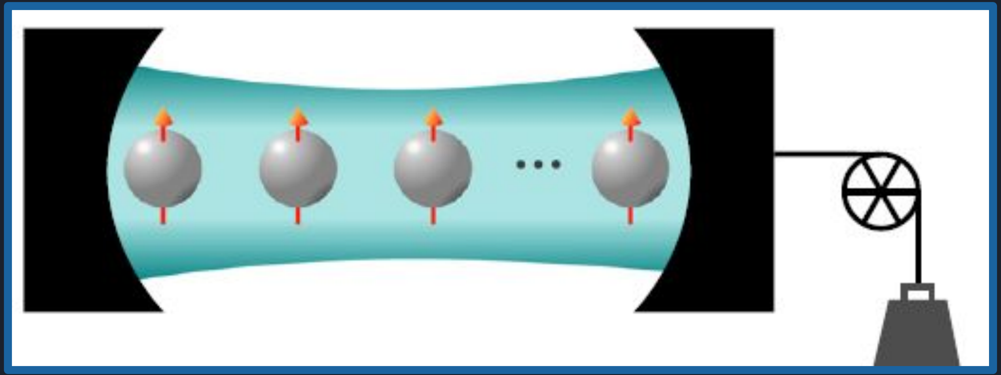

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QB: a simplified proof of the quantum advantage

A few remarks:

In general, the charged and discharged states are mixed, and rather called the *active* and *passive* states.

N-qubit (i.e. highly non-local) interactions are necessary for ideal QA. Hard to engineer.

QA requires non-local operations *but not necessarily entanglement*: QA is possible even if all the intermediary states are separable.

QB: orbits of separable states

[Campaioli et al, 2017]

Considering the same internal Hamiltonian $H_q = -\epsilon\sigma_z$

Passive state

$$\omega_\beta = \frac{e^{-\beta H}}{Z}$$

$U(T)$ $U(T)$

Active state

$$\rho = \frac{e^{\beta H}}{Z}$$

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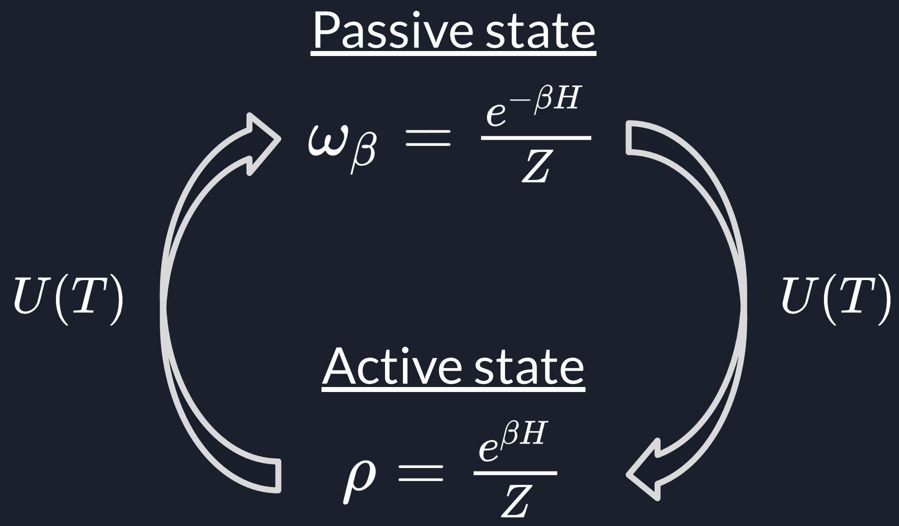

This is achieved with the same local or global external potentials, yielding the same speedup N for the former.

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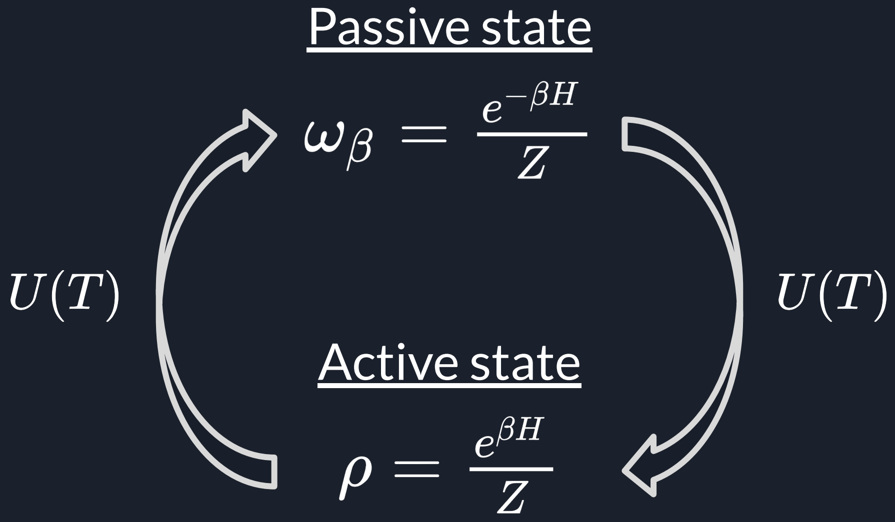

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Unitary operations don't affect the purity of states.

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Yet there is a minimal purity below which all states are separable.

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Yet there is a minimal purity below which all states are separable.

Hence for β small enough, the orbit do not leave a separable region.

QB: tighter bounds on the quantum advantage

[Campaioli et al, 2017]

A superextensive power QA might be in practice unachievable, because of the difficulty of engineering *N*-fold interaction.

In [Campaioli et al, 2017], a tighter bound for the power QA was found:

$$\Gamma = \frac{P_{\#}}{P_{\|}} < \gamma[k^2(m-1)+k]$$

with *k*<*N* the interaction order and *m* the participation number.

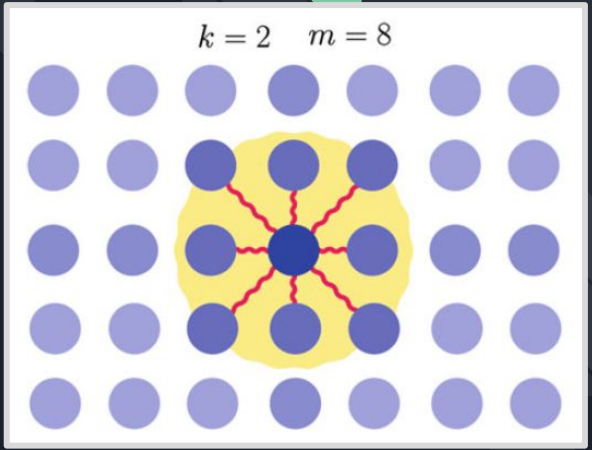


Figure from [Campaioli et al, 2018]

QB: the Dicke model as a promising candidate

Generate entanglement amongst the qubits, without the need of a qubit interaction of order N.

Phenomenon of *Dicke superradiance*, where the spontaneous relaxation of the qubits by light emission occurs in a pulse of duration $\propto N$.

Well-known model with numerous possible realization (SC qubits coupled with SC line resonators, quantum dots with SC microwave circuits, photonic crystals, ...)

QB: the Dicke model as a promising candidate

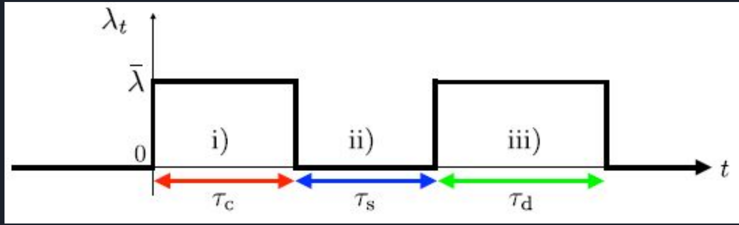
[Ferraro et al, 2018]

N photons coupled with N qubits: $|\Psi_N\rangle = \underbrace{|N\rangle}_{\text{Photons}} \otimes \underbrace{\left(|0\rangle^{\otimes N}\right)}_{\text{Qubits}}$

$$J_\alpha \propto \sum_{\mu=1}^{N} \sigma_\alpha^\mu$$

Dicke Hamiltonian:

$$H_D = \underbrace{\omega a^\dagger a}_{\text{Field}} + \underbrace{\omega J_z}_{\text{Qubits}} + \underbrace{2\omega_c \lambda_t J_x (a + a^\dagger)}_{\text{Interaction}}$$



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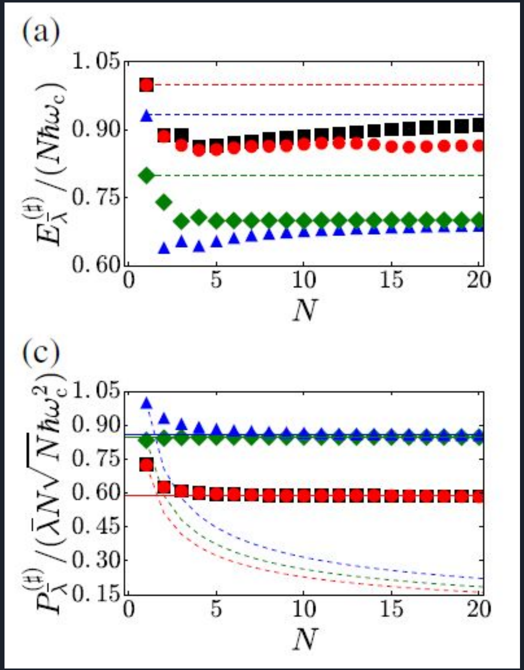

Figure from [Ferraro et al, 2018].

Black squares: λ=0.05 (TC)
Red circles: λ=0.05
Blue triangles: λ=0.5
Green diamonds: λ=2.0

Dashed lines: parallel-charging.

$$P_{\#} \propto N^{3/2} \longrightarrow \Gamma \propto \sqrt{N}$$

QB: the Dicke model as a promising candidate

Already an experiment have been performed that tends to confirm the superextensive scaling of the power [Quach et al, 2022]. It consists of a microcavity coupled with an organic SC (LFO *molecular dye*).

QB: ergotropy and stability in micromasers

[Shaghaghi et al, 2023]

Micromaser is described by the Rabi model (Dicke with N=1)

- Single-mode EM field cavity as a unique battery.
- Charged through sequential collisions with nonequilibrium qubits.

QA was found in the absence of collective effect, due to interference effects inside the battery (similar to quantum random walk). Such QA is not extensive.

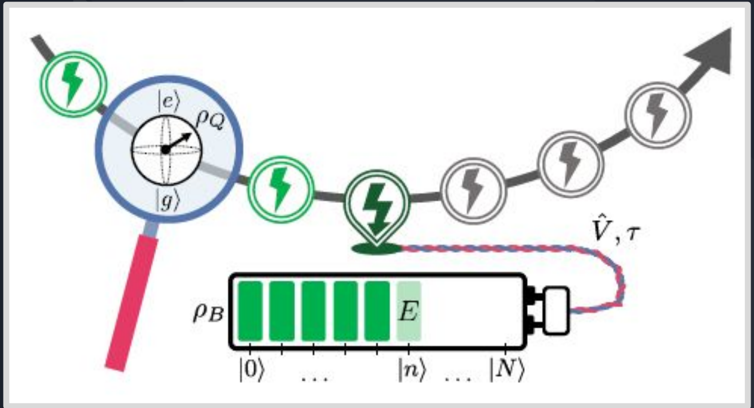

Figure from [Seah et al, 2021].

QB: ergotropy and stability in micromasers

[Shaghaghi et al, 2023]

Rabi Hamiltonian:

$$H_D = \omega a^\dagger a + \frac{1}{2}\omega\sigma_z + g\sigma_x(a + a^\dagger)$$
$$= \omega(a^\dagger a + \frac{1}{2}\sigma_z) + g(\sigma_+ + \sigma_-)(a + a^\dagger)$$

Frequency modulation of the qubit and field pulsations allow to retrieve an effective Jaynes-Cumming evolution operator:

$$U_I = \exp(g\tau(a\sigma_+ + a^\dagger\sigma_-))$$

State of incoming qubits:

$$\rho_q = q|g\rangle\langle g| + (1-q)|e\rangle\langle e| + c\sqrt{q(1+q)}(|g\rangle\langle e| + |e\rangle\langle g|)$$

c is the degree of coherence.

QB: ergotropy and stability in micromasers

[Shaghaghi et al, 2023]

c=0, lossless cavity.

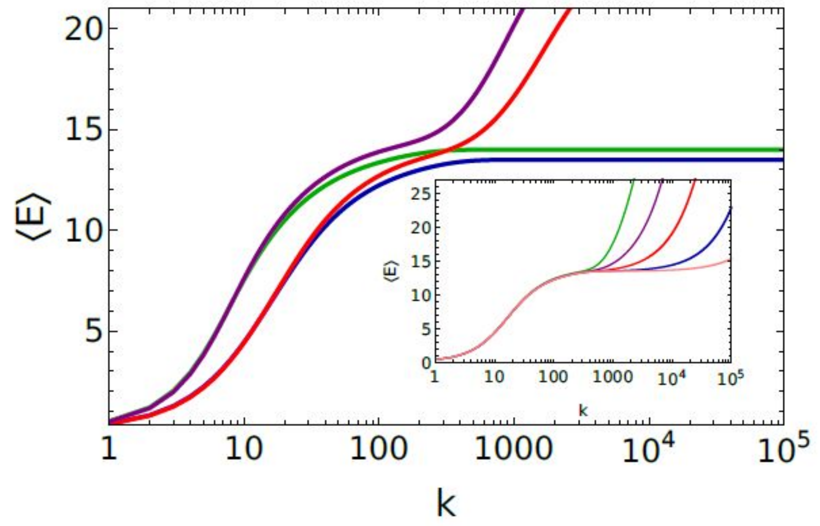

Figure 1. Energy of the battery vs. the number of battery–qubit collisions, for $q = 0.25$ and $\theta = \frac{\pi}{\sqrt{15}}$ (blue line), $q = 0.25$ and $\theta = \frac{\pi}{\sqrt{15.6}}$ (red line), $q = 0$ and $\theta = \frac{\pi}{\sqrt{15}}$ (green line:), $q = 0$ and $\theta = \frac{\pi}{\sqrt{15.6}}$ (purple line). Inset: Energy of the battery vs. the number of battery–qubit collisions, for $q = 0.25$ and $\theta = \frac{\pi}{\sqrt{15.01}}$ (pink line), $\theta = \frac{\pi}{\sqrt{15.02}}$ (blue line), $\theta = \frac{\pi}{\sqrt{15.05}}$ (red line), $\theta = \frac{\pi}{\sqrt{15.1}}$ (purple line), $\theta = \frac{\pi}{\sqrt{15.2}}$ (green line).

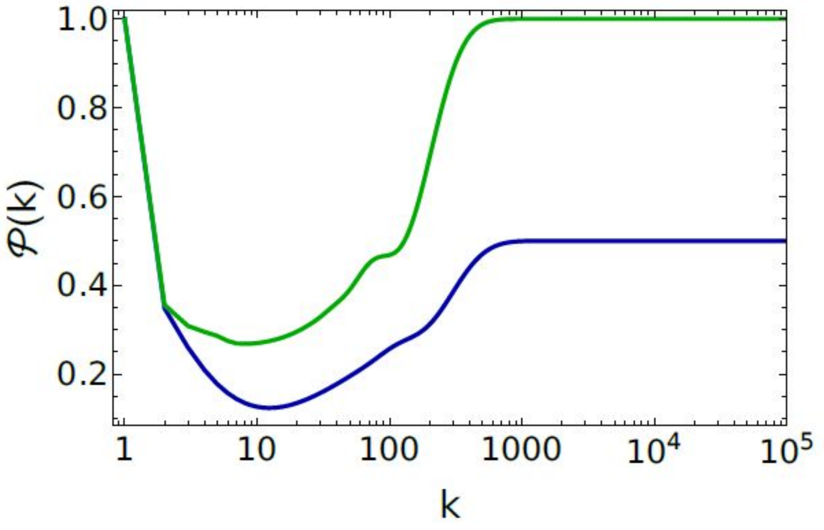

Figure 2. Purity of the battery vs. the number of battery–qubit collisions, for the fine-tuned value $\theta = \frac{\pi}{\sqrt{15}}$, $q = 0.25$ (blue line) and $q = 0$ (green line).

Figures from [Shaghaghi et al, 2023].

QB: ergotropy and stability in micromasers

[Shaghaghi et al, 2023]

<u>c=1, lossless cavity.</u>

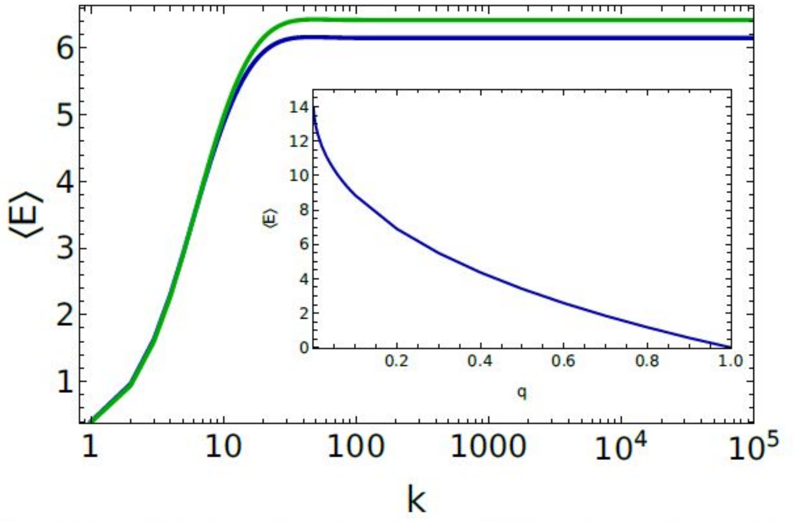

Figure 3. Energy of the battery vs. the number of battery–qubit collisions, for $q = 0.25$, $\theta = \frac{\pi}{\sqrt{15}}$ (blue line) and $\theta = \frac{\pi}{\sqrt{15.6}}$ (green line). Inset: the steady-state energy for the case $\theta = \frac{\pi}{\sqrt{15}}$ as a function of the parameter q, as computed by means of Equation (15).

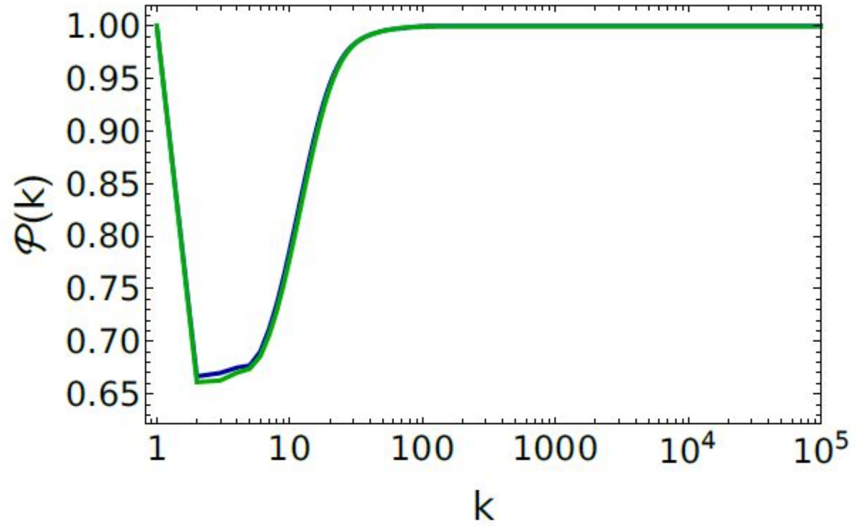

Figure 4. Purity of the battery vs. the number of battery–qubit collisions, for the same parameter values as in Figure 3.

Figures from [Shaghaghi et al, 2023].

QB: ergotropy and stability in micromasers

[Shaghaghi et al, 2023]

<u>c=1, lossy cavity.</u>

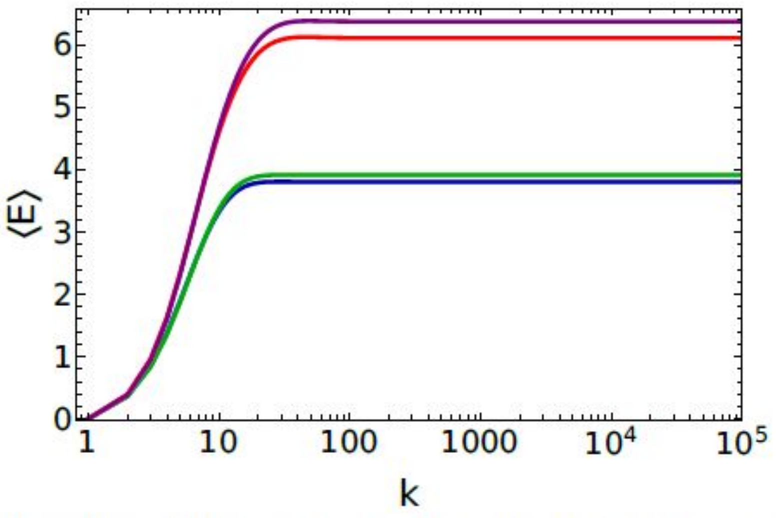

Figure 5. Energy of the battery vs. the number of battery–qubit collisions, in the presence of dissipation, at $q = 0.25$ and $\bar{n} = 0.15$, $\theta = \frac{\pi}{\sqrt{15}}$ and $\gamma t_r = 0.1$ (blue line), $\theta = \frac{\pi}{\sqrt{15.6}}$ and $\gamma t_r = 0.1$ (green line), $\theta = \frac{\pi}{\sqrt{15}}$ and $\gamma t_r = 0.001$ (red line), $\theta = \frac{\pi}{\sqrt{15.6}}$ and $\gamma t_r = 0.001$ (purple line).

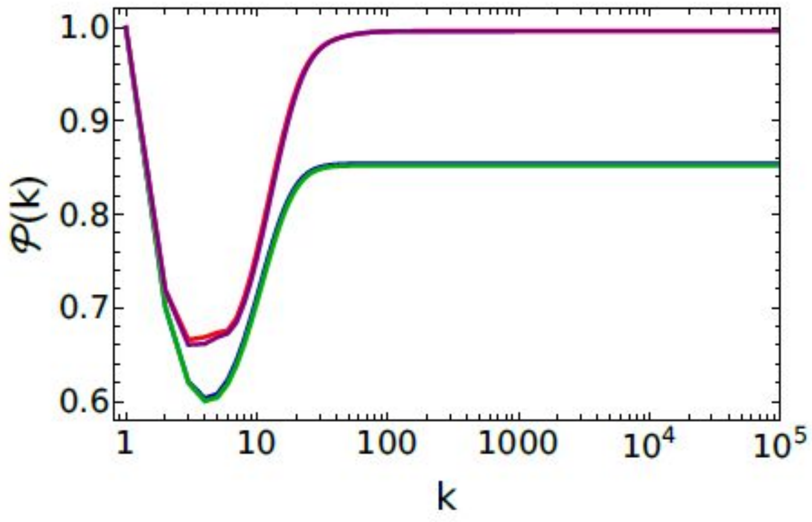

Figure 6. Purity of the battery vs. the number of battery–qubit collisions, for the same parameter values as in Figure 5. Notice the almost perfect overlap of curves in pairs.

Figures from [Shaghaghi et al, 2023].

QB: ergotropy and stability in micromasers

As essentially pure steady states are reached, great ergotropy: most of the energy can in principle be extracted.

Possible experimental realizations include SC quantum circuits, an existing technology in quantum computers.

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Thanks for your attention.